A Appendix: Formal Description of Market Equilibrium

A.1 Set up

As we explained in the main text, we assume the constant population of workers. All the participants in the labor market are risk-neutral and use the common discount rate r.

We consider two types of jobs, type c and type s.The productivity of both jobs depend on the state of the *industry.* There is a continuum of the *industry* and the state of each *industry* evolves according to the following simple stochastic process. There are two possible states, 'g' (good) and 'd' (depressed). The collection of the *industries* in each state is called sector. Thus g-[d-] sector collects all the g-*[d-] industries.* Each *industry* in state g turns to the state d with probability ρ, representing the permanent productivity shock. In order to prevent all the existing *industries* becoming d, we assume the continuum of the *industry* expands over time so that there always exists strictly positive mass of the industries in state g. Let q^c [q^s] denote the output per unit of time of a type c[s] job in g-sector filled by a properly trained worker. As we explained in the main text, each worker is endowed with innate aptitudes for type c jobs and we represent by z the worker continuum in terms of the training cost needed. The value z represents the cost of the first training. We assume that such a cost is payable in once and each training is complete once the cost is incurred. We denote by F the distribution of z.

The basic building blocks of the model are the firms' job offers and workers' application strategies. The job offer must stipulate employment and compensation in all payoff relevant contingencies: i.e., required type of the worker, the type of job slot, the *industry*, and wage schedule which can be contingent on the state of *industry.* Such a job offer maximizes the profit given the application strategies of each type of unemployed workers. Workers are distinguished by training cost z and the *last* training she has taken. As we will see shortly, no worker receives training for the first time at a type c job which is *already* in the state d. Still, an unemployed worker could have received training at a job which is *currently* in g-sector, or, in d-sector, or, never been trained before. Although a worker can search for a job in any *industry,* if the job where he received the training is currently in g sector, he has no reason to search in a different *industry* because all the *industries* in the g sectors are identical because of the imposed symmetry of the industries. A worker may move out, however, from the *industry* where he received training if the *industry* has moved to the d-sector and job productivity is lower. Therefore a sub market in d - sector is populated only by those who received the training in the same *industry,* whereas sub markets for jobs in g sector can be populated by workers with training in g as well as d.

Let $e = g, d$, denote the current state of the *industry* where he received training and denote by $e = n$ for the untrained. Let $T \equiv \{g, d, n\} \times Z$ denote the set of types of unemployed worker. Although a worker can be trained more than once, what matters is the most recent training[31]. Thus the notation of the type of training refers to the most recent one. Note that the training type of the worker can change over time for two reasons. First the state of the *industry* where he received training can change. Second, once he is employed, he receives re-training.

The job offer can be identified by the pair of wage schedule $w = (w_g, w_d)$ and contract types, wherein w_g [w_d]denotes the wage offer contingent upon the *industry* being in the state g [d]. For each cohort of workers with given training cost index

---

[31] As no one choose to go back to the *industry* now in the state $d,$ all the past trainings before the last must have been in the *industries* which are now all in d sector.

z, the following types of contract types are possible. Let us start with contract types in type c jobs. First, the offer can be contingent upon the state of the *industry.* By contingent contract, we mean that the employment is terminated once the industry is hit by the shock. For simplicity, we also assume that in this type of contract the job slot itself is self-destroyed when hit by the productivity shock[32]. Since we only consider a permanent shock, the state d is permanent. A job in g-sector can also offer an unconditional employment contract, except for the exogenous separations. Namely, an unconditional contract guarantees employment after the shock (but a worker can of course walk away any time they want). Since the state d is permanent, all the employment offer is unconditional guarantee. Hence we need to consider contingent employment offer only for jobs currently in g-sector. By the same token, non-trivial *contingent wage schedule* applies only to one type, i.e., the one that promises unconditional employment.

For untrained workers, no offers from jobs in d sector can compete with those in g sector because, for the untrained workers, the only difference between the two is lower productivity in $d.$ Thus all the untrained workers will choose *industries in g* sector. We impose symmetry on the selection among the totally homogenous industries in the g sector so that each *industry* is populated with the same density of the untrained workers.

We now consider contract types for type s jobs. It is easy to see that there can be only two types of contracts, conditional and unconditional employment in g-sector because unfilled job slots are immediately discarded as soon as the industry is hit by the shock. This is the case because type s jobs do not need any training. Thus there is no advantage of retaining the vacancies in d-sector. Consequently there is no active job slot that will post an offer. Again we assume each industry in g sector has the same size of the workers searching for a job[33].

Consequently we need to consider the following five types of contracts: unconditional type c in the g-sector($\bar{g}$), conditional type c in the g-sector ($\hat{g}$), (unconditional) type c in the d-sector (d), unconditional type s ($\bar{s}$), and conditional type s ($\hat{s}$). While unconditional employment contract is never terminated by the employer, conditional employment contract is terminated when the sector is hit by a permanent productivity shock without any compensations.

Since the firm can make an offer contingent on the type of worker, we can define the type contingent employment contract by the pair $\{\mathcal{C}, \mathcal{W}\}$, where $\mathcal{C} : T \to C$ and $\mathcal{W} : T \to \mathbb{R}_+^2$ are mapping from worker type to offer types and wage schedule, respectively. $C \equiv \{\bar{g}, \hat{g}, d, \bar{s}, \hat{s}\}$ is the set of type of contract. Thus each sub-market is fully specified by the job offer. In each sub market, all the job offers are of the same type, and all the workers are of the same type. Denote by x the ratio of job searchers to the vacancy, which we call queue length. Then the probability that a vacancy receives at least one application is given by $\psi(x)$, which is increasing and strictly concave in x.

Now that the description of each sub market is complete, an allocation of the economy can be fully specified by a tuple $\{\mathcal{C}, \mathcal{W}, x\}$ where $x : T \to \mathbb{R}_+$ is the queue length in each sub market. Given the allocation $\{\mathcal{C}, \mathcal{W}, x\}$, we can compute the expected present value of profit stream for a job slot and the corresponding value of

---

[32] This simplifying assumption is used only to avoid further crowding of notations. As we impose the zero profit condition, all the existing vacancy at equilibrium will have zero value anyway.

[33] Notice that because of the free access to the underlying technology, single industry can accommodate any size of job vacancies and workers. Although uneven distribution across *industries* are immaterial as far as they remain in g sector, the impact of the technology shock on the aggregate labor market obviously will depend upon the size of the industry hit by the shock. We avoid this unnecessary complications by imposing symmetry.

the expected income stream for a worker. Let $V_c(w,x)$ be such value for a vacancy that post the offer (c,w), where $c \in C$ is type of offer. Then we have

$$rV_c(w,x) = -p(c) + \psi(x)[J_c(w) - V_c(w,x)]. \tag{A1}$$

where $p(c) = p_c$ if $c = \bar{g}, \hat{g}, d$ and $p(c) = p_s$ otherwise. Viz, the cost of maintaining a job slot is $p_c\,[p_s]$ for type $c\,[s]$ jobs. The value of filled job depends upon the state contingent wage schedule $w = (w_g, w_d)$ and the type of contract c and given by

$$\begin{aligned}
rJ_d(w) &= \theta q^c - w_d - p_c - (\delta_c + d)J_d(w),\\
rJ_{\bar{g}}(w) &= q^c - w_g - p_c - (\delta_c + d)J_{\bar{g}}(w) + \rho[J_d(w) - J_{\bar{g}}(w)],\\
rJ_{\hat{g}}(w) &= q^c - w_g - p_c - (\delta_c + d + \rho)J_{\hat{g}}(w),\\
rJ_{\bar{s}}(w) &= q^s - w_g - p_s - (\delta_s + d)J_{\bar{s}}(w) + \rho[J^s_d(w) - J_{\bar{s}}(w)],\\
rJ_{\hat{s}}(w) &= q^s - w_g - p_s - (\delta_s + d + \rho)J_{\hat{s}}(w),
\end{aligned} \tag{A2}$$

where $J^s_d(w)$ is given by

$$rJ^s_d(w) = \theta q^s - w_d - p_s - (\delta_s + d)J^s_d(w)$$

In words, the first is the value function for an active contract in state d. The first three terms sum up to the net flow profit, and the last term corresponds to the capital loss upon worker retirement or exogenous separations. The second and the third are the value function of type c jobs for contracts in g state. The last two equations are for type s jobs in the state g. Note that a permanent shock occurs only to jobs in g industries. The unconditional employment contract continues even after the sector is hit by permanent shock but it incurs the capital loss due to the change of state. On the other hand, the conditional employment contract terminates and the job slot is destroyed if the sector is hit by permanent shock. Thus the value of job turns to be zero.

Next, we consider the value of unemployed worker given the allocation and the equilibrium value of each type of unemployed worker. The rational unemployed workers choose their application strategies taking their future change of training into account. Here, we incorporate the equilibrium value into the value function instead of considering the future decision directly. Let $U_c(t,w,x,U^*)$ be the value of type t unemployed worker given the wage schedule, contract type, queue length, and equilibrium value of unemployed worker, which we denote by U^*. In this formulation, we treat as given the value of different types of unemployed workers, even though in the future he may become one of those types. We thus focus on the optimal choice of the current application strategy. We have

$$\begin{aligned}
rU_c(t,w,x,U^*) =& \phi(x)[E_c(z,w,U^*) - U_c(t,w,x,U^*) - \kappa(c,t)]\\
&- dU_c(t,w,x,U^*) + I(e)\rho[U^*(d,z) - U_c(t,w,x,U^*)],
\end{aligned} \tag{A3}$$

where $\phi(x) \equiv \frac{\psi(x)}{x}$ is the probability of being matched for a worker, and $I(\cdot)$ is indicator function that takes value one if $e = g$ and zero otherwise. The last term in the square bracket is the capital loss associated with productivity shock which is applicable only for a worker trained in g-sector.

The training cost $\kappa(c,t)$ is given by[34]

$$\kappa(c,t) = \begin{cases} 0 & \text{if} \quad c = \bar{s}, \hat{s} \\ z & \text{if} \quad e = n \text{ and } c \neq \bar{s}, \hat{s} \\ \epsilon z & \text{if} \quad (e,c) = (d,d), (g,\bar{g}), (g,\hat{g}) \\ (\epsilon + m)z & \text{otherwise} \end{cases} \tag{A4}$$

In words, the first case applies to type s jobs irrespective of contract types as no training is required for this type of job. If a worker is never trained and he applies for a type c job, full training cost must be incurred, irrespective of the contract type. In the third line, we show the following: a worker have to pay ϵz if he decides to search and apply for a type c job in the same industry where he received the last training. Finally the same worker has to incur $(\epsilon + m)z$ if he decides to move out of the current industry and being matched to a type c job in the g-sector.

