Macroeconomic Implications of Low Japanese Government Bond Yields

Masaya SAKURAGAWA*, Yukie SAKURAGAWA**

Abstract

One major puzzle of the Japanese economy at present is that low government bond yields coincide with a high level of government debt. This paper investigates the general equilibrium mechanism underlying this relationship and examines its macroeconomic implications. We argue that when investors fail to achieve international risk diversification in their asset portfolios because of home bias, they have no assets to hedge fiscal risk. The bond yield is then less sensitive to default risk, and the government can sustain a high level of debt. However, such sustainability is only possible at the expense of a reduction in the real interest rate, which weakens policy effects and ultimately leads to secular stagnation.

JEL Codes: E00, G12, H63
Keywords: Fiscal risk, Safe assets, Risk premium, Sustainability, Real interest rates

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* Professor, Keio University, 2-15-45, Mita, Minato-ku, Tokyo, 108-8345, (Tel) 81-3-5427-1832.
** Associate Professor, Atomi University, 1-9-6, Nakano, Niiza-shi, Saitama, 352-8501, (Tel) 81-48-478-4110.
政府債務残高の水準が極めて高いにもかかわらず、なぜ国債利回りが低いままなのかは、現在の日本経済における主要なパズルの一つである。本稿は、このパズルを明らかにする理論メカニズムを示し、そのマクロ経済へのインプリケーションを考察する。

分析のカギとなるのは、ホームバイアスが強く、投資家が資産ポートフォリオの国際的なリスク分散を達成できない場合、財政リスクをヘッジする代替的な資産を保有できないという視点である。なぜなら、財政破たんが起きると、国内のいかなる資産もその収益の減少を免れなければならないからである。

この結果、国債の利回りは財政のデフォルトリスクに対して非感応的になり、政府は多額な債務発行が可能になる。しかしながら、こうした持続可能性は、実質金利の下落という犠牲によってのみ実現する。このことは日本経済を回復させようとする政策効果を弱めてしまい、結局のところ長期停滞をもたらすことになる。

JEL 分類コード：E00, G12, H63
キーワード：財政リスク、安全資産、リスクプレミアム、持続可能性、実質金利
1. Introduction

One major puzzle of the Japanese economy at present is that low government bond yields coincide with a high level of government debt\(^1\). From the perspective of conventional asset pricing theory, the bond yield should reflect the fiscal risk of large outstanding debt, currently more than 200 percent of GDP. Even more surprisingly, and as illustrated in Figure 1, among Organization for Economic Co-operation and Development (OECD) countries, the nominal bond yield in Japan is the lowest, but the debt-to-GDP ratio remains the highest. The Japanese government can sustain this high level of debt because of these low yields. However, this may have serious consequences for the Japanese macro economy. This paper aims to effectively understand the macroeconomic implications of the large stock of Japanese government bonds (JGBs) by investigating the general equilibrium mechanism underlying the low yields on JGBs.

There are three possible explanations concerning why the Japanese government can sustain a high level of debt at such low yields. The first is that there is substantial scope in the future for increasing the consumption tax rate and that domestic investors...
trust government actions\textsuperscript{2}. The second relates to monetary policy. Since 2001, the Bank of Japan (BoJ) has set the nominal interest rate close to zero\textsuperscript{3}. In 2013, the BoJ commenced quantitative easing (QE) that involved large-scale purchases of long-term government bonds. It is widely believed that the recent decline in long-term government bond yields is attributed to that policy.

The third and final explanation is that the cumulative domestic savings surplus and the persistent strong home bias in the asset portfolios of domestic investors induce them to hold almost all JGBs on issue\textsuperscript{4}. In support of this, Tomita (2001), Tokuoka (2010), and Ito (2011) emphasize the strong demand for JGBs by domestic financial institutions, which do not seem willing to diversify portfolio risk across international borders, but instead prefer yen-denominated assets. In Japan, domestic residents currently hold more than 95 percent of the JGBs on issue, a very high share compared with other advanced countries. However, the mechanism through which this home bias lowers the yields on JGBs is unclear. Indeed, while the large amount of domestic savings can lower the riskless component of these yields, any forces that reduce the risky component are not well known.

Hence, we construct a theoretical mechanism to explain why investors do not require a risk premium on government bonds under adverse fiscal conditions when there is a strong home bias. To understand the mechanism that links the home bias to the risk premium, we examine the features of JGB market. Table 1 details the portfolio characteristics of primary holders of JGBs in 2012. As shown, the largest bondholders are the central bank (the BoJ), private depository institutions, private insurance companies, public financial institutions, and social security funds, all of which are either owned or regulated by the Japanese government and considered to be “stable” holders\textsuperscript{5,6}. Of particular note in terms of home bias is that these large bondholders

\textsuperscript{2} This explanation is reasonable, but on the other hand, the pace of fiscal consolidation is too slow to maintain optimism among Japanese investors.

\textsuperscript{3} This monetary easing has been conducive to lowering long-term interest rates through the channel of forward guidance of future policy rates. For example, Oda and Ueda (2007) find that the zero interest-rate policy lowered the 10-year interest rate by approximately 0.3 percentage points. The quantitative impacts are significant but are not large enough to explain all low yields, at least until 2012.

\textsuperscript{4} A growing number of studies have investigated the determinants of home bias from both rational and behavioral perspectives. The determinants proposed by these extensive studies include transaction costs (Glassman and Riddick, 2001), foreign exchange risks (Fidora, Fratzscher and Thimann, 2007), information barriers (Ahearne, Griever and Warnock, 2004), corporate governance issues (Dahlquist et al., 2003), and lack of familiarity (Portes and Rey, 2005).

\textsuperscript{5} Public financial institutions include Japan Post Bank and Japan Post Insurance. Social security funds
hold small amounts of foreign assets in their portfolios.

