Exchange Rate Determination in the EMS:  
An Econometric Model

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Exchange rate determination in macroeconometric models has so far been discussed and implemented largely on a country by country basis, and to the best of our knowledge there has been no multi-country or world econometric model that explicitly incorporates the EMS exchange rate arrangement. This paper reports an attempt to fill the gap by extending the EPA World Economic Model developed at the Economic Planning Agency of Japan.

This model is consisted of nine individual country models (seven "Summit" countries plus Australia and Korea) and one regional model, which are linked together by a trade linkage model.\(^1\) In the current version, exchange rates of European countries are endogenized by what we call the Flex method, and they are determined as if these countries were floating independently against the US dollar. Briefly speaking, the Flex method of exchange rate determination may be characterised as an application of the stock/flow interaction model a la Niehans (1977). There is no exchange rate equation in a country model, however, and exchange rates are solved for implicitly by an interactive procedure so as to satisfy general equilibrium.

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\(^1\) For the EPA World Economic Model, see Amano, Sadahiro, and Sasaki (1981) and Amano, Maruyama, and Yoshitomi (1982).
In the solution process, therefore, possible limitations to exchange rate variations can be imposed.

For example, let $S_i$ be the spot exchange rate of country $i$, i.e., the price of US dollar in terms of country $i$’s currency units, and $B_i$ be the permissible limit to quarter-to-quarter exchange rate movements for country $i$’s dollar-rate. Then, the regular Flex solution procedure can impose the following constraints:

\[(1) \quad (1 - B_i) \cdot S_i(t-1) \leq S_i(t) \leq \frac{1}{(1 - B_i)} \cdot S_i(t-1).\]

Under the managed floating-rate regime, $B_i$ is usually set sufficiently large (say, 10 to 15 per cent per quarter) so that the above constraints are not binding; but it appears that $B_i$ assumed rather low values once in a while, especially in an earlier stage of generalized floating, reflecting heavy official interventions in foreign exchange markets.3)

Now, it may seem a straight-forward application for the Flex method to incorporate the EMS constraints on the cross-rates of European currencies. Let $C_{ij}$ be the central rate in the EMS arrangement expressed as the price of currency $j$ in terms of currency $i$, and $B_{ij}$ be the permissible exchange rate band between the two currencies. Then, variations of cross-rates must be limited according to the conditions:

\[(2) \quad (1 - B_{ij}) \cdot C_{ij} \leq \frac{S_i}{S_j} \leq \frac{1}{(1 - B_{ji}) \cdot C_{ji}}.\]

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2) This type of approach is called the structural balance of payments approach by Hooper, Haas, Symansky, and Stekler (1982).

3) For details of the Flex method, see Amano (1981).
Alternatively, let

\begin{align*}
\text{BL}_t &= \max \{ (1 - B_i) \cdot S_i(t-1), \\
&\quad \max_{j \neq i} \{ (1 - B_{ij}) \cdot C_{ij} \cdot S_j(t) \} \} \\
\text{BU}_t &= \min \{ (1/(1 - B_i)) \cdot S_i(t-1), \\
&\quad \min_{j \neq i} \{ 1/((1 - B_{ij}) \cdot C_{ij}) \cdot S_j(t) \} \}.
\end{align*}

Then, \( S_t \) must satisfy

\begin{equation}
\text{BL}_t \leq S_t \leq \text{BU}_t.
\end{equation}

Thus, modifying conditions (1) of the Flex solution procedure as (4), we may be able to impose the EMS constraints on the European exchange rates.

Unfortunately, however, this method has one shortcoming as illustrated in Figure 1. Suppose there are only two EMS currencies (1 and 2). Their dollar-rates are measured along the two axes in Figure 1. The cone designates the area in which the two dollar-rates must lie to accord with the EMS arrangements. In the two attached figures are drawn excess supply curves of foreign exchange, \( E_i \), for relevant countries. (For simplicity of exposition, we assume that the position of \( E_i \) is independent of the level of exchange rate in the other country.) Should both countries float independently, the dollar-rates would be determined at \( Q \), which lies outside the EMS cone. How can we, then, contain them within the cone? In the EPA World Economic Model, each country model is solved iteratively one by one in a larger iteration loop until all endogenous variables in the system converge. In Figure 1, let \( I \) represent the initial estimates. Then, if country 1 is solved first, constrained equilibrium rates of the current period will converge to point
A. On the other hand, if country 2 is solved first, they will converge to point B. Clearly, starting from the initial estimates, currency 1 must appreciate whereas currency 2 must depreciate; and, as far as the maintenance of parity grids is concerned, the rate of depreciation of currency 2 can be smaller as currency 1 appreciates more. The present method cannot determine how to divide the adjustment pressure between the two currencies.