The value of employed worker depends on the training cost index z and wage. They are represented by the following value functions for employment.

$$\begin{aligned} rE_d(z,w,U^*) =& w_d - \delta_c[E_d(z,w,U^*) - U^*(d,z)] - dE_d(z,w,U^*), \\ rE_{\bar{g}}(z,w,U^*) =& w_g - \delta_c[E_{\bar{g}}(z,w,U^*) - U^*(g,z)] - dE_{\bar{g}}(z,w,U^*) \\ & + \rho\left[E_d(z,w,U^*) - E_{\bar{g}}(z,w,U^*)\right], \\ rE_{\hat{g}}(z,w,U^*) =& w_g - \delta_c[E_{\hat{g}}(z,w,U^*) - U^*(g,z)] - dE_{\hat{g}}(z,w,U^*) \\ & + \rho\left[U^*(d,z) - E_{\hat{g}}(z,w,U^*)\right], \\ rE_{\bar{s}}(z,w,U^*) =& w_g - \delta_s[E_{\bar{s}}(z,w,U^*) - U^*(n,z)] - dE_{\bar{s}}(z,w,U^*) \\ & + \rho\left[E^s_d(z,w,U^*) - E_{\bar{s}}(z,w,U^*)\right], \\ rE_{\hat{s}}(z,w,U^*) =& w_g - \delta_s[E_{\hat{s}}(z,w,U^*) - U^*(n,z)] - dE_{\hat{s}}(z,w,U^*) \\ & + \rho\left[U^*(n,z) - E_{\hat{s}}(z,w,U^*)\right], \end{aligned} \tag{A5}$$

where

$$rE^s_d(z,w,U^*) = w_d - \delta_s[E^s_d(z,w,U^*) - U^*(n,z)] - dE^s_d(z,w,U^*)$$

This completes the specifications of all the value functions for potentially active contract and job types.

A.2 Market equilibrium

We now complete the specification of the model by imposing the zero profit condition for all the active vacancies. Note by the complementary slackness, expected net value of inactive vacancies must be non-positive.

We are ready to define the market equilibrium.

Definition 1. *The market equilibrium is defined by the set* $\{\mathcal{C}^*,\mathcal{W}^*,x^*,U^*,V^*\}$ *that satisfies the following conditions.*

1. *For any type $t \in T$, firms post vacancies so as to maximize their values under the constraint that the offer must guarantee the equilibrium value $U^*(t)$,*

---

[34] Who actually pays the training cost is immaterial. If you so wish, we could add another dimension in the contract type, depending upon who pays the training cost. Since training cost is independent from the state (although its consequence does depend upon the state in the future), contracts stipulations on who pays (how much) for the training is redundant. To put it differently, if a firm offers a contract in which they pay the training cost, the equilibrium value of the offer will be the same as the current one in that wage schedule will be adjusted accordingly. Needless to say, who pays the training cost *does* matter at least potentially, if we allow incompleteness of the contract. See section 5 of this appendix below.

i.e., given that queue lengths are determined so as to be consistent with the equilibrium value of unemployment $U^*(t)$. Therefore, we have

$$(\mathcal{C}^*(t), \mathcal{W}^*(t), x^*(t)) \in \arg\max_{c,w,x} \; V_c(w, x^*) \equiv V^*(t)$$

subject to

$$U^*(t) \geq U_c(t, w, x, U^*)$$

and $x \geq 0$ with complementary slackness, where U^* solves

$$\forall t \in T \quad U^*(t) = U_{\mathcal{C}^*(t)}(t, \mathcal{W}^*(t), x^*(t), U^*)$$

2. *By the free entry condition, the maximized value of active vacancies, $V^*(t)$, must be equal to zero.*

In equilibrium, a job slot computes the value of deviation based on the belief that the queue length that corresponds to alternative job offer should be adjusted so as to guarantee to the unemployed workers the market determined present value of the expected income stream. In order to obtain the market equilibrium, we can solve the problem,above, or, equivalently, we can solve the dual. That is, we maximize the value of the unemployed worker conditional under the zero profit condition.

$$\begin{aligned} U^*(t) = &\max_{c,w,x} \; U_c(t, w, x, U^*) \\ &s.t. \quad V_c(w, x) = 0 \end{aligned} \tag{A6}$$

Before we move on to analyze the market equilibrium defined above, we offer verbal explanations why the market equilibrium defined above coincides with that of a social planner. Given the full array of contractual arrangements, the market equilibrium defined above evidently solves the resource allocation problem for a social planner endowed with the same search technology, technology evolutions underlying productivity shocks to job slots. In order to see through the logic behind, let us assume for the time being that both job slots and workers are identical among themselves and assume away also the productivity shocks, etc. None of these additional factors matters for this explanation. Under this simplified setting, an individual offer is simple a wage rate at matched worker receives. The competitive search equilibrium is such that the individual agents take the value of the unemployed worker, N^* given. From the viewpoint of each job slot, its own wage offer must satisfy the constraint

$$\phi(x)M(w) = N^*$$

Namely the product of the probability that an application results in an offer ,$\phi(x)$, and the value of the offer, $M(w)$,should be equal to the market determined N^*. Since

$$\phi(x) \equiv \frac{\psi(x)}{x}$$

we have

$$N^* = \frac{\psi(x)M(w)}{x},$$

namely, this is the trade off between more attractive (hence higher wage) offer and the probability that a job is filled. Since the offer competition guarantees that N^*

is the shadow price of the unemployed worker, the profit maximization condition ensures that the optimal choice of queue length, x, coincides that of a social planner. [35].

We now solve the problem (A6) in two steps. First, we calculate the value of unemployment worker given the type of employment contract, and then compare these contracts. To be specific, we derive the following value at first

$$\begin{aligned} U_c(t, U^*) \equiv & \max_{w,x} U_c(t, w, x, U^*) \\ & s.t. \quad V_c(w, x) = 0 \end{aligned}$$

then solve the functional equations

$$U^*(t) = \max_{c} U_c(t, U^*).$$

In order to derive market equilibrium, we must look for the optimal employment contract for each type of worker. The following points that we already made are helpful in deriving the desired functional. First, recall that the untrained or those trained in the g sector should apply to the job in the g-sector since they have no advantage to work in d-sector. In addition, the worker who search for type s jobs should also apply to the job in the g-sector. The crucial remaining problem is whether or not to move for those who received training in the *industry* which now belongs to d-sector. To put this question in terms of contract choice, the question is : when the unemployed worker applies to the job in the g-sector, should she choose the unconditional employment contract or not?

Next, note also that the trained workers never apply to type s job in the steady state equilibrium. This is evident from the stationarity of the optimal policy: the fact that he received trained in the past implies that it was optimal to apply for a type c job. Then, it should be optimal to do so now, as well. Therefore the third remaining question is which type of untrained worker should apply to type s job.

The following proposition gives the answer to the second. For the last question, we have to wait until section A.4.

Proposition 1. *The comparison between the value of unconditional contract and conditional contract can be implemented by the checking the following inequality.*

$$U_{\bar{g}}(t, U^*) \gtreqless U_{\hat{g}}(t, U^*) \Leftrightarrow (r + d)U^*(d, z) \lesseqgtr \theta q^c - p_c$$

The intuition behind the Proposition 1 is as follows. From (A2) and (A5), the joint surplus that is gained by a match in d-sector is given by

$$E_d(z, w, U^*) + J_d(z, w, U^*) - U^*(d, z) = \frac{\theta q^c - p_j - (r + d)U^*(d, z)}{R_j}$$

Therefore, Proposition 1 says that the unconditional employment contract is preferred to the conditional contract if and only if the joint surplus from declining job is positive. The proposition also implies that the comparison between unconditional contract and conditional contract hinges only upon innate trait z, it does not change according to career path of worker. The unemployed worker trained in the [current]

---

[35] See Moen and Rosen (2004) for a more formal proof in a similar model. The formal proofs (omitted) for our case involves straight forward but lengthy derivations of optimal policy for a social planner that solves the corresponding Hamiltonian defined upon the net social output. The solutions of course coincide with those given here for market equilibrium.

g-sector always should apply to the type c job in the g-sector. Hence the equilibrium value must satisfy

$$U^*(g,z) = \begin{cases} U_{\bar{g}}((g,z),U^*) \text{ if } (r+d)U^*(d,z) \leq \theta q^c - p_j \\ U_{\hat{g}}((g,z),U^*) \text{ if } (r+d)U^*(d,z) > \theta q^c - p_j \end{cases}$$

Therefore, the optimal application of type (g,z) unemployed worker depends on the expected income of type (d,z) unemployed worker. We solve the optimal application problem of type (d,z) worker, which is the most important decision in our model, in the following section.

A.3 The optimal policy for trained workers

In order to derive the optimal policy of type (d,z) [those trained in the *industry* which is currently in d-sector] unemployed worker, we consider the case wherein the worker always chooses the same type of contract irrespective of his past choices. The option values that correspond to these strategies can be defined recursively

$$\begin{aligned} \tilde{U}_c(t) \equiv U_c(t,\tilde{U}_c) = & \max_{w,x} \; U_c(t,w,x,\tilde{U}_c) \\ & s.t. \quad V_c(w,x) = 0 \end{aligned}$$

Here we can find the equilibrium value of type (d,z) unemployed worker by comparing these values. If the type (d,z) worker should apply to the job in the d-sector, then his re-training (costing ϵz) would not change his training type. Therefore stationarity implies that he should apply to the same type of job in the future as well. On the other hand, by Proposition 1, if it is optimal to move out and search for a new job in g sector, he should choose the same policy in the future even though re-training could change the type of worker. Therefore, in equilibrium, type (d,z) unemployed worker should apply to the same type of contract even after he receives re-training. That is, we have

$$U^*(d,z) = \max_{c\in\{d,\bar{g},\hat{g}\}} \tilde{U}_c(d,z)$$

We can show that the optimal contract for type (d,z) worker is monotone in z.

Lemma 1. *Let $x_c(t)$ be the queue length that maximizes the present value of the expected income stream of type t unemployed given the type of contract c and the subsequent value $U(t)$. Then the value can be written as the function of queue length,*

$$U_c(t,U) = \frac{\Delta(x_c(t))p(c) + I(e)\rho U(d,z)}{r+d+I(e)\rho}$$

where $\Delta(x) = \psi'(x)/(\psi(x) - \psi'(x)x)$. Moreover, if $U(t) = U_c(t,U)$, then we have $x_c(g,z) \leq x_c(d,z)$ for any c and z. That is, type (g,z) worker has larger probability to receive an offer than type (d,z) worker if he seek the same type of job.