Figure 2 depicts the nominal returns on 10-year government bonds for four countries, denominated in their own currencies. As shown, the returns on government bonds in the US, the UK, and Germany move similarly over the entire period, whereas the return on Japanese bonds is substantially and persistently lower. In line with uncovered interest rate parity, this is because domestic bondholders perceived a significant risk of an appreciation of the Japanese yen but of much larger magnitude than its actual movement. Hence, Japanese bondholders have overestimated the foreign exchange risk and, therefore, have failed to gain from the holding of a globally diversified portfolio of assets.

One reason for this is that safe assets are internationally limited; Gourinchas and Jeanne (2012) and Caballero and Farhi (2014) propose the notion of a shortage of safe assets, arguing that only some developed countries have the ability to supply safe assets. The number of safe assets available to Japanese investors is even more limited because of the flight to quality with the yen, given Japan’s enormous holding of foreign assets, such that the yen tends to appreciate when adverse shocks hit the global economy. Remarkably, the Japanese yen appreciated substantially relative to both the US dollar and the euro for several years following the recent global financial crisis.

Table 1: Portfolio characteristics of the primary holders of government bonds

<table>
<thead>
<tr>
<th>sector</th>
<th>market share of national debt</th>
<th>national debt/financial assets</th>
<th>foreign assets/financial assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central bank</td>
<td>11.6%</td>
<td>53.3%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Private depository institutions</td>
<td>21.5%</td>
<td>11.6%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Private insurance</td>
<td>16.4%</td>
<td>40.6%</td>
<td>15.8%</td>
</tr>
<tr>
<td>Public financial institutions</td>
<td>25.3%</td>
<td>31.0%</td>
<td>2.8%</td>
</tr>
<tr>
<td>Social security funds</td>
<td>8.7%</td>
<td>33.4%</td>
<td>16.9%</td>
</tr>
<tr>
<td>Private pension funds</td>
<td>3.5%</td>
<td>22.7%</td>
<td>20.1%</td>
</tr>
<tr>
<td>Households</td>
<td>3.0%</td>
<td>1.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Overseas</td>
<td>4.4%</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Others</td>
<td>5.6%</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: National debt comprises the central government securities and Fiscal Investment and Loan Program (FILP) bonds. Foreign assets comprise outward investment in securities.

Data source: National Accounts (Cabinet office).

include Government Pension Investment Funds (GPIF).

6 Onji, Kameda, and Akai (2012) propose the hypothesis that the presence of the public sector as a large holder of JGBs reduced the risk premium and estimate the effects of the change in the policy on the yields of JGBs.
The failure to achieve international risk diversification is key to understanding why Japanese investors have not looked for a higher risk premium on JGBs. If investors have easy access to foreign markets, they can hedge fiscal risk by holding foreign safe assets; otherwise, they would be obliged to hold only domestic assets. Unfortunately, domestic assets such as stocks and loans to domestic firms are also vulnerable to fiscal risk. Investors then face an “absence of safe assets” in the sense that they have no assets available for hedging fiscal risk. In this context, Sakuragawa and Sakuragawa (2016) construct a two-period model that explains low yields on government bonds under adverse fiscal conditions.

We extend the study by Sakuragawa and Sakuragawa (2016) to a dynamic framework and investigate fiscal sustainability and its macroeconomic implications. We construct a closed economy model of overlapping generations populated by two-period-lived agents. These agents hold two assets: government bonds and private capital. The central component of the model is that a fiscal crisis produces a negative externality on the return on capital. Agents have no access to any asset that hedges fiscal risk.
When there is an asset that hedges fiscal risk, the standard argument on asset pricing applies, namely, the yield on government bonds rises to reflect the default risk (e.g., Uribe, 2006). However, when there is no asset that hedges fiscal risk, the bond yield is not sensitive to the default risk. Intuitively, when fiscal default occurs, the return on government bonds declines as does the opportunity cost of holding the bonds. Given that bond yield does not sensitively react to default risk, the government can sustain a high level of debt under even quite serious fiscal conditions. This explains why Japan benefits from low government bond yields and why it has not experienced a fiscal crisis, despite its debt-to-GDP ratio being the highest in the OECD.

This model has several salient dynamic features. As debt increases, bond yield and the probability of default increase, reaching a debt limit (that is, the maximum sustainable debt), followed by default on the debt. The dynamic paths of debt and the bond yield differ depending on the presence of an asset to hedge fiscal risk. When capital is an alternative asset that hedges fiscal risk, the bond yield starts to rise at a low debt level, and the debt reaches the debt limit early. In contrast, when there is no asset for hedging fiscal risk, the bond yield rises only slowly, and it takes much more time to reach the debt limit. Fiscal default can then take place before any hike in bond yields.

This model also has several interesting macroeconomics implications. First, while the absence of safe assets allows the government to sustain a high level of debt, this sustainability arises only with the sacrifice of a reduction in the real interest rate. As long as the bond yield is insensitive to fiscal risk, the reduction in the real interest rate absorbs fiscal risk. The return on the stock market then negatively correlates with the level of outstanding debt.

Second, expansionary government expenditure financed by public debt can either stimulate or repress investment, but it increases the risk of default, followed by a reduction in the real interest rate. This contrasts with the results of IS–LM analysis in which the financing of government expenditure raises the real interest rate and crowds out capital investment.

Finally, reforms in financial markets and/or policies for enhancing growth positively affect investment, but worsen fiscal sustainability, and thus have an ambiguous effect on welfare. An increase in anxiety about fiscal balances can reduce the real interest rate, which in turn weakens the policy effects intended by the government and eventually leads to secular stagnation. The government should simultaneously
implement policies for enhancing economic growth and pursuing fiscal reforms.

This paper belongs to the literature that explains low real interest rates. Over the last two decades, we have witnessed the secular decline in real interest rates around the world. For instance, Caballero and Farhi (2014) use the notion of a shortage of safe assets backed by sound collateral to explain the low yields on US government bonds. In contrast, we treat government bonds as risky assets including default risk and explain the low yields on JGBs and the low real interest rates using the argument of the absence of safe assets. Barro (2006) explains low real interest rates by incorporating low-probability disasters in an asset pricing model, whereas Kitagawa (1994) explains the low interest rate from the perspective of a world of risky capital. This argument for low real interest rates is along the lines suggested by the extensive discussions on secular stagnation (e.g., Krugman, 2013, Summers, 2013).