We can avoid this sort of indeterminacy by modeling the actual practice of the EMS system a little more closely.
Let $A_i$ be the amount of currency $i$ contained in one unit of ECU (European Currency Unit). Column 1 of Table 1 shows the actual number of currency units in the ECU basket. Since the EPA World Economic Model involves only the first four European countries, we constructed a mini-ECU, which we shall hereafter call "ecu", by using the relative weights of these four currencies computed from parities as of March 13, 1978. Table 1 indicates the method of constructing ecu, and the last column shows the number of currency units included in it. Note that Column 2 is used only to calculate Column 5, and actual parities are used in model simulations.

Now, the price of US dollar in terms of ecu, $S_{ecu}$, is determined by

\begin{equation}
S_{ecu} = \frac{1}{(\sum_i A_i/S_i)}
\end{equation}

<table>
<thead>
<tr>
<th>Currency</th>
<th>Number of currency units in ECU basket</th>
<th>Parity as of March 13, 1978 (Units per ECU)</th>
<th>Relative weight</th>
<th>Relative weight of the first four currencies</th>
<th>Number of currency units in mini-ECU basket</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.M.</td>
<td>0.828</td>
<td>2.51064</td>
<td>0.329796</td>
<td>0.435950</td>
<td>1.0945</td>
</tr>
<tr>
<td>Stg. £</td>
<td>0.0885</td>
<td>(0.663247)</td>
<td>0.133434</td>
<td>0.176384</td>
<td>0.1170</td>
</tr>
<tr>
<td>F.Fr.</td>
<td>1.15</td>
<td>5.79831</td>
<td>0.198337</td>
<td>0.262173</td>
<td>1.5202</td>
</tr>
<tr>
<td>Lit.</td>
<td>109.</td>
<td>1148.15</td>
<td>0.094935</td>
<td>0.125493</td>
<td>144.09</td>
</tr>
<tr>
<td>D.Gls.</td>
<td>0.286</td>
<td>2.72077</td>
<td>0.105117</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.Fr.</td>
<td>3.80</td>
<td>39.4582</td>
<td>0.096304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D.Cr.</td>
<td>0.217</td>
<td>7.08592</td>
<td>0.030624</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. £</td>
<td>0.000759</td>
<td>0.662638</td>
<td>0.011454</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
so that if we substitute the unconstrained Flex solution values, $S_i^u$, for $S_i$, we obtain the ecu/dollar rate $S_{ecu}^u$. In Figure 1, the curve U passing through Q shows the combination of $S_i$ and $S_2$ that makes $S_{ecu} = S_{ecu}^u$.

Next, let $P_i$ be the parity rate of currency i against ecu, and compute

\[ S_i^0 = P_i \cdot S_{ecu}^u. \]

Since $C_{ij} = P_i/P_j$ and $\Sigma_i A_i/P_i = 1$, the intersection of curve U and the ray from the origin, $C_{2j}$, gives $S_i^0$ and $S_j^0$ at point R. We then search equilibrium exchange rates in the following way.

For the EMS member countries (Germany, France, and Italy in our model) additional upper and lower limits to their dollar-rates are imposed in each iteration:

\[ (7) \quad BL_i = (1 - \text{MU}_i) \cdot S_i^0 \]
\[ BU_i = (1 + \text{MU}_i) \cdot S_i^0 \]

where MU$_i$ is initially set equal to 0.01 and is adjusted after every iteration unless conditions (2) are satisfied for all $j \neq i$.

The adjustment rules for MU$_i$ in the k-th iteration are as follows:

(i) If $S_i$ is smaller than the permissible lower limit, i.e., if

\[ S_i < \max_j \left\{ (1 - B_{ij}) \cdot C_{ij} \cdot S_j \right\}, \]

then MU$_i$ is reduced according to

\[ (8a) \quad \text{MU}_i^{(k)} = \text{MU}_i^{(k-1)} - \beta \left\{ \max_j \left\{ (1 - B_{ij}) \cdot C_{ij} \cdot S_j \right\} / S_i - 1 \right\}, \]
where $\beta$ is an adjustment coefficient to be determined experimentally to ensure smooth convergence. (In simulations to be reported later, we set $\beta = 0.5$.)