Proposition 2. *Suppose that type (d,z^u) unemployed worker is indifferent between staying in the d-sector and re-entering the g-sector, i.e., $\tilde{U}_d(d,z^u) = \tilde{U}_g(d,z^u)$, where $\tilde{U}_g(d,z) = \max\{\tilde{U}_{\bar{g}}(d,z), \tilde{U}_{\hat{g}}(d,z)\}$. Then, we have*

$$U^*(d,z) = \begin{cases} \tilde{U}_d(d,z) \; if \quad z \geq z^u \\ \tilde{U}_g(d,z) \; if \quad z < z^u \end{cases}$$

In addition $U^*(d,z)$ is decreasing in z.

Proposition 2 can be stated in words as follows. The difference in z yields the difference in value of unemployed worker which is proportional to the additional training cost in future. Thus the impact of z is smaller if you stay at d-sector because re-training cost is proportionally larger if you move out to g-sector, i.e., the negative slope of U is steeper if you move out, than at d-sector. Hence the intersection at z^u of the two value function is unique. Consequently, only the high ability worker $(z > z^u)$ should move out to a new *industry* in the g-sector after the productivity shock, whereas the low ability worker $(z < z^u)$ should stay in the same *industry* after the shock.

We can also show that the choice between unconditional contract and conditional contract is monotone in z because $U^*(d, z)$ is decreasing in z. Let define z^e by $U^*(d, z^e) = (\theta q^c - p_c)/(r + d)$. Then those with lowest training cost $(z < z^e)$ should move every time after the shock , i.e., they choose type $\hat{g}$ job if $z^e < z^u$. (see Figure 5)

A.4 The optimal policy for untrained workers

Given the optimal choice of experienced workers, as summarized in the two thresholds, z^u and z^e, we can finally solve the optimal strategies of untrained workers. The untrained workers must incur the same amount of training cost z regardless of the state of sector as long as they apply to type c job, thus they should apply to the job in the g-sector if they apply to type c job. The choice between unconditional contract and conditional contract is determined by z^e as in the case of type (g, z) worker. That is, we have

$$U_g(n, z) \equiv \max\{U_{\bar{g}}(n, z), U_{\hat{g}}(n, z)\} = \begin{cases} U_{\bar{g}}((n, z), U^*) \text{ if } z \geq z^e \\ U_{\hat{g}}((n, z), U^*) \text{ if } z < z^e \end{cases}$$

The last remaining problem is who should apply for a type s job. Since this type of job does not require training, the choice should be unanimous for those who choose type s. Let $U_s = \max\{U_{\bar{s}}, U_{\hat{s}}\}$ denote the value of unemployed worker who searches for type s job. If $U_s > U_g(n, z)$, type z worker should apply to type s job when he enters the labor market. Let z^s be the threshold that satisfies $U_s = U(u, z^s)$. Since we can show that $U_g(n, z)$ is decreasing in z, the worker should apply to type s job if $z > z^s$.

Suppose that $z^e < z^u < z^s$. Then, we can summarize the optimal strategy of each ability of unemployed worker as follows. First, the most talented workers $(z < z^e)$ always apply to type c job in the g-sector and leave the job if the sector is hit by permanent shock. Second, $z \in [z^e, z^u)$ workers also apply to the type c job in the g-sector but they stay in the d-sector as long as they are employed. Third, type $z \in [z^u, z^s)$ workers apply to the type c job in the g-sector when they enter the labor market and stay in the same sector even though they become unemployed. Finally, the least adaptable $z \geq z^s$ worker always apply to the type s job in the g-sector.

Now we have made the full circle and the market equilibrium is completely determined except for the evolution of the state variables, which are shown in section A.6. Notice that we have the complete system of equations which jointly determine the equilibrium values of unemployment and employment. See the last section of this appendix for the details of derivations for the optimal queue lengths. These values are independent from the dynamics through which state variables converge to the steady state.

A.5 Ex Post Optimality and Incomplete Contract

In the definition of market equilibrium, we assumed that job slots offer two types of contract: contingent and non-contingent employment contracts. The former stipulates that the employment is terminated at the moment of the productivity shock. On the other hand, in unconditional contract, wage is made contingent upon the state. If we deprive of full commitment ability, and assume instead agents are restricted to offer non-contingent wage and employment. In that case, time inconsistency problem may arise: namely, when they post the vacancy, their optimal choice of offer entails unconditional employment at a wage rate which is also unconditional. Ex post, when the job slot filled by a worker is hit by the productivity shock, the job slot may well find it optimal to renege on the promise as the expected return under the depressed state may well be negative.

Now let us return to the contingent wage schedule contract. We show that firms can avoid the time inconsistency problem if the state contingent wage schedule is available. Whether wage schedule is state contingent or not matters only for the case in which a job slot posts an unconditional employment contract. In the proof of Lemma 1, we have shown how the queue length for each type of worker is determined by the optimal (first order) condition, whereas the optimal wage schedule is derived by substituting the queue length into the zero profit conditions.

Since single zero profit condition can pin down only the discounted sum of the state contingent wage, there are (infinitely) many wage schedules that satisfy this condition. That is, the optimal wage schedule of the unconditional employment contract is not unique. The conditions, by which the retention of employment in d-sector is made *ex post* optimal for both sides, are:

$$\frac{\theta q^c - w_d - p_c}{R_c} \geq 0, \tag{A7}$$

$$\frac{w_d + \delta_c U^*(d,z)}{R_c} \geq U^*(d,z). \tag{A8}$$

which are equivalent (respectively to)

$$J_d(w) \geq 0,$$
$$E_d(z,w,U^*) \geq U^*(d,z).$$

where $R_c = r + d + \delta_c$. Therefore, if w_d satisfies

$$(r+d)U^*(d,z) \leq w_d \leq \theta q_c - p_j,$$

then the unconditional contract is *ex post* optimal. We can show that this condition can be satisfied when the joint surplus from the retention of job in d-sector is non-positive. This always holds true if type $\bar{g}$ contract is *ex ante* optimal.

Proposition 3. *In equilibrium, the firm can make the type $\bar{g}$ contract ex post optimal by the appropriate state contingent wage schedule if the type $\bar{g}$ contract is ex ante optimal.*

A.6 Steady State

Given the equilibrium allocation $\{\mathcal{C}^*, \mathcal{W}^*, x^*\}$, we consider the flow and distribution of workers at steady state. Let $e_s(z)$ and $u(e,z)$ denote the proportion of employment in the state s sector and the proportion of unemployed with experience e among trait z workers, respectively. Let define function $\iota_e(z)$ and $\iota_u(z)$ as

$$\iota_e(z) = \begin{cases} 1 & \text{if } z \geq z^e \\ 0 & \text{if } z < z^e \end{cases}, \quad \iota_u(z) = \begin{cases} 1 & \text{if } z \geq z^u \\ 0 & \text{if } z < z^u \end{cases}$$

Then the flows of workers that apply to type c jobs ($z \leq z^s$) are given by[36] (see Figures 6-8)

$$\begin{aligned} \dot{u}(n,z) &= d - (\phi_n + d)u(n,z) \\ \dot{u}(g,z) &= \delta_j e_g(z) - (\phi_g + \rho + d)u(g,z) \\ \dot{u}(d,z) &= \rho(u(g,z) + (1-\iota_e(z))e_g(z)) + \delta_j e_d(z) - (\phi_d + d)u(d,z) \\ \dot{e}_d(z) &= \iota_e(z)\rho e_g(z) + \iota_u(z)\phi_d u(d,z) - (\delta_j + d)e_d \\ \dot{e}_g(z) &= \sum_{e\in\{n,g\}} \phi_e u(e,z) + (1-\iota_u(z))\phi_d u(d,z) - (\rho + \delta_j + d)e_g(z) \end{aligned} \tag{A9}$$

where $\phi_e = \phi(x^*(e,z))$. From (A9), at steady state, we have

$$\begin{aligned} u(n,z) &= \frac{d}{d+\phi_n} \\ u(g,z) &= \frac{d\delta\phi_n\omega_2(z)}{(d+\phi_n)\Omega(z)} \\ u(d,z) &= \frac{d\phi_u\rho\omega_1(z)}{(d+\phi_n)\Omega(z)} \\ e_d(z) &= \frac{d\phi_u\rho[\iota_e(z)(d+\rho+\phi_g)\omega_2(z) + \iota_u(z)\phi_d\omega_1(z)]}{(d+\delta_j)(d+\phi_n)\Omega(z)} \\ e_g(z) &= \frac{d(d+\rho+\phi_g)\phi_n\omega_2(z)}{(d+\phi_n)\Omega(z)} \end{aligned}$$

where

$$\begin{aligned} \Omega(z) &= (d+\rho)(d+\rho+\delta_j+\phi_g)\omega_2(z) - (1-\iota_u(z))\rho\phi_d\omega_1(z) \\ \omega_1(z) &= (d+\delta_j)[(1-\iota_e(z))(d+\rho+\phi_g)+\delta] + \delta_j\iota_e(z)(d+\rho+\phi_g) \\ \omega_2(z) &= d(d+\delta_j+\phi_d) + (1-\iota_u(z))\delta\phi_d \end{aligned}$$

On the other hand, low ability $z > z^s$ worker always apply type s job. The steady state distribution for these worker, which is independent of trait of worker, is simply given by

$$\begin{aligned} u(n,z) &= \begin{cases} \frac{d+\delta_s}{d+\delta_s+\phi_n} & \text{if } U_{\bar{s}} \geq U_{\hat{s}} \\ \frac{d+\delta_s+\rho}{d+\delta_s+\rho+\phi_n} & \text{if } U_{\bar{s}} < U_{\hat{s}} \end{cases} \\ e_s &= \begin{cases} \frac{\phi_n}{d+\delta_s+\phi_n} & \text{if } U_{\bar{s}} \geq U_{\hat{s}} \\ \frac{\phi_n}{d+\delta_s+\rho+\phi_n} & \text{if } U_{\bar{s}} < U_{\hat{s}} \end{cases} \end{aligned}$$

where e_s is the share of employment at type s job, which is independent of z, for $z > z^s$.

---

[36] Simple but extremely tedious computations will show that the state variable subsystem is locally stable. It involves confirming for each sub-case the linearized transition matrix to have non-positive eigen values only. We have not encountered any (non-local) instability in numerical computations we used for the analysis in Section 4 of the main text.