Japan’s debt problem has been extensively studied from the perspective of fiscal sustainability. These studies include those by Doi and Ihori (2003), Dekle (2005), Broda and Weinstein (2005), Ihori et al. (2006), Sakuragawa and Hosono (2010, 2011), Doi, Hoshi, and Okimoto (2011), Hoshi and Ito (2013), Hansen and Imrohoroglu (2015), Matsuoka (2015), Miyazawa and Yamada (2015), and others. Many studies present a pessimistic scenario for fiscal sustainability in Japan. For example, Hansen and Imrohoroglu (2015) argue that to sustain its debt, the Japanese government must increase the consumption tax rate to 35 percent, a figure almost impossible to realize from a politico-economic perspective in Japan. In other study, Ghosh et al. (2013) compute the fiscal space, defined as the difference between projected debt limits and actual debt in advanced countries, and find that Japan has very little fiscal space.

Furthermore, a huge body of literature exists on sovereign default. Most of these studies examine the government’s strategic default in developing countries (e.g., Calvo, 1988, Cole and Kehoe, 2000, Arellano, 2008, and others). In contrast, we model fiscal default as an inability-to-pay problem which is triggered by a stochastic shock to the primary balance. We believe that inability to pay rather than strategic default is more likely to be relevant for analyzing public debt in countries where domestic residents hold almost all the debt (see Ghosh et al, 2013).

Our analysis also relates to several studies examining the overvaluation of asset prices or asset bubbles. Heterogeneous beliefs generated by overconfidence with short-sale constraints can account for bubbles in asset prices (e.g., Harrison and Kreps, 1978, Scheinkman and Xiong, 2003). Rational investors have an incentive to
exploit arbitrage opportunities when the asset is mispriced, thereby allowing for the overvaluation of prices relative to fundamentals to persist over several periods (De Long et al., 1990, Abreu and Brunnermeier, 2003). However, our finding contrasts with the findings of those studies in that we never attribute the seemingly overvalued bond price to heterogeneity in beliefs or short-sale constraints.

The remainder of the paper is organized as follows. Section 2 constructs our model, and Section 3 details its features. Section 4 provides some policy implications, and Section 5 discusses several related issues. Section 6 concludes the paper.

2. The Model

Consider an economy of overlapping generations populated by agents who are born and live for two periods. This economy is a closed economy in which agents do not have access to foreign markets. The young receive labor income $W$ in the form of an endowment, pay tax at rate $\tau$, and save the remaining income by holding $k$ units of capital and the one-period bond $d$ with a price $P_A(=1/(1+R_{t+1}))$. The young’s budget constraint is given by

\begin{equation}
(1-\tau)W = k + P_A d
\end{equation}

The old receive interest income and transfers, and consume.

The central component of the model is that the fiscal crisis produces a negative externality on the return on capital. The return on capital is $1+\theta$ in the nondefault state and falls to $(1-\theta)(1+\theta)$ in the default state, with $0 \leq \theta \leq 1$. Cohen and Sachs (1996), Cole and Kehoe (2000), and Arellano (2008) model fiscal default costs as having negative implications for output. We can think of $\theta$ as a no-risk hedge measure because $\theta$ captures the degree to which capital does not hedge fiscal risk.

The decline in output arises in several ways. As banks use government bonds as collateral for repo transactions among themselves, fiscal default leads to a shortage of

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7 Braun and Nakajima (2011) demonstrate that in an environment similar to theirs, the bond yield can be lower compared with the case of frictionless financial markets.

8 In recent fiscal crises, GDP declined by 15.9 percent in Argentina during 2000–2002 and 24.8 percent in Greece during 2010–2013. In general, fiscal default negatively impacts output accompanied by inflation, reductions in government expenditure, and the associated financial turmoil. Reinhart and Rogoff (2010), in analyzing 224 domestic crises, document that the declines in output around the period of a domestic debt crisis are significant, amounting to 8 percent on average.
liquidity in the interbank market, followed by a contraction of economic activity. The fiscal default forces the central bank to finance the deficit through money creation, which will trigger inflation and further harms the economy. The decline in bond prices also affects the balance sheet of banks holding government bonds, and forces those banks to improve their balances at the sacrifice of their lending to the private sector.

We now turn to the behavior of the government. Given the outstanding debt $d_t$, the government budget constraint is standard:

$$d_{t+1} - d_t = R_{t+1} d_t - s_{t+1}, \tag{2}$$

where $s_t$ is the primary surplus (tax revenue minus government expenditure). The government is committed to follow a fiscal reaction function:

$$s_{t+1} = \mu + f(d_t) + \varepsilon_{t+1}, \tag{3}$$

where the parameter $\mu$ captures the systematic determinants of the primary surplus balance other than the past debt, and $f(\cdot)$ is the response of the primary surplus to the debt, which is a continuously differentiable and nondecreasing function. The stochastic variable $\varepsilon_{t+1}$ follows the distribution function $G(\varepsilon_{t+1})$, which is continuously differentiable with a positive density defined over the interval $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$. The stochastic primary surplus reflects uncertainty over the tax revenues and/or unexpected government expenditure. The government levies tax $\tau_i W$ on the young.

We obtain the debt dynamics from (2) and (3) by

$$\text{(DD)} \quad \mu + f(d_t) + \varepsilon_{t+1} = (1 + R_{t+1}) d_t - d_{t+1},$$

where the left-hand side (LHS) is the fiscal surplus and the right-hand side (RHS) is the ”net” interest payment. To capture the idea of a fiscal crisis, the function has the property that there exists a debt $d^m$ such that, for any $d_t > d^m$,

$$\mu + f(d^m) + \varepsilon_{\text{min}} < (1 + R_{t+1}) d^m - d^m \text{ and } f'(d^m) < R_{t+1}, \tag{A}$$

given $R_{t+1}$. At $d_t = d^m$ and with the worst shock, the primary surplus cannot cover the interest payment; once the economy falls into this situation, the response of the primary balance is so weak that the government cannot escape from this difficulty.