(ii) If $S_i$ exceeds the permissible upper limit, i.e., if

$$S_i > \min_{j \neq i} \left[ \frac{S_j}{(1 - B_{ji}) \cdot C_{ji}} \right],$$

then $MU_i$ is reduced according to

$$(8b) \quad MU_i^{(k)} = MU_i^{(k-1)} - \beta \left\{ 1 - \min_{j \neq i} \left[ \frac{S_j}{(1 - B_{ji}) \cdot C_{ji}} \right] / S_i \right\}.$$ 

(iii) If $S_i$ exceeds the lower limit with a positive excess supply of foreign exchange, i.e., if

$$S_i > \max_{j \neq i} \left[ (1 - B_{ij}) \cdot C_{ij} \cdot S_j \right]$$

and

$$E_i > 0,$$

then $MU_i$ is increased according to

$$(8c) \quad MU_i^{(k)} = MU_i^{(k-1)} + \beta \left\{ S_i / \max_{j \neq i} \left[ (1 - B_{ij}) \cdot C_{ij} \cdot S_j \right] - 1 \right\}.$$ 

(iv) If $S_i$ is smaller than the permissible upper limit with a negative excess supply of foreign exchange, i.e., if

$$S_i < \min_{j \neq i} \left[ \frac{S_j}{(1 - B_{ji}) \cdot C_{ji}} \right]$$

and

$$E_i < 0,$$
then $MU_i$ is increased according to

$$(8d) \quad MU_i^{(k)} = MU_i^{(k-1)} + \beta \left\{ \min_{j \neq i} \left[ S_j / ((1 - B_{ji}) \cdot C_{ji}) \right] / S_i - 1 \right\}.$$ 

The iteration for each period terminates when the $MU_i$'s and other endogenous variables of the system converge to stationary values.

In terms of Figure 1, we start with a small box one corner of which is fixed at $R$. Then, the width and the height of the box are adjusted simultaneously in such a way that the other, diagonal corner of the box comes closer and closer to $Q$ until it hits an edge of the cone. Although we have not formally modeled the divergence indicator system of the EMS, the above method of adjustment incorporates one essential element: exchange rates tend to lose flexibility as they approach their limits.

How does the above "Flex·EMS" method work, then? We applied it to the EPA World Economic Model, using its October 1982 ex ante forecast as a standard case. Figure 2 compares two solution paths for each currency price of the US dollar, solid lines indicating the Flex·EMS solution and dotted lines the unconstrained solution (i.e., the standard forecast). Curves relating to the US dollar depict movements of the index of effective exchange rate for US dollar ($1975 = 100$), expressed as the price of composite foreign currencies (Deutsche Mark, French Franc, Pound Sterling, Canadian Dollar, and Japanese Yen) in terms of US dollar. It can be seen from the compari-

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4) See Amano, Kagawa, and Yoshitomi (1983).
sons of two lines that in 1982 three European currencies all exhibit slightly weaker position vis-à-vis the US dollar under the EMS arrangements as compared with the unconstrained Flex solution. In 1983 solid lines display more or less milder movements with lesser appreciation of Deutsche Mark and slight appreciation rather than sharp depreciation of French Franc and Italian Lira in the last two quarters.

More insight will be gained by examining the movements of bilateral rates among European currencies, which are shown in Figure 3, where parities are shown by broken lines. In 1982 there were two multilateral currency re-alignments in the EMS: one on February 22 and the other on June 14. For the purpose of present simulation, however, we applied the February 1982 parities to the first two quarters of 1982 and the June 1982 parities to the rest of the simulation period. In the latter re-alignment Deutsche Mark was revalued and French Franc and Italian Lira were devalued against other EMS currencies. Our unconstrained Flex solution for the second and the third quarters of 1982 clearly showed the tendency for French
Figure 2
Figure 3

- F. Fr./D. M.
- Lit./D. M.
- Lit./F. Fr.

Unconstrained Solution
Parity
Flex-EMS Solution
Franc and Italian Lira to depreciate against Deutsche Mark, but the extents of depreciation were somewhat smaller than implied by actual parity changes. This explains why our EMS solution, which constrains the exchange rates within the new set of currency bands, shows larger appreciation of Deutsche Mark and larger depreciation of Italian Lira.

For the year 1983, our standard forecast showed that French Franc and Italian Lira would weaken quite a bit towards the end of the year. With the EMS constraints, however, depreciation of these currencies against Deutsche Mark will be suppressed, provided that parities are maintained and that reserve losses are within a tolerable range.

Although these experiments are still tentative, and the EPA World Economic Model itself needs further improvements in many respects, the above results indicate that the present method can help evaluate the consequences of maintaining or re-aligning current parities in the EMS system.
REFERENCES