A.7 Proofs

Proof of Proposition 1

From (A3), we have

$$\begin{aligned} U_c(t,U^*) = &\max_{w,x} \ \frac{\phi(x)[E_c(z,w,U^*)-\kappa(c,t)]+I(e)\rho U^*(d,z)}{r+d+\phi(x)+I(e)\rho} \\ &s.t. \quad V_c(w,x)=0 \end{aligned}$$

Note that $\kappa(\bar{g},t)=\kappa(\hat{g},t)$ for any $t \in T$. From (A4),

$$\begin{aligned} E_{\bar{g}}(z,w,U^*) &= \frac{R_c w_g + \rho w_d + \delta(R_c U^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)} \\ E_{\hat{g}}(z,w,U^*) &= \frac{w_g + \delta U^*(g,z) + \rho U^*(d,z)}{R_c+\rho} \end{aligned}$$

Use the zero profit condition to substitute for w, we get

$$U_c(t,U^*) = \max_{x} \ \frac{\phi(x)[\tilde{E}_c(z,x,U^*)-\kappa(c,t)]+I(e)\rho U^*(d,z)}{r+d+\phi(x)+I(e)\rho}$$

where

$$\begin{aligned} \tilde{E}_{\bar{g}}(z,x,U^*) &= \frac{\tilde{q}-p_c}{R_c} + \frac{\delta(R_c U^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)} - \frac{p_c}{\psi(x)}, \\ \tilde{E}_{\hat{g}}(z,x,U^*) &= \frac{q^c - p_c + \delta U^*(g,z) + \rho U^*(d,z)}{R_c+\rho} - \frac{p_c}{\psi(x)} \end{aligned}$$

where $\tilde{q} \equiv \frac{R_j+\rho\theta}{R_j+\rho} q^c$. Since we have the following, the proof is complete.

$$\forall x \quad \tilde{E}_{\bar{g}}(z,x,U^*) \lesseqgtr \tilde{E}_{\hat{g}}(z,x,U^*) \Leftrightarrow (r+d)U^*(d,z) \gtreqless \theta q^c - p_c$$

Proof of Lemma 1

The optimal queue length solves the following problem

$$U_c(t,U) = \max_{x} \frac{\phi(x)[\tilde{E}_c(z,x,U)-\kappa(c,t)]+I(e)\rho U(d,z)}{r+d+\phi(x)+I(e)\rho}$$

where $\tilde{E}_c(z,x,U^*)$ is the value of employment that satisfies the zero profit condition. By the first order condition, we have

$$(r+d+\psi'(x)+I(e)\rho)p_c = \gamma(x)[(r+d+I(e)\rho)(\tilde{E}_c(z,x,U)+p(c)/\psi(x)-\kappa(c,t))-I(e)\rho U(d,z)]$$

where $\gamma(x)=\psi(x)-\psi'(x)/x$. By arranging terms, we get

$$\tilde{E}_c(z,x,U)-\kappa(c,t) = \frac{\Delta(x)x}{\psi(x)}p(c) + \frac{[\psi'(x_c(t))/\gamma(x_c(t))]p(c)+I(e)\rho U(d,z)}{r+d+I(e)\rho}$$

By substituting this into the objective function, we complete the proof of the first half of the proposition.

Second, we will show the second half of the proposition. By using the results above, we write

$$\begin{aligned} U_c((g,z),U) &= \frac{\Delta(x_c(g,z))p(c)+\rho U_c((d,z),U)}{r+d+\rho}, \\ U_c((d,z),U) &= \frac{\Delta(x_c(d,z))p(c)}{r+d}. \end{aligned}$$

Since $U_c((g,z),U) \geq U_c((d,z),U)$ and $\Delta'(x)<0$, we have $x_c(g,z) \leq x_c(d,z)$.

Proof of Proposition 2

Let define $\bar{z}^u$ and $\hat{z}^u$ by $\tilde{U}_d(d,\bar{z}^u)=\tilde{U}_{\bar{g}}(d,\bar{z}^u)$ and $\tilde{U}_d(d,\hat{z}^u)=\tilde{U}_{\hat{g}}(d,\hat{z}^u)$, respectively. We will prove that $z \gtreqless \bar{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\bar{g}}(d,z)$ and that $z \gtreqless \hat{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\hat{g}}(d,z)$, which will complete the proof if we let $z^u=\max\{\bar{z}^u,\hat{z}^u\}$.

First, we show that $\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z \le \partial\tilde{U}_d(d,\bar{z}^u)/\partial z < 0$, which assures that $z \gtreqless \bar{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\bar{g}}(d,z)$. Since $I(d)=0$ and $p(c)=p_c$ for $c=d,\bar{g},\hat{g}$, lemma 1 implies $x_d(d,\bar{z}^u)=x_{\bar{g}}(d,\bar{z}^u)$ and $x_{\bar{g}}(g,\bar{z}^u)\ge x_{\bar{g}}(d,\bar{z}^u)$. Let $x'=x_d(d,\bar{z}^u)=x_{\bar{g}}(d,\bar{z}^u)$ and $x''=x_{\bar{g}}(g,\bar{z}^u)$.

From (A3) and (A5), $\tilde{U}_d(d,z)$ is written by

$$\begin{aligned}\tilde{U}_d(d,z) = &\max_{w_d,x}\frac{w_d-R_c\epsilon z}{(r+d)(R_c+\phi(x))}\phi(x)\\ &s.t.\quad V_d(w,x)=0\end{aligned}$$

By the envelope theorem, we have

$$\frac{\partial\tilde{U}_d(d,\bar{z}^u)}{\partial z}=-\frac{R_c\epsilon\phi(x')}{(r+d)(R_c+\phi(x'))}$$

Similarly, we have

$$\begin{aligned}\tilde{U}_{\bar{g}}(d,z) = &\max_{w_g,w_d,x}\frac{R_cw_g+\rho w_d-R_c(R_c+\rho)(\epsilon+m)z+\delta_cR_c\tilde{U}_{\bar{g}}(g,z)}{(r+d)(R_c+\rho)(R_c+\phi(x))+\phi(x)\delta_cR_c}\phi(x)\\ &s.t.\quad V_{\bar{g}}(w,x)=0\end{aligned}$$

and

$$\begin{aligned}\tilde{U}_{\bar{g}}(g,z) = &\max_{w_g,w_d,x}\frac{\phi(x)(R_cw_g+\rho w_d-R_c(R_c+\rho)\epsilon z)+\rho[R_c(R_c+\rho)+\phi(x)\delta_c]\tilde{U}_{\bar{g}}(d,z)}{R_c(r+d+\rho)(R_c+\rho+\phi(x))}\\ &s.t.\quad V_{\bar{g}}(w,x)=0\end{aligned}$$

Again by the envelope theorem, we have

$$\frac{\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)}{\partial z}=\frac{-(R_c+\rho)R_c(\epsilon+m)+\delta_cR_c\cdot\partial\tilde{U}_{g^u}(g,z)/\partial z}{(r+d)(R_c+\rho)(R_c+\phi(x'))+\phi(x')\delta_cR_c}\phi(x') \tag{A10}$$

$$\frac{\partial\tilde{U}_{\bar{g}}(g,\bar{z}^u)}{\partial z}=\frac{-R_c(R_c+\rho)\epsilon\phi(x'')+\rho[R_c(R_c+\rho)+\phi(x'')\delta_c]\partial\tilde{U}_{g^u}(d,z)/\partial z}{R_c(r+d+\rho)(R_c+\rho+\phi(x''))} \tag{A11}$$

By substituting (A11) into (A10), we have

$$\frac{\partial\tilde{U}_{\bar{g}}(d,z)}{\partial z}=-\frac{R_c\phi(x')[(r+d+\rho)(R_c+\rho+\phi(x''))(\epsilon+m)+\delta_c\phi(x'')\epsilon]}{(r+d)[(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x''))+\delta\phi(x')(R_c+\phi(x''))]}$$

Taking the ratio

$$\frac{\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z}{\partial\tilde{U}_d(d,z_u^u)/\partial z}=\frac{(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x''))(\epsilon+m)/\epsilon+\delta_c\phi(x'')(R_c+\phi(x'))}{(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x''))+\delta_c\phi(x')(R_c+\phi(x''))}$$

Since $m\ge 0$ and $x''\ge x'$, the numerator is not less than the denominator. Therefore, we have $\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z \le \partial\tilde{U}_d(d,\bar{z}^u)/\partial z<0$.

Since we can also show $\partial\tilde{U}_{\hat{g}}(d,\hat{z}^u)/\partial z \le \partial\tilde{U}_d(d,\hat{z}^u)/\partial z < 0$ by the same procedure, the first part of the proposition is proved.

The second part of the proposition is obvious because $\partial\tilde{U}_c(d,z)/\partial z$ is non-positive for $c=d,\bar{g},\hat{g}$.

Proof of Proposition 3

If type $\bar{g}$ contract is *ex ante* optimal, we have $(r+d)U^*(d,z) \leq \theta q^c - p_c$ from Proposition 1. Therefore by setting w_d so as to satisfy (8), the firm can make the contract *ex post* optimal.

A.8 The Optimal Queue Lengths

We briefly indicate how to solve for the optimal queue lengths. As in the proof of Lemma 1, the optimal queue length satisfies

$$\begin{aligned}&(r+d+\psi'(x)+I(e)\rho)p_c\\&=\gamma(x)[(r+d+I(e)\rho)(\tilde{E}_c(z,x,U)+p(c)/\psi(x)-\kappa(c,t))-I(e)\rho U(d,z)]\end{aligned}$$

As an example, we consider the case in which the unemployed trained workers choose to stay in the same *industry*, i,e, $z \in (z^s, z^e)$. The other cases can be solved in a similar fashion.