We use a simple default rule: the government makes the promised payment if it can pay, otherwise it pays nothing. Behind the government’s default rule, we assume that if the government defaults on the debt, it returns the collected tax to the taxpayers\(^9\).
The government aims to maximize investor welfare and there is no conflict of interest between them. Investors who are domestic citizens hold all government bonds, and thus, the government has no incentive for strategic default.

For the default state, we assume that once default has occurred, the government cannot issue government bonds, at least during this period. The young’s budget constraint in the default state is given by $W = k$. Table 2 summarizes the returns on the government bonds and capital.

### 3. The Model Features

We first characterize the fiscal default. In doing so, we consider the “debt limit,” denoted $d_{t+1}$, as the maximum level of debt under which investors are willing to buy the bonds. For the present, we proceed to the analysis by assuming that there is a debt limit and that the government can roll over debt until it reaches this limit. If the shock $\varepsilon_{t+1}$ is so small that the debt that is necessary to cover the interest payment exceeds the debt limit, $\overline{d}_{t+1} < d_{t+1}$, the government cannot roll over the debt and must default.

Given the debt limit $\overline{d}_{t+1}$, there exists a cutoff variable $\varepsilon_{t+1}$ below which the debt that the government must issue to avoid default exceeds the debt limit:

$$\varepsilon_{t+1} = (1 + R_{t+1})d_t - \mu - f(d_t) - \overline{d}_{t+1} = \varepsilon(R_{t+1}, d_t, \overline{d}_{t+1}).$$

From (3) and (4), the default probability is defined by

$$\pi_{t+1} = \text{prob.}[\varepsilon_{t+1} < \varepsilon_{t+1}] = G(\varepsilon(R_{t+1}, d_t, \overline{d}_{t+1})).$$

which has the following properties: the higher the bond yield, the larger the current debt and the smaller the debt limit, the higher the default probability.

Agents should be indifferent to holding capital and bonds. The no-arbitrage condition is written as

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We focus on the case of full default. Examining the case of partial default is a promising direction of this research.
\( (1 - \pi_{t+1})(1 + R_{t+1}) = (1 - \pi_{t+1})(1 + A) + \pi_t(1 - \theta)(1 + A), \)

for \( d > 0 \). The LHS is the expected return on government bonds such that agents receive payment only if the government rolls over the debt. The RHS is the expected return on capital such that agents receive dividend \( 1 + A \) if the government rolls over the debt and receive \( (1 - \theta)(1 + A) \) if fiscal default occurs. The distinguishing feature is that a fiscal default produces a negative externality on the return on capital.

We rearrange (6) to yield the equation for the determination of the bond yield:

\[ 1 + R_{t+1} = (1 + A) + \frac{\pi_{t+1}}{1 - \pi_{t+1}}(1 - \theta)(1 + A). \]

The second term on the RHS represents the risk premium in terms of \( 1 + A \). As \( \theta \) increases, the risk premium tends to decrease for a given \( \pi_{t+1} \) and is less likely to reflect the default risk fully. This is because when a fiscal default occurs, the return on government bonds declines, as does the opportunity cost of holding bonds. At the extreme for \( \theta = 1 \), the bond yield does not react to the fiscal risk. At the other extreme for \( \theta = 0 \), (7) reduces to the standard expression, such that the bond yield fully reflects the risk.

In this model, there is no safe asset, so we define the real interest rate by the expected return on bonds, or equivalently, the expected return on capital. Letting \( r^*_{t+1} \) denote the real interest rate, it follows from (6) that

\[ (1 - \pi_{t+1})(1 + R_{t+1}) = (1 - \theta \pi_{t+1})(1 + A) = 1 + r^*_{t+1}. \]

The fiscal risk captured by the increase in the default probability \( \pi_{t+1} \) negatively correlates with the real interest rate.

We now turn to the formal analysis. We use (5) and (6) to define the net return function from government bonds by

\[ \Omega(R_{t+1}, d_n, \bar{d}_{t+1}, \theta) = (1 + R_{t+1}) \{ 1 - G(\varepsilon(R_{t+1}, d_n, \bar{d}_{t+1})) \} - (1 + A) \{ 1 - \theta G(\varepsilon(R_{t+1}, d_n, \bar{d}_{t+1})) \}, \]

for \( 0 \leq \theta < 1 \), where the first term is the return from government bonds and the second is the return from capital\(^{10} \). The function has three properties: (i) with the bond yield, the function at first increases, peaks, and then decreases——it has a maximum, suggesting that there may or may not be a bond yield at which agents are willing to

\(^{10}\) We exclude the case for \( \theta = 1 \). When \( \theta = 1 \), \( \Omega(\cdot) = (R_{t+1} - A)(1 - G(\cdot)) \) holds. Then \( \Omega(\cdot) = 0 \) if and only if \( R_{t+1} = A \).
buy government bonds; (ii) the function decreases as the actual debt increases; and 
(iii) the function increases as the debt limit increases. When there exists any bond 
yield satisfying $\Omega(\cdot)>0$, there are two bond yields that satisfy $\Omega(\cdot)=0$, as illustrated in 
Figure 3\textsuperscript{11}. Among these two, the lower bond yield is chosen as the equilibrium, but the 
higher bond yield is not\textsuperscript{12}. Given the actual debt $d_t$ and the debt limit $\overline{d}_{t+1}$, there is an 
$R_{t+1}$ that satisfies 
\begin{equation}
\Omega(R_{t+1}, d_t, \overline{d}_{t+1}, \theta)=0,
\end{equation}
as long as $d_t<\overline{d}_t$. Accordingly, the default probability is given by 
\begin{equation}
\pi_{t+1}=G\{((1+R_{t+1})d_t-\mu-f(d_t)-\overline{d}_{t+1})\}.
\end{equation}
Conversely, when $\Omega(\cdot)<0$, for any $R_{t+1}$, there is no bond yield at which agents are 
willing to lend to the government. As is familiar from the literature on equilibrium 
credit rationing (e.g., Stiglitz and Weiss, 1981, Williamson, 1986), agents are never 
willing to hold government bonds irrespective of the bond yield. We now establish the 
determination of the bond yield.