Let x_d^* be the optimal queue length for type d worker. Since the unemployed worker chooses the type d contract, $c=d$ and $\kappa(c,t)=\epsilon z$, then x_d solves

$$\begin{aligned}&(r+d+\psi'(x_d^*))p_c\\&=\gamma(x_d^*)(r+d)[(\tilde{E}_d(z,x,U)+p(c)/\psi(x_d^*)-\epsilon z]\\&=\gamma(x_d^*)(r+d)\left[\frac{\theta q^c-p_c+\delta_c U^*(d,z)}{R_c}-\epsilon z\right]\end{aligned}$$

From Lemma 1, we have $U^*(t)=\frac{\Delta(x^*(t))p(c)+I(e)\rho U^*(d,z)}{r+d+I(e)\rho}$, define, thus

$$\begin{aligned}&(r+d+\psi'(x_d^*))p_c\\&=\gamma(x_d^*)(r+d)\left[\frac{\theta q^c-p_c}{R_c}+\frac{\delta_c}{R_c}\frac{\Delta(x_d^*)p_c}{r+d}-\epsilon z\right]\end{aligned}$$

Since $\Delta(x)=\psi'(x)/\gamma(x)$, we have

$$p_c(R_c+\psi'(x_d^*))=\gamma(x_d^*)(\theta q^c-p_c)$$

On the other hand, the optimal queue length for type g worker, x_g^*, satisfies

$$\begin{aligned}&(r+d+\psi'(x_g^*)+\rho)p_c\\&=\gamma(x_g^*)[(r+d+\rho)(\tilde{E}_{g^u}(z,x,U)+p(c)/\psi(x_g^*)-\epsilon z)-\rho U^*(d,z)]\\&=\gamma(x_g^*)\left[(r+d+\rho)\left(\frac{\tilde{q}-p_c}{R_c}+\frac{\delta_c(R_cU^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)}-\epsilon z\right)-\rho U^*(d,z)\right]\\&=\gamma(x_g^*)\left[(r+d+\rho)\left(\frac{\tilde{q}-p_c}{R_c}-\epsilon z\right)+\frac{\delta_cR_c\Delta(x_g^*)p_c-\rho(r+d)(r+d+\rho)U^*(d,z)}{R_c(R_c+\rho)}\right]\end{aligned}$$

By rearranging terms, we have

$$R_c(R_c+\rho+\psi'(x_g^*))p_c=\gamma(x_g^*)\left[(R_c+\rho)(\tilde{q}-p_c-R_c\epsilon z)-\rho\Delta(x_d^*)p_c\right]$$

Similarly, the optimal queue length for type n worker, x_n^*, satisfies

$$\begin{aligned}&(r+d+\psi'(x_n^*))p_c\\&=\gamma(x_n^*)(r+d)[\tilde{E}_{g^u}(z,x,U)+p(c)/\psi(x_n^*)-z]\\&=\gamma(x_n^*)(r+d)\left[\frac{\tilde{q}-p_c}{R_c}+\frac{\delta_c(R_cU^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)}-z\right]\\&=\gamma(x_n^*)(r+d)\left[\left(\frac{\tilde{q}-p_c}{R_c}-z\right)+\delta_c\frac{(r+d)R_c\Delta(x_g^*)p_c+\rho(r+d+\rho+R_c)\Delta(x_d^*)p_c}{R_c(R_c+\rho)(r+d+\rho)(r+d)}\right]\end{aligned}$$

Thus, we have

$$(r+d+\psi'(x_n^*))p_c$$
$$= \gamma(x_n^*)\left[(r+d)\left(\frac{\tilde{q}-p_c}{R_c}-z\right)+\delta_c\frac{(r+d)R_c\Delta(x_g^*)p_c+\rho(r+d+\rho+R_c)\Delta(x_d^*)p_c}{R_c(R_c+\rho)(r+d+\rho)}\right]$$

(The end of the Appendix)

References

[1] Anton Braun, R., J. Esteban-Pretel, T. Okada, and N. Sudou (2006) "A comparison of the Japanese and US business cycles," *Japan and The World Economy*,Vol. 18, No. 4, pp. 441-463.

[2] Ariga, K., G. Brunello and Y. Ohkusa (2000), *Internal Labor Market in Japan*, Cambridge University Press, Cambridge.

[3] Ariga, K., M. Kurosawa, and F. Ohtake (2006) "Human Resource Management Practices and Firm Level Training," in Hayashi, F. (eds) *Keizai Teitai no Genin to Seido*, (in Japanese) Keiso-Syobou, Tokyo.

[4] Ariga, K., M. Kurosawa, F. Ohtake, and M. Sasaki (2009), "Productivity and Job Training: Evidence from Japanese Automobile Makers," a paper presented at Second Trans-Pacific Labor Seminar, Tokyo.

[5] Bassanini, A., A. L. Booth, G. Brunello, M. De Paola, and E. Leuven (2005), "Workplace Training in Europe," IZA Discussion Paper 1640

[6] Caballero, R.J., T. Hoshi, and A.K. Kashyap (2008) "Zombie lending and depressed restructuring in Japan," *American Economic Review*, Vol. 98, pp. 1943-1977.

[7] Esteban-Pretel, R. Nakajima, and R. Tanaka (2009) "Are Contingent Jobs Dead Ends or Stepping Stones to Regular Jobs? Evidence from a Structural Estimation," RIETI working paper No. 09-E-002.

[8] Fujita, S. and G. Ramey (2007) "Job matching and propagation," *Journal of Economic Dynamics and Control*, Vol. 31, No. 11, pp. 3671-3698.

[9] Fukao, K., Y. Kim, and H. U. Kwon (2008) 'Why has the Japanese TFP growth recovered?," (in Japanese) RIETI, DP, 08-J-050.

[10] Fukao, K. and H. U. Kwon (2006) "Why Did Japan's TFP Growth Slow Down in the Lost Decade? An Empirical Analysis Based on Firm-Level Data of Manufacturing Firms," *Japanese Economic Review*, Vol. 57, No. 2, pp. 195-228.

[11] Fukao, K. and M. Otaki (1993) "Accumulation of Human Capital and the Business Cycle," *Journal of Political Economy*, Vol. 101, No. 1, p. 73-99.

[12] Genda, Y., A. Kondo, and S. Ohta (forthcoming) "Lon*g*-term effects of a recession at labor market entry in Japan and the United States," forthcoming in *Journal of Human Resources.*

[13] Genda, Y. and M. Kurosawa (2001) "Transition from School to Work in Japan," *Journal of The Japanese and International Economies*, Vol. 15, No. 4, pp. 465-488.

[14] Genda, Y., S. Ohta, and H. Teruyama (2008) "The Japanese Unemployment after 90's," (in Japanese) Bank of Japan, DP, 08-J-4.

[15] Hornstein, A., P Krusell, and G.L. Violante (2007) "Technology-Policy Interaction in Frictional Labour-Markets," *Review of Economic Studies*, Vol. 74, No. 4, pp. 1089-1124.

[16] Ito, K and S. Lechevalier (2008) "The evolution of the productivity dispersion of firms-A reevaluation of its determinants in the case of Japan," RIETI, DP, 08-E-014.

[17] Kato, T. and R. Kambayashi "The Japanese Employment System after the Bubble Burst: New Evidence." this conference.

[18] Kambayashi, R. and Y. Ueno (2006) "Vacancy Market Structure and Matching Efficiency,"" ESRI, DP, No. 160.

[19] Kano, S. and M. Ohta (2005) "Estimating a matching function and regional matching efficiencies: Japanese panel data for 1973-1999," *Japan and The World Economy*, Vol. 17, No. 1, pp. 25-41.

[20] Koike, K. (2005) *Economics of Work* (in Japanese), Toyo-Kiezai, Tokyo.

[21] Kondo, A. (2007) "Does the first job really matter? State dependency in employment status in Japan," *Journal of The Japanese and International Economies*, Vol. 21, No. 3, pp. 379-402.

[22] Kuroda, S. (2003) "Analysis of Changes in Japan's Unemployment Rate Using Gross Flow Data," *Monetary and Economic Studies*, Vol. 21, pp. 69-104.

[23] Miyagawa, T. (2005), *Economics of longrun stagnation* [*Choki Teitai no Keizaigaku*] (in Japanese), University of Tokyo Press

[24] Miyagawa, T., Y. Sakuragawa, and M. Takizawa (2006) "The impact of technology shocks on the Japanese business cycle - An empirical analysis based on Japanese industry data," *Japan and The World Economy*, Vol. 18, No. 4, pp. 401-417.

[25] Moen, E.R. and A. Rosen (2004) "Does Poaching Distort Training?" *Review of Economic Studies*, Vol. 71, No. 4, pp. 1143-1162.

[26] Mortensen, D.T. and C.A. Pissarides (1998) "Technological Progress, Job Creation, and Job Destruction," *Review of Economic Dynamics*, Vol. 1, No. 4, pp. 733-753.

[27] Muramatsu, K., 1995, "Employment Adjustments in Japan: A Review," in Inoki and Higuchi (eds.) *The Japanese Employment System and the Labor Market* (in Japanese), Nihon-Keizai-Shinbun, Tokyo

[28] Nishimura, K.G., and K. Minetaki (2004) *Information Communication Technology and the Japanese Economy* [*Jyoho Kijyutsu Kakushin to Nhon Keizai*] (in Japanese), Yuhikaku

[29] Ohtake, F. (2005) *A Study of Inequality in Japan* (in Japanese), Toyo-Kiezai, Tokyo.

[30] Shinada, N. (2003) "Decline in Productivity in Japan and Disparities between Firms in the 1990s: An Empirical Approach Based on Data Envelopment Analysis", Development Bank of Japan Research Report, No.38, Development Bank of Japan, Tokyo.

	industry type	
Worker type	g-sector	d-sector
not trained	(n,g)	not active
trained in the same industry	(g,g)	(d,d) if $z \geq z^u$
trained elsewhere	(d,g) if $z \leq z^u$	not active

Table 1: Table of sub-markets

parameter	value	discription
r	0.005	interest rate (annual 2%)
d	0.0063	exiting rate (annual 2.5%)
μ_z	9.0	mean of training cost
σ_z	12.1	standard deviation of training cost
ϵ	0.15	across firm mobility
m	0.42	across industry mobility cost
ρ	0.006	the frequency of productivity shock (annual 2.4%)
θ	0.88	the scale of negative shock
δ^j	0.003	separation rate of type j job (annual 1.2%)
δ^s	0.037	separation rate of type s job (annual 14.8%)
q^c	1.5	productivity of complicated task
q^s	1.0	productivity of simple task
p^j	0.42	rental cost of type j capital
p^s	0.23	rental cost of type s capital
A	0.48	efficiency of matching function
η	0.3	parameter of matching function

Table 2: Benchmark Parameter Values

	benchmark	θ shock	ρ shock	both shock
Unemployment rate (%)	3.1	3.6	3.6	4.5
Vacancy /Unemployment	1.00	1.01	1.00	1.02
Thresholds				
z^s	26.7	25.3	25.1	22.6
z^u	24.6		18.2	
z^e	11.8	18.0	7.6	12.4
Job Finding Rate	0.48	0.48	0.48	0.48
part-time job	0.51	0.51	0.51	0.51
full-time job	0.47	0.47	0.46	0.46
Annual worker turnover rate (full-time %)	9.9	11.3	10.6	13.1
Annual worker turnover rate (part-time %)	39.4	39.4	44.2	44.2
Share of accessions with industry change (%)	60.0	64.9	69.1	77.7
Employment share (%)				
at part-time job	8.4	10.6	10.8	15.7
at full-time job in d	17.5	7.1	35.1	19.3
at full-time job in g	74.1	82.3	54.2	65.0
Gross output	1.38	1.38	1.33	1.31
Aggregate training cost	0.085	0.091	0.079	0.084
Average value of untrained	74.1	73.6	71.5	70.4