\textbf{Fig. 3: The net return function}

\textsuperscript{11} This function is globally concave in terms of $R$ if the functional form on $G(\cdot)$ satisfies $2G(\cdot) + G'(\cdot)(R-A)>0$. Specifically, this restriction is not necessary for all the main results to hold. Without this assumption, the number of equilibria may be more than three, and the analysis will be a little complicated.

\textsuperscript{12} See the explanation in Section 3 of the study by Sakuragawa and Sakuragawa (2016) for the proof.
**Proposition 1**: (i) There exists a finite bond yield at which the government can issue debt for \( d \in [0, \bar{d}] \), and (ii) There exists no finite bond yield at which the government can issue debt for \( d \in (\bar{d}, +\infty) \).

The first property implies that if debt remains at or below the debt limit, agents are willing to lend to the government at a positive finite bond yield. The second property implies that if debt exceeds its limit, there is no finite bond yield that can compensate for the default risk to agents. As a result, the government faces a complete loss of access to the bond market.

We now turn to the determination of the debt limit \( \bar{d}_{t+1} \). Consider the function \( \Omega(\overline{R}_{t+1}, d_t, \bar{d}_{t+1}, \theta) \) when \( d_t \) reaches \( \bar{d}_{t+1} \). By construction, for the debt limit to exist, there are some large \( \bar{d}_{t+1} \), above which \( \Omega(\overline{R}_{t+1}, \bar{d}_{t+1}, \bar{d}_{t+1}, \theta) \) is decreasing in \( \bar{d}_{t+1} \), or equivalently,

\[
(B) \quad R_{t+1} - f'(\bar{d}_{t+1}) > 0.
\]

Condition (A) guarantees that the debt limit satisfies (B) for any \( \bar{d}_{t+1} > d^m \). There exists a debt limit \( \bar{d}_{t+1} \) and a “bond yield limit” \( \overline{R}_{t+1} \) that satisfy the following two equations:

\[
(12) \quad \Omega(\overline{R}_{t+1}, \bar{d}_{t+1}, \bar{d}_{t+1}, \theta) = 0, \quad \text{and}
\]

\[
(13) \quad \partial \Omega(\overline{R}_{t+1}, \bar{d}_{t+1}, \bar{d}_{t+1}, \theta) / \partial R_{t+1} = 0.
\]

Under Condition (A), the \( \Omega(\cdot) \) function is decreasing in \( \bar{d}_{t+1} \). As shown in Figure 4, at the debt limit \( \bar{d}_{t+1} \), the bond yield limit is the unique yield at which agents are willing to

**Fig. 4**: Bond yield and debt limits

![Figure 4: Bond yield and debt limits](image-url)
hold the bonds. These two variables are time invariant; hereafter, we denote \( \overline{d}_{t+1} = \overline{d} \) and \( \overline{R}_{t+1} = \overline{R} \) without the time subscript. The default probability at the debt limit, denoted as \( \pi = G(\overline{d}) \), is generally less than unity so that if the debt were to increase beyond the limit \( \overline{d} \), the default probability would suddenly jump to unity.

We define the temporary equilibrium in period \( t \) by a set of variables \{s_t, d_t, k_t, R_{t+1}, \pi_{t+1}\} that satisfies five equations, (1), (2), (3), (10), and (11), given \( \tau_t \) and the fiscal shock \( \varepsilon_t \). At the beginning of period \( t \), \( d_{t-1} \) and \( R_t \) have been already determined. Once the fiscal shock \( \varepsilon_t \) is revealed, default occurs if \( \varepsilon_t < \overline{\varepsilon}_t \), otherwise default does not occur. When default occurs, the economy shifts to the regime without debt. When default does not occur, (2) and (3) determine the fiscal surplus \( s_t \) and the new debt \( d_t \). Equation (10) determines the bond yield \( R_{t+1} \). Equation (11) determines the default probability \( \pi_{t+1} \). Once \( d_t \) and \( R_{t+1} \) are determined, (1) determines \( k_t \).

We now investigate several features of the model. Consider first the effects of an increase in debt.

**Proposition 2**: When debt \( d_t \) increases, the bond yield \( R_{t+1} \) and the default probability \( \pi_{t+1} \) increase and the real interest rate \( r_{t+1}^* \) decreases for \( d_t \in (0, \overline{d}] \).

**Proof**: See the Appendix.

The relation among debt, bond yield, and default probability is a standard one. Additionally, debt negatively relates to the real interest rate. The increase in debt followed by a high default probability reduces the expected return on capital due to the negative externality. We next investigate the effects of a change in the no-risk hedge measure \( \theta \).

**Proposition 3**: As the no-risk hedge measure \( \theta \) increases, the bond yield \( R_{t+1} \) and the default probability \( \pi_{t+1} \) decrease for \( d_t \in (0, \overline{d}] \) and the debt limit \( \overline{d} \) increases.

**Proof**: See the Appendix.

As the no-risk hedge measure \( \theta \) increases, the bond yield tends to be lower given the debt level. This arises because when there is fiscal default, the return on the government bond declines as does the opportunity cost of holding these bonds.

This finding suggests that the dynamics of debt and bond yield can differ across...
economies depending on the no-risk hedge measure $\theta$. This explains why Japan, even with the OECD’s highest debt-to-GDP ratio, enjoys low bond yields and no experience to date of a fiscal crisis. It also explains why several euro countries have suffered from fiscal difficulties with much lower debt-to-GDP ratios than that of Japan. The parameter $\theta$ for Japan is supposedly high in that many large bondholders are domestic financial intermediaries lacking the superior knowledge and skills needed to manage international portfolios. In contrast, $\theta$ in the euro countries is supposedly low in that high-debt countries such as Italy, Spain, Portugal, and Greece have adopted the common euro currency, freeing their investors of the foreign exchange risk associated with buying the safe assets of other euro countries.

Figure 5 plots bond yields as debt increases for three different values of $\theta$ (0, 0.5, and 0.8). We use the uniform distribution function $G(\varepsilon) = (\varepsilon + 0.5\varepsilon^m) / \varepsilon^m$ over the interval $[-0.5\varepsilon^m, 0.5\varepsilon^m]$, with the mean $E[\varepsilon] = 0$, and the response function $f(d) = \rho d$. The chosen parameters are $\varepsilon^m = 10$, $A = 0.02$, $\rho = 0.01$, and $\mu = 5$. When $\theta$ is zero, the yield starts to rise at a low debt level and then the debt reaches the debt limit $d_{\theta=0}$. As $\theta$ increases, the yield rises only after some large debt level has been reached and the debt limit becomes large. In this example, the debt limit for the high-$\theta$ ($\theta = 0.8$) economy is about four times larger than that for the zero-$\theta$ economy. The absence of safe assets contributes to the high level of sustainability. The bond yield limits also

![Fig.5: The bond yields for different $\theta$'s](image_url)
differ. In fact, the yield limit of 12.1 percent for the zero-$\theta$ economy is nearly double that of the 6.5 percent for the high-$\theta$ economy. In our simulation, a low yield limit accompanies a large debt limit. Fiscal default can then take place before any hike in bond yield.

The next concern is to understand the relation between bond yield and real interest rate. Figure 6 illustrates the relation among bond yield, real interest rate, and default probability when $\theta = 0.5$. As shown, the bond yield does not rise sufficiently as the default probability increases but the real interest rate declines. While the bond yield remains low, the reduction in the real interest rate absorbs the fiscal risk. As $\theta$ increases, this tendency becomes even stronger such that the bond yield increases slightly and the real interest rate declines significantly.

Caballero and Farhi (2014) uses the notion of a shortage of safe assets backed by sound collateral to explain the low yields on US government bonds. In contrast, we treat government bonds as risky assets with default risk and explain the low yields on JGBs by relying on a mechanism in which the bond yield is insensitive to fiscal risk. Instead, the reduction in the real interest rate absorbs the fiscal risk in an economy without safe assets. Our finding relates to the study by Kitagawa (1994) in explaining the low real interest rate in a world of risky capital.

The finding that the real interest rate declines as fiscal risk increases suggests that the stock market return negatively correlates with the outstanding debt to the extent that bond yields are insensitive to the fiscal risk. We can define the stock price as

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**Fig. 6 : The relation among bond yield, real interest rate, and default probability**

$R, r^*, \pi$

Note: The chosen parameters are the same as the case for $\theta = 0.5$ in Figure 5.
which is equal to unity, as in the one-good production framework. However, the observed "price dividend" ratio, given that fiscal default has not occurred, is $Q/(1+A) = (1-\theta \pi_{t+1})/(1+r^n_{t+1})$, which makes investors believe that the stock price is undervalued. When we observe two economies with the same bond yields but different debt levels, observing stock market prices may reveal what is actually going on.

**Welfare**

A low bond yield is desirable from the viewpoint of fiscal sustainability, but it reduces the real interest rate when the debt is large. We examine the welfare of a generation using the expected consumption in old age:

$$W = (1-\tau)(1+r^n_t)W.$$  

Here, welfare decreases as the debt $d_t$ increases (Proposition 2) or the tax rate $\tau$ increases. Fiscal reform can then have an ambiguous effect on welfare. The trade-off is stringent when the actual debt is close to the debt limit. An increase in tax can reduce after-tax labor income, which is a source of worsening welfare, while a rise in the real interest rate follows the debt reduction, which is a source of improving welfare. Figure 7 plots the effects of the tax reform on welfare when $\theta = 0.5$. The horizontal axis represents the debt level before the reform. The bold unbroken line plots the case

![Fig.7: Tax reform and welfare](image)

Note) The chosen parameters are the same as the case for $\theta = 0.5$ in Figure 5.
before the reform \( (\tau_i=0) \). As shown, welfare starts to decline at about \( d_i = 14.94 \) and reaches a minimum at a debt limit of \( d_i = 15.086 \). The bold dashed line plots the case for a tax reform where \( \tau_i = 0.01 \). When debt is small, post-reform welfare is lower; but as the debt goes beyond \( d_i = 14.98 \), welfare is higher. The thin dashed line plots the case for reform where \( \tau_i = 0.02 \). As shown, the reform gain becomes larger as debt approaches its limit.

4. Policy Implications

Our analysis provides several implications for fiscal sustainability and the macroeconomy. In this section, we select expansionary fiscal policy, financial market reform, and growth-enhancing policies for consideration.

*Expansionary Fiscal Policy*

Consider the effect of government expenditure financed by government debt. Assume that labor income increases as government expenditure \( g_i \) increases. This is motivated by the idea that government expenditure yields positive externalities on the aggregate demand and output. The young’s budget constraint is rewritten as

\[
(1-\tau_i)W(g_i) = k_i + P_id_i,
\]

where \( W'(g_i)>0 \). Fiscal policies influence capital through three channels. First, an increase in government expenditure decreases the surplus \( s_i \) and increases debt \( d_i \), which in turn has a crowding-out effect on capital. Second, the increase in debt increases the default probability followed by a decrease in the bond price, which has the effect of mitigating this crowding-out. Finally, the externality on output positively affect stimulating capital. The overall effects of these three channels on capital are ambiguous. However, as Proposition 2 states, the increase in debt \( d_i \) will decrease the real interest rate.

**Proposition 4:** When there is an increase in government expenditure financed by government debt, capital can increase or decrease, the bond yield \( R_{i+1} \) increases, but the real interest rate \( r_{i+1} \) decreases.

In a world of no safe assets, a decrease in the real interest rate follows expansionary fiscal policy. This finding contrasts sharply with Hick’s mechanism in the famous
IS–LM analysis, in which fiscal policy increases the interest rate that crowds out investment.