Table 3: Steady States before and after Two Macro Shocks

	θ shock	
	50% benchmark	75% benchmark
Unemployment rate	1.5	1.75
Employment share (%)		
at part-time job	27	50
at full-time job in g	0.5	0.5

	ρ shock	
	50% benchmark	75% benchmark
Unemployment rate	1	24
Employment share (%)		
at part-time job	29	56
at full-time job in g	9	18

Table 4: Transition paths after macro shocks (unit: year)

	both shock	capital subsidy	firing cost	training (untrained)	training (trained)
Unemployment rate	4.5	-0.2	-0.2	-0.2	+0.1
Vacancy /Unemployment	1.02			-0.01	
Job Finding Rate	0.48				
Thresholds					
z^s	22.6	+0.3		+0.9	
z^u					
z^e	12.4	-0.9	-1.5		+1.1
Annual worker turnover (full-time %)	13.1	-0.4	-0.7	-0.1	+0.6
Change of industry (%)	77.7	-0.5	-0.8	-0.1	+0.5
Employment share (%)					
at part-time job	15.7	-0.8		-1.9	
at full-time job in d	19.3	+2.6	+3.4	+1.1	-2.6
at full-time job in g	65.0	-1.8	-3.4	+0.8	+2.5
Gross output	1.31			+0.01	+0.01
Average value of untrained	70.4	-0.1	-0.3	+0.2	

Table 5: Policy effects

Figures show net changes from the benchmark values in the first column

	benchmark	Case 1	Case 2
	$\epsilon = .15$	$\epsilon = 0$	$\epsilon = .15$
	$m = .42$	$m = .42$	$m = .21$
Unemployment rate	1.4	1.5	1.6
Annual worker turnover (full-time %)	3.2	4.2	4.7
Change of location (%)	17.7	16.3	14.9
Employment share (%)			
at part-time job	7.3	7.2	6.7
at full-time job in d	1.8	-1.0	-2.1
at full-time job in g	-9.1	-6.2	-4.6
Gross output (% change)	-4.9	-4.3	-3.9
Average value of untrained (% change)	-5.0	-4.6	-4.2

Table 6: The impact of Macro shocks

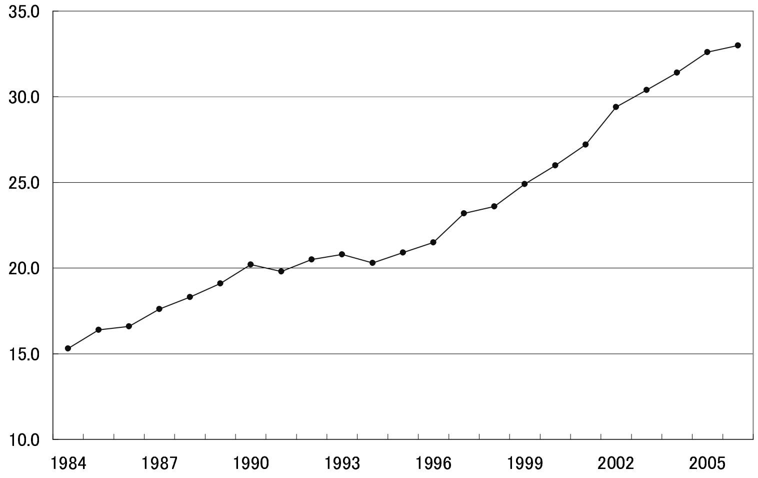

Figure 1: The Share of Temporary and Parttime Employment

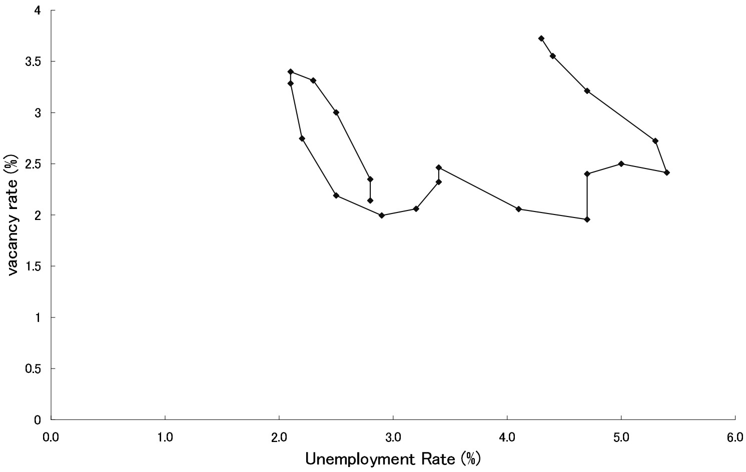

Figure 2: Beveridge Curve

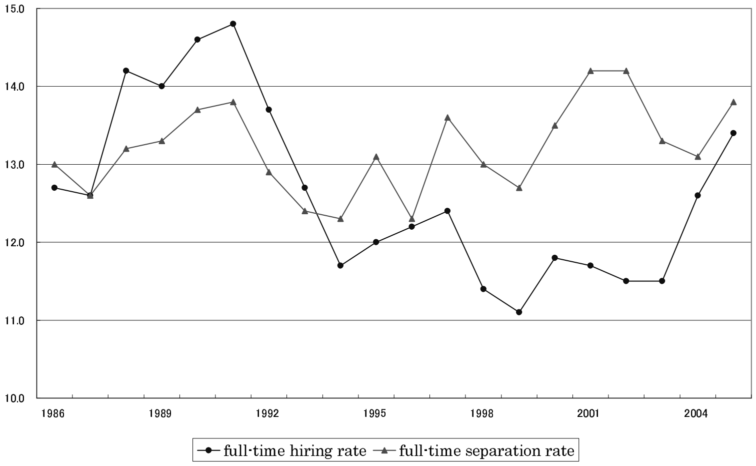

Figure 3: Job Accession and Separations: Full time employees

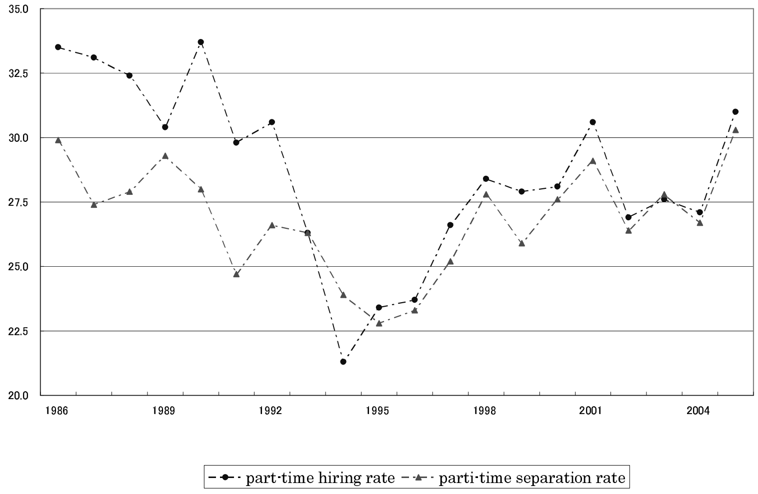

Figure 4: Job Accession and Separations: Part time employees

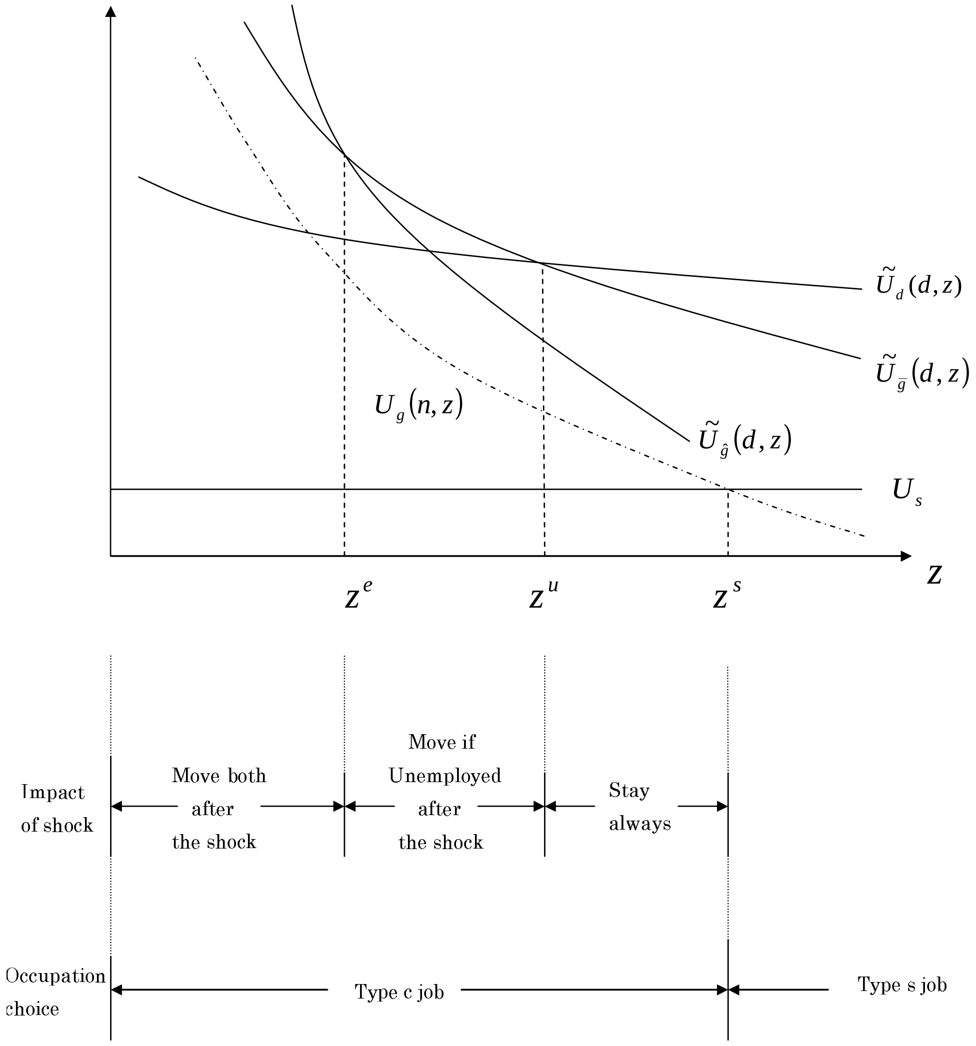

Figure 5: Optimal Job Search Strategy

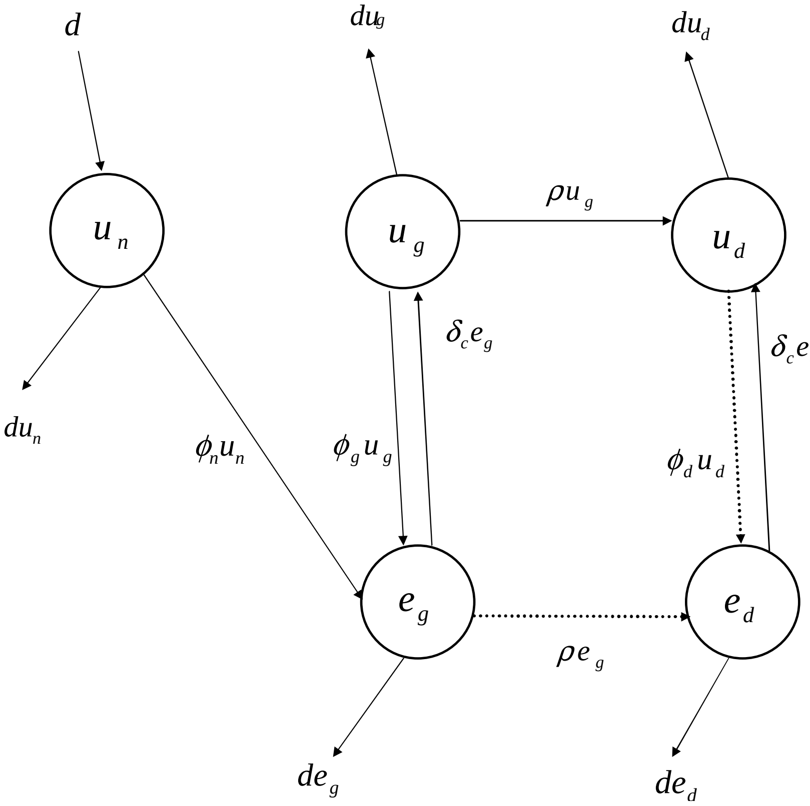

Figure 6: Worker flows $z^u < z < z^s$

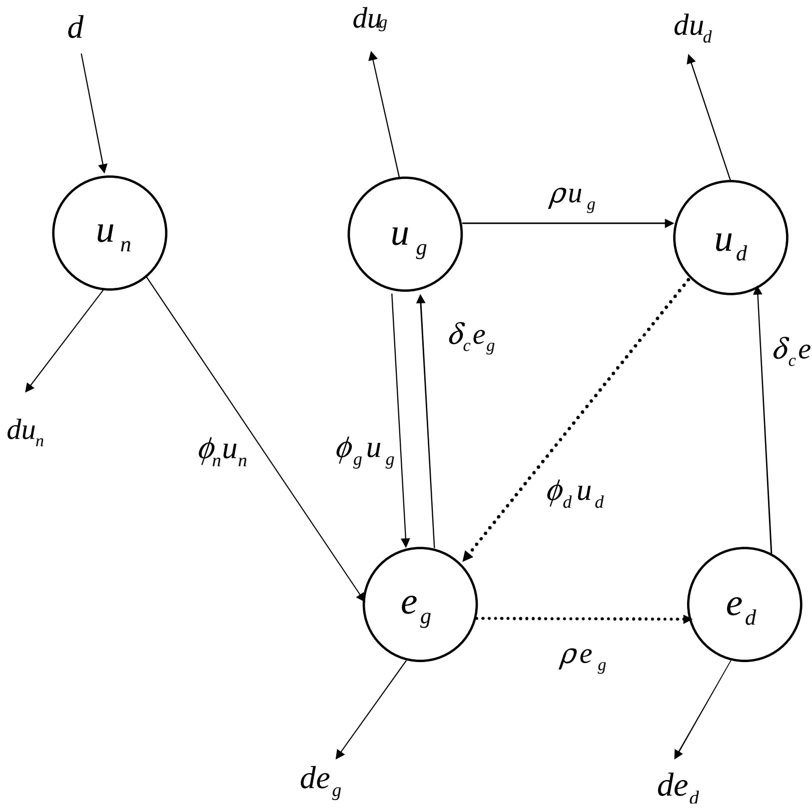

Figure 7: Worker Flows $z^e < z < z^u$

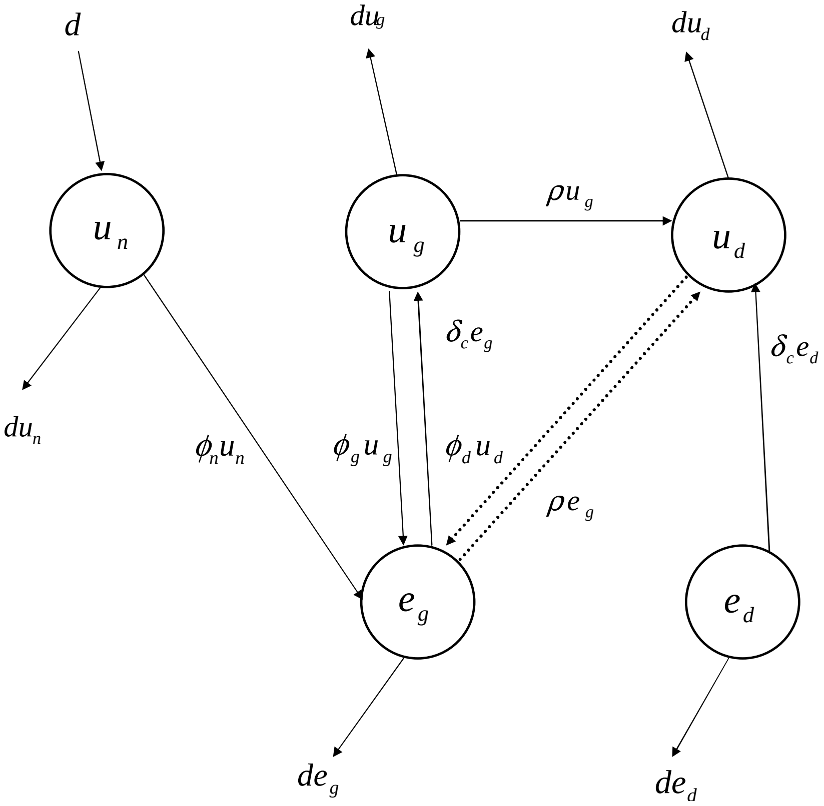

Figure 8: Worker Flows $z < z^e$

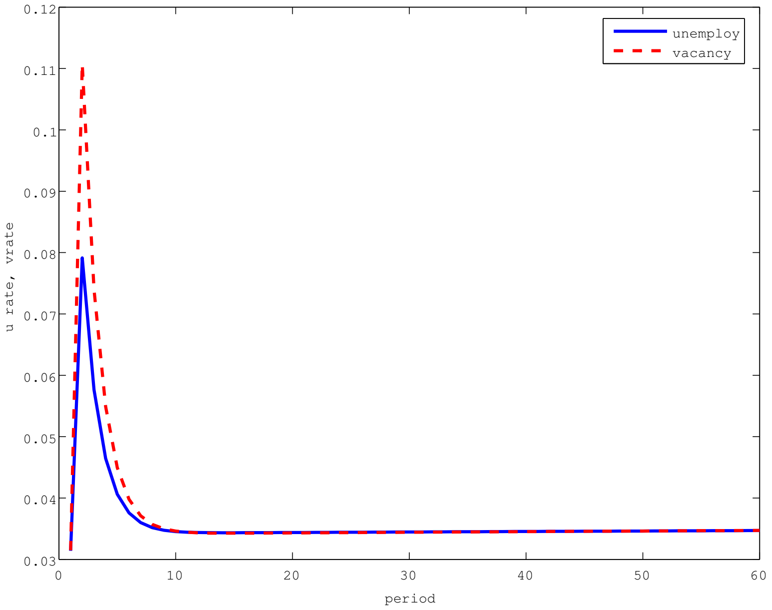

Figure 9: Simulated Dynamics of Unemployed and Vacancy: θ shock

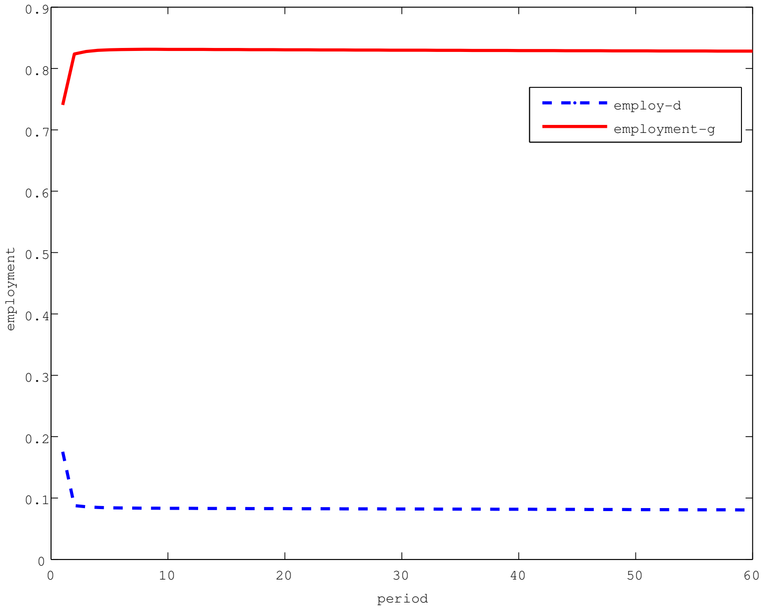

Figure 10: Simulated Dynamics of Employment Composition: θ shock