Financial Market Reform

In Japan, the government either owns or regulates several large holders of JGBs; hence, regulations and misguided governance may induce them to hold larger amounts of government bonds. We describe this environment as one where there are barriers that prevent investors from holding capital. Assume that a proportion $\eta$ of the return on capital is lost because of these barriers. In this environment, we can rewrite the no-arbitrage condition between bonds and capital as

$$\left(1 - \pi_{t+1}\right)\left(1 + R_{t+1}\right) = (1 - \theta \pi_{t+1})(1 - \eta)(1 + A).$$

We then describe financial market reform as a reduction in $\eta$. Several effects of the reform are similar to those of a reduction in $\theta$. For example, the bond yield and default probability are higher, and the debt limit is lower (Proposition 3). The higher bond yield reduces the bond price, which in turn allows capital to increase. The direction of the real interest rate is ambiguous; the higher debt probability tends to reduce the real interest rate, while the higher return on capital tends to increase the real interest rate. The direction of the effect on welfare is also ambiguous.

Financial market reform contributes to stimulating investment in capital but worsens fiscal sustainability. Welfare may be better or worse. Specifically, given that the debt is already close to the debt limit, the reform may make it difficult or impossible to sustain the debt, and this will make agents worse off. The government then must understand the potential trade-off between investment boom and fiscal difficulty in conducting financial market reform.

Growth-Enhancing Policy

We describe the policy for enhancing economic growth as an increase in the return on capital $A$. The effects are similar to those of financial market reform. When the

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13 Private depository institutions are subject to international standard for capital requirements and have a strong incentive to hold government bonds excessively because of the zero risk weight for evaluating the asset risk. Insurance companies are regulated to eliminate the maturity mismatch of the balance sheet given the solvency margin ratio and to induce investors to hold long-term public bonds excessively. Japan Post Bank was privatized in 2005 but still has a weak governance structure in its asset portfolio and holds more than 80 percent of public debt in its total assets. Owing to the weak governance structure, GPIF holds more than half of its assets in the form of public debt and is currently being reformed.
government implements a growth-enhancing policy, capital, the bond yield, and the default probability increase and the debt limit declines. The direction of its effect on the real interest rate and welfare are ambiguous. When the policy increases anxiety about fiscal sustainability, the stock market may remain stagnant. The government should then propose a policy package that advances economic growth and fiscal sustainability at the same time.

5. Discussion

Several related issues require further discussion.

*The Role of Monetary Easing*

We have thus far investigated one possible mechanism that explains the low yields of JGBs. It would be meaningful to discuss the role of monetary policy as one of other channels for explaining the low yields. As of 2016, the BoJ has been conducting QE that involves large-scale purchases of long-term government bonds. Since the beginning of this policy implementation, the nominal yield on 10-year government bonds has declined from 0.9 percent to 0.4 percent as of 2015 end. Certainly, the QE policy has contributed to lowering yields on JGBs.

The BoJ is in a special position relative to private investors in that it does not aim to earn profits from holding JGBs. Assume that the central bank does not request positive yields and purchases bonds at a higher price than the market price. When the central bank holds a share $\phi (0<\phi<1)$ of the outstanding debt, the government budget constraint is rewritten by

$$d_{t+1} - d_t = (1-\phi)R_{t+1} d_t - s_{t+1}.$$  \hspace{1cm} (18)

This expression states that the government can economize on the expense of bond yields to the extent that the central bank holds the bonds. Taking other things as given, as $\phi$ increases, the bond yield and the default probability decline. QE can improve fiscal sustainability and play a complementary role in strengthening the effects of financial market reforms and growth-enhancing policy. However, this policy is only effective if people value the money issued in exchange of purchasing bonds.

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14 From a theoretical perspective, the effects of open market operations of this kind are neutral in standard macroeconomic models unless there are restrictions on asset trading. Wallace (1981) was the first to indicate this neutrality result.
The Setting of a Closed Economy

We use the framework of a closed economy because this setting well approximates the financial environment surrounding the JGB market, in which primary market participants find it costly to access foreign markets. However, our intention is not to suggest that investors would hold domestic assets for any real interest rate; in fact, there may be a threshold level of real interest rate under which a flight to foreign assets would arise. As shown in Figure 6, as the fiscal condition deteriorates, the real interest rate falls. Domestic investors would switch to an international portfolio at some instance, which would accelerate the increase in the default probability and the bond yield. The central bank would then attempt to avoid any fiscal difficulty by purchasing JGBs directly; however, the effectiveness of that policy depends on how people continue to hold the money issued as a means of purchasing these bonds. Both money and bonds are government liabilities, and it may be difficult to imagine that in the face of a fiscal default, government bonds would become valueless, but money would retain its confidence. In particular, if people perceive the return on money to be lower than the threshold interest rate, nobody will hold money and the direct purchases will lead only to fiscal default followed eventually by hyperinflation.

People’s Beliefs about Rare Risk

What is happening in the model is that when people perceive a massive economic downturn in the face of fiscal default, they find government bonds attractive assets relative to real assets, which in turn enables the government to roll over the debt and delays the period of fiscal default.

Some readers might question if in reality investor expectations are formed this way. What makes things complicated is that both optimistic and pessimistic beliefs coexist about the pricing of JGB yields. On the one hand, people may be optimistic about fiscal sustainability from the low bond yields on JGBs. On the other hand, they may hold a pessimistic belief that conjecturing from the rapidly increasing debt–GDP–ratio, the government will default on its debt in the future with high probability and that the fiscal default will lead to the end of the Japanese economy. Our approach goes only one step toward describing people’s beliefs in some consistent and simple manner. A natural next step would be to consider how people form expectations in the face of rare risk that they have not experienced15.
**Determinants of the No–Risk Hedge Measure \( \theta \)**

We now discuss the foundation of the no–risk hedge measure \( \theta \). The determinants of \( \theta \) include not only domestic but also international factors. The domestic factors include regulations for capital control and the bank’s skill necessary to attain an internationally diversified portfolio. Regulations and misguided governance of domestic financial institutions can strengthen financial autarky and increase the value of \( \theta \). International factors include the exchange rate regime and extent of a firm’s globalization. The value of \( \theta \) is high when domestic and foreign asset returns move procyclically, while it is low when they move countercyclically. The Japanese yen also plays a role in the flight to quality and appreciates in response to adverse global shocks. The value of \( \theta \) in Japan is supposedly high. The international location of domestic companies also influences the value of \( \theta \). For example, multinational companies would suffer a small damage from the fiscal crisis; thus, a country that possesses relatively many of these companies would have a small \( \theta \).

6. Conclusion

We analyze a mechanism that explains the low yields of JGBs given a high level of debt and provide macroeconomic implications. The key idea is that if fiscal default leads to an overall reduction in asset returns, domestic investors fall into a situation of having no assets for hedging fiscal risk. In the absence of safe assets, government bonds become more attractive relative to other real assets, and this enables the government to roll over its debt and delay the period of fiscal default. This explains why bond yields do not react sensitively to default risk. However, while the government can sustain a high level of debt, it is only sustainable given a reduction in the real interest rate, which in turn weakens policy effects and ultimately leads to secular stagnation.

**Appendix : Proof of Propositions 2 and 3**

Equations (12) and (13) are continuous in \( \bar{R}_{t+1}, \bar{d}_{t+1}, \) and \( \theta \), and there exist time invariant continuous functions \( \bar{R}_{t+1}=\bar{R}(\theta) \) and \( \bar{d}_{t+1}=\bar{d}(\theta) \) that satisfy \( \Omega(\bar{R}(\theta), \bar{d}(\theta), \bar{d}(\theta), \theta)=0 \) and \( \partial \Omega(\bar{R}(\theta), \bar{d}(\theta), \bar{d}(\theta), \theta)/\partial \bar{R}_{t+1}=0 \).

Differentiating (12) totally and using (13), we obtain

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\[
\frac{dd}{d\theta} = -\frac{\partial \Omega}{\partial \theta} \frac{\partial \Omega}{\partial d} \left|_{d=\pi} \right. > 0.
\]

where \( \frac{\partial \Omega}{\partial \theta} = (1 + A) \pi > 0 \) and

\[
\frac{\partial \Omega}{\partial d} \bigg|_{d = \pi} + \frac{\partial \Omega}{\partial d} = -(1 + \bar{R}) - \theta (1 + A) \frac{G'(\varepsilon)}{\bar{R} - f'(d)} < 0 \quad \text{because} \quad (1 + \bar{R}) > \theta (1 + A)
\]

from (8) and \( \bar{R} > f'(d) \) from Condition (A). This establishes the third part of Proposition 3.

We can rewrite (10) as \( \Omega(R_{t+1}, d, \bar{d}(\theta), \theta) = 0 \). There exists a continuous function \( R_{t+1} = R(d, \theta) \) that satisfies \( \Omega(R_{t+1}, d, \bar{d}(\theta), \theta) = 0 \). Totally differentiating the latter equation, we obtain

\[
dR = -\frac{\Omega_s}{\Omega_K} dd - \frac{\Omega_s}{\Omega_K} d\theta - \frac{\Omega_s}{\Omega_K} d\theta
\]

\[
= -\frac{\Omega_s}{\Omega_K} dd + \left\{ \frac{\Omega_s}{\Omega_K} \frac{\partial \Omega}{\partial \theta} \bigg|_{d=\pi} + \frac{\partial \Omega}{\partial d} \right\} d\theta,
\]

where \( \Omega_s = \frac{\partial \Omega(R_{t+1}, d, \bar{d}, \theta)}{\partial i} (i = R_{t+1}, d, \bar{d}, \theta) \), and \( \Omega_K = 1 - G(\varepsilon) - \{(1 + R_{t+1}) - \theta (1 + A)\} G'(\varepsilon) \bar{d} \) should be positive at the equilibrium. \( \Omega_d = -\{(1 + R_{t+1}) - \theta (1 + A)\} G'(\varepsilon) \{1 + R_{t+1} - f'(d_i)\} < 0 \), \( \Omega_x = \{(1 + R_{t+1}) - \theta (1 + A)\} G'(\varepsilon) \bar{d} > 0 \), and \( \Omega_x = (1 + A) G(\varepsilon) \bar{d} > 0 \). As the actual debt increases, the no-risk hedge measure increases, the bond yield decreases. This establishes the first part of Proposition 2. As the no-risk hedge measure increases, the bond yield decreases. This establishes the second part of Proposition 2. As \( \theta \) increases, the default probability decreases. This establishes the second part of Proposition 3.

We can rewrite (11) as \( \pi_{t+1} = G\{(1 + R(d, \theta)) d_i - \mu - f(d_i)\} = \pi(d, \theta) \). We have

\[
\frac{\partial \pi_{t+1}}{\partial d_i} = G'(\varepsilon) \left\{ \frac{\partial R_{t+1}}{\partial d_i} d_i + (1 + R_{t+1}) - f'(d_i) \right\} > 0 \quad \text{and} \quad \frac{\partial \pi_{t+1}}{\partial \theta} = G'(\varepsilon) \left\{ \frac{\partial R_{t+1}}{\partial \theta} d_i - \frac{\partial \bar{d}}{\partial \theta} \right\} < 0.
\]

As the actual debt increases, the default probability increases. This establishes the second part of Proposition 2. As \( \theta \) increases, the default probability decreases. This establishes the second part of Proposition 3.

Finally, we can rewrite (8) as \( 1 + r^*_{t+1} = (1 - \theta \pi(d, \theta))(1 + A) \). We have

\[
\frac{\partial (1 + r^*_{t+1})}{\partial d_i} = -\theta \frac{\partial \pi_{t+1}}{\partial d_i} (1 + A) < 0.
\]

As the actual debt increases, the natural interest rate decreases. This establishes the third part of Proposition 2. Q.E.D.
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