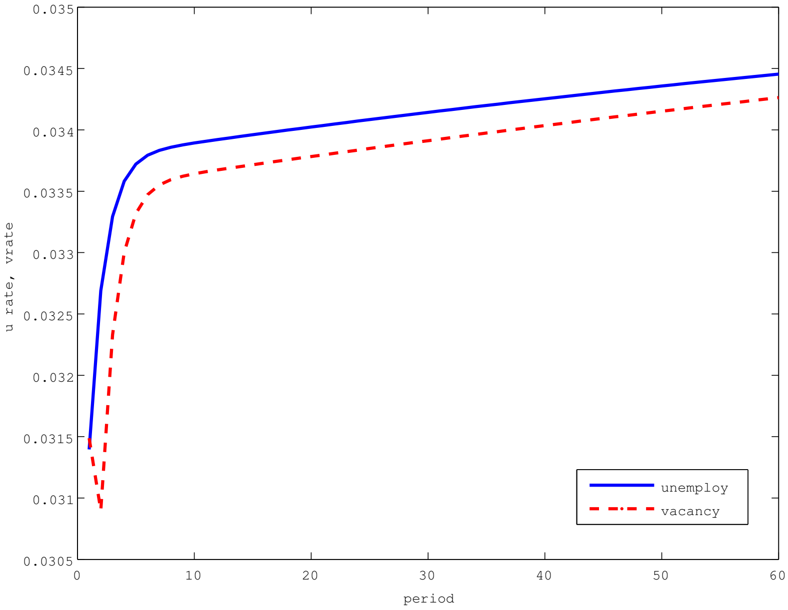

Figure 11: Simulated Dynamics of Unemployed and Vacancy: ρ shock

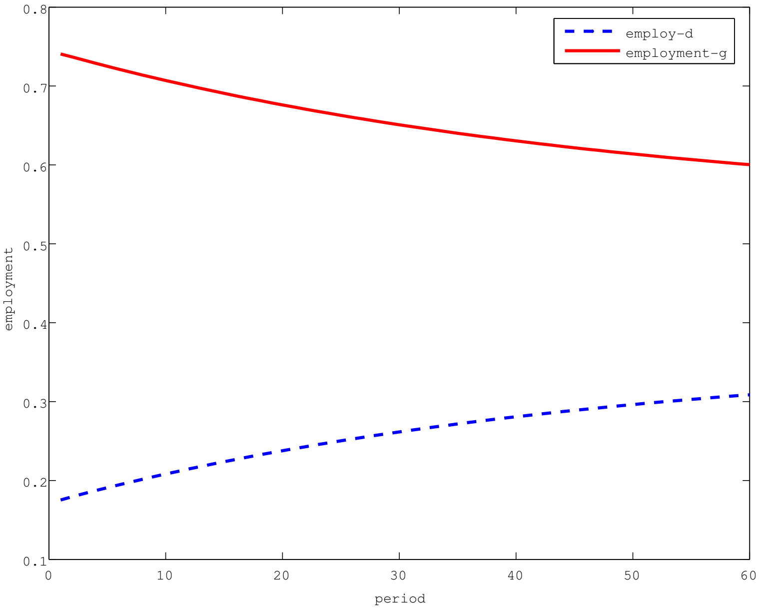

Figure 12: Simulated Dynamics of Employment Composition: ρ shock

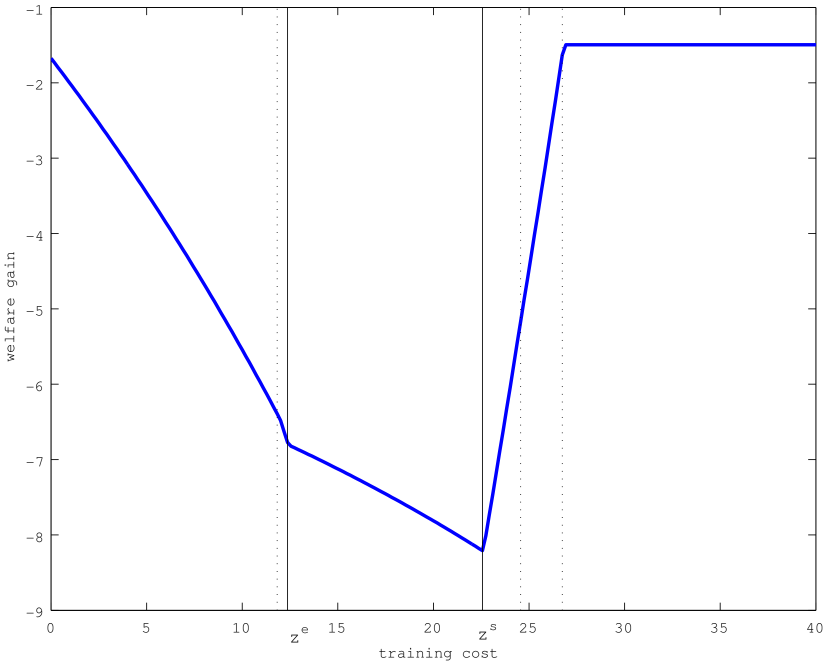

Figure 13: Welfare loss by combined macro shocks (%)

Dashed (real) lines indicate threshold values before (after) the macro shocks.

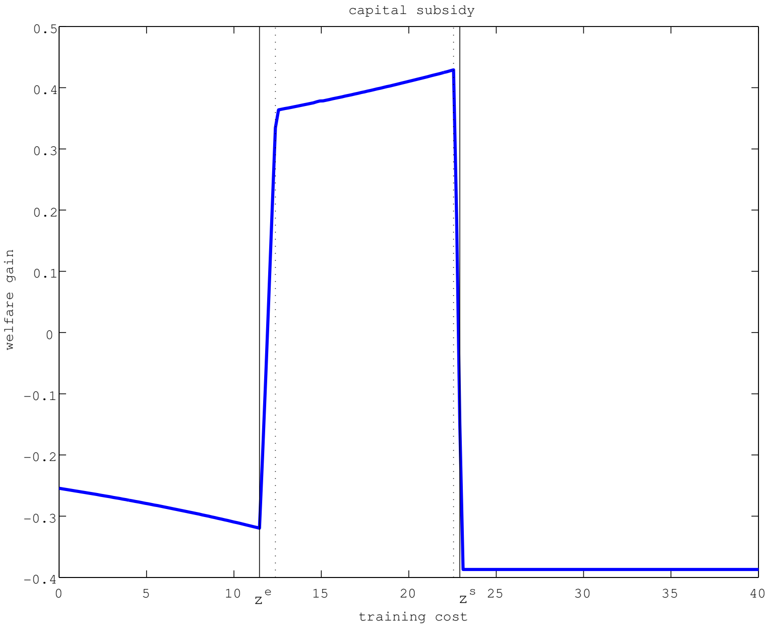

Figure 14: Net Welfare gains: Capital subsidy (%)

In Figures 14 through 17, dashed (real) lines indicate threshold values without (with) respective policy injections.

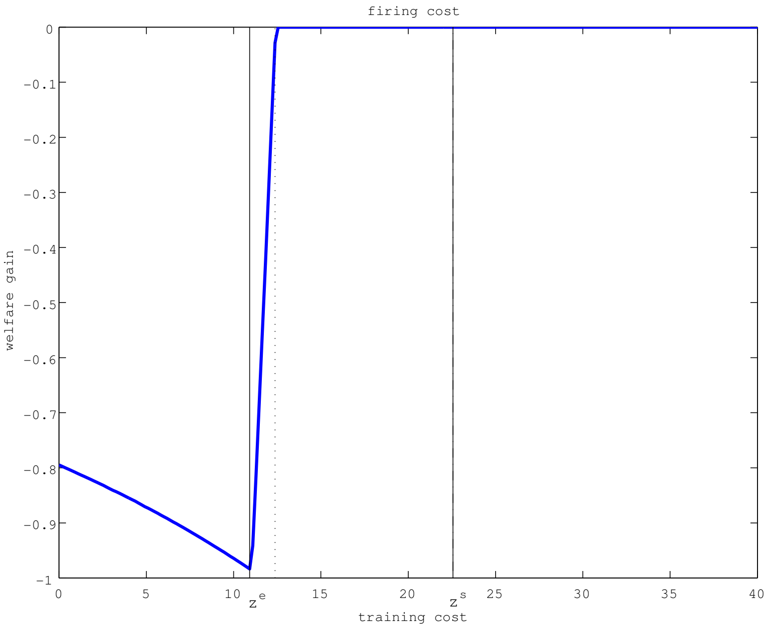

Figure 15: Net Welfare gains: Firing cost (%)

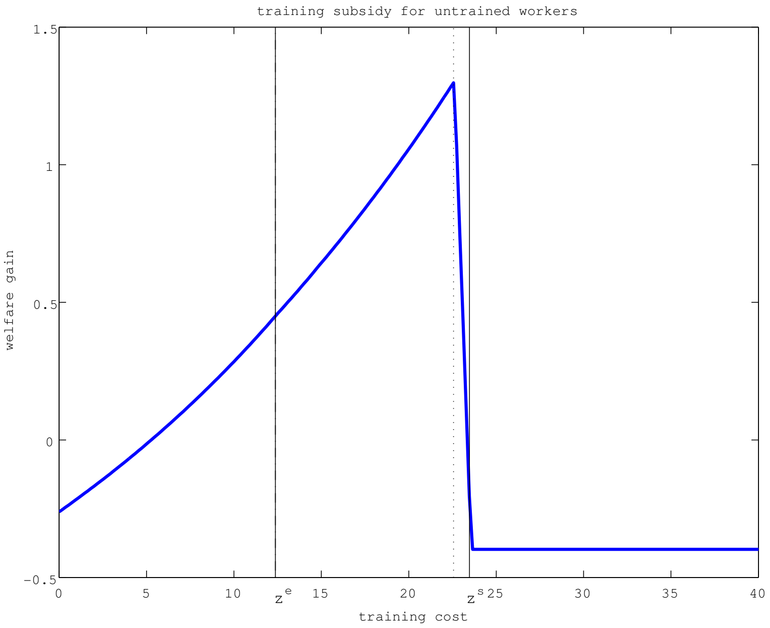

Figure 16: Net Welfare gains: Training subsidy for untrained (%)

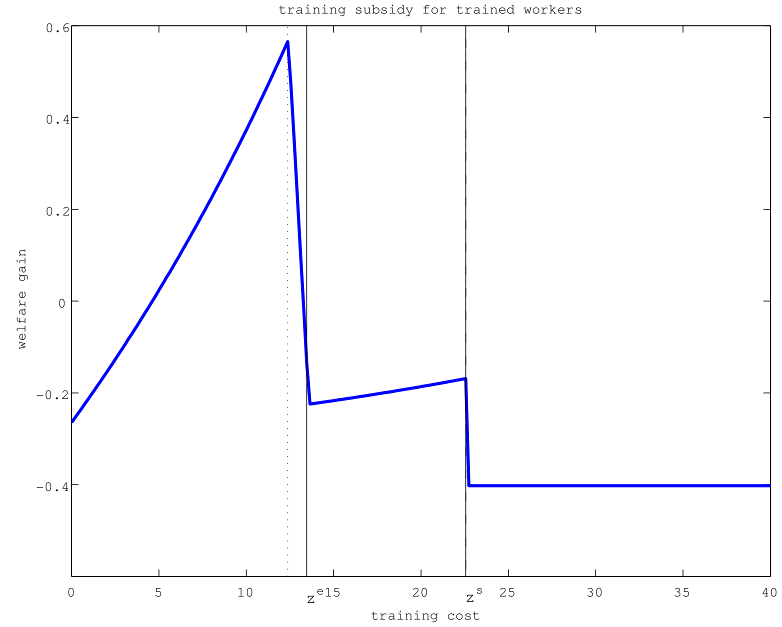

Figure 17: Net Welfare gains: Re-training subsidy for trained (%)