The Endogenous Economic Growth under the declining Population Growth:
Simulation Analysis by the Overlapping Generations Model

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Abstract

We analyze future Japanese economic situation, which is facing with decreasing of population growth, by using Computable Overlapping Generations (OLG) model based on the endogenous growth theory. We incorporate Human Capital Stock Sector into the traditional (Auerbach and Kotlikoff type) Computable OLG model. So, endogenous growth is generated by the process of the accumulation of physical and human capital stock in this model. In this paper, we conduct some simulations under the alternative population growth patterns and some the sensitivity analysis with respect to the some key parameters of our model. And also, we show the simulation results of the long-run growth path of the Japanese economy through 2050 under the recent demographic projections by the Japanese Government.

According to it, in terms of population growth rate, it will decrease by 0.5% per annum in 2007 to 2050. The aged ratio was 16.7% in 1999, and will increase sharply to the ratio of 25.2% in 2015 and 28% in 2030. The main results of this paper are as follows; First, the importance of continuous human capital investment was confirmed in order to assure the sustainable economic growth. The endogenously determined growth rate of human capital more than offsets the negative population growth rates, which, in turn, assures, the positive aggregate economic growth rate as well as per positive capita growth rate. Second, the individual has an incentive to allocate his/her time into schooling investment for accumulating his/her human capital in the phase of declining population growth. Third, there is the negative correlation between the growth rate of labor supply and human capital. From these simulation results, we can conclude that in order to attain the sustainable economic growth, more policy priority should be put on the schooling activity and training activity of the individuals, which in turn increases the level of human capital.

Key words: Endogenous growth, Human capital accumulation, Computable OLG model.
JEL classification: O0, E0, C0
1. Introduction

The macro economic growth is basically determined by such three factors as physical capital, labor and technical progress. Especially, as clarified by a series of neoclassical growth model initiated by Solow (1956) and Swan (1956), technical progress is the most critical factor for the sustainable economic growth of the country.

Japan is going into the “population declining society” in a near future. Under this prospect, it is the most urgent issues for Japan to show how Japan to sustain the positive economic growth.

Sadahiro and Shimasawa (1999b) showed the simulation results of the impact of declining population on the Japanese economy by using the Solow-Swan type of neoclassical model and OLG model. There, the main conclusion is that “the growth rate of technical progress must be higher than the declining growth rate of the population in order to maintain the national economic welfare for the long-run.”

However, in their paper, the model was based on the traditional growth model, where technical progress was treated as exogenous variable with neglecting the internal feedback mechanism between declining population and technical progress. And these traditional neoclassical growth models can not explain why the growth rates are difference between various countries (growth rate divergence), and/or why the rich countries and the poor countries coexist in a world economy. The endogenous growth models, which are explored by Romer (1986), Lucas (1988) and Rebelo (1991), have been developed with the intention of overcoming this deficiency of the traditional neoclassical growth models. Namely, these endogenous growth models have tried to clarify the fundamental factors of the growth rate divergence.

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4 The declining population phenomenon is not only Japan's issue but also common challenge among the developed countries.

In the case of Japan, total population in 1999 was 126.7 million and is expected amount to the peak level of 127.8 million in 2007. And after that, it will decrease to the level of 100 million in 2050 (medium variants of “Population Projection for Japan” (January 1997) by National Institution of Population and Social Security Research). In terms of population growth rate, it will decrease by 0.5% per annum in 2007 to 2050. The aged ratio (rate of those aged 65 and above to the total population) was 4.9% in 1950, and 16.7% in 1999, and will increase sharply to the ratio of 25.2% in 2015 and 28% in 2030. In the case of other developed countries, the total population will decreases in 2045 to 2050 except United States (United Nations, "World Population Prospects; 1996). As for the aged ratio in other countries, while U.S will be relatively low level of 21.7% in 2050, Italy is 34.9% in 2050. Finally, total fertility rate (TFR) in 1955 was 1.42 in Japan, which is the third lowest country in the G7 following the 1.19 in Italy and 1.25 in Germany.

5 Sadahiro and Shimasawa (1999b) left this issue for the future works.

6 In the neoclassical growth models, the growth differential among countries reduces to; i) the difference in initial endowments of production factors. ii) the difference in fundamental parameters.
with incorporating the internal mechanism through which the technical progress is endogenously determined as “an engine of economic growth”.

And a series of research works in this field came to the conclusion that the accumulation of human capital, based on the continuous R&D and schooling investment, has the crucial role for augmenting the technical progress and economic growth.

The initial trial of the endogenous economic growth model dates back to Arrow (1962) and Uzawa (1965) in 1960’s. After these works, research on this field could not get the main stream among the theoretical economists, because there were some feelings of hatred concerning the very technical assumption in the model specification.

However, since pioneering papers of Romer (1986, 1990) and Lucas (1988), the research on the endogenous growth models got the momentum for the main stream in the field of the theoretical economics.

The model structure of Lucas (1988) is summarized as follows. The model assumes the perfect competition with the rational agents living in the closed economy.

The representative individual maximizes his/her inter-temporal utility functions:

\[
U = \int_0^\infty e^{-\rho t} \frac{N}{1-\sigma} \left[ c_{t+1}^{1-\sigma} - 1 \right] dt
\]

where \( c \) is per capita consumption. \( \rho \) is time preference. \( \sigma \) is the degree of relative risk aversion.

The accumulation of human capital is described as:

\[
h_t = h_t \delta u_t
\]

where \( \delta \) is 1-replacement rate. \( u \) is schooling investment.

The representative firm maximizes its profit under the Cobb-Douglass determining the production function and utility function.

7 In Romer (1986, 1990), the core part of the model was the non-rival of the knowledge and the resulting externality and increasing returns.

8 In Lucas (1988), the human capital was endogenously determined.

9 Here, for simplify, externality in relating to population growth and human capital accumulation is ruled out.

10 In Romer (1990), the growth engine is the increase of blue print of intermediate goods, which is proportional to the level of human capital engaging in R&D activity. Therefore, the level of human capital stock has the crucial role for the sustainable economic growth.
production function, and derives physical capital stock (K) and effective labor (L^e).
Here, total population (N) and labor supply (L_t) are assumed constant.

(1-3) \[ Y_t = A K_t^\alpha L_t^{1-\alpha}, \quad \text{where} \quad L_t^e = (1-u_t)\lambda L_t, \ L_t = N \]

The accumulation of the physical capital is described as:

(1-4) \[ \dot{K}_t = AK_t^{\alpha}L_t^{1-\alpha} - NC_t \]

The representative individual maximizes equation (1-1) under the constraint of equation (1-2) and equation (1-4). The current value Hamiltonian is described in equation (1-5). And by maximizing equation (1-5) with respect to the endogenous arguments, the following equations are described as:

(1-5) \[ H(K, h, \lambda_1, \lambda_2, c, u_t, t) = \frac{N}{1-\sigma}\left(c_t^{1-\sigma} - 1\right) + \lambda_1\left[AK_t^\alpha(1-u_t)\lambda L_t^{1-\alpha} - NC_t\right] + \lambda_2(\delta h_t u_t) \]

(1-6) \[ \frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\sigma}, \quad \text{where} \quad r_t = \frac{\alpha Y_t}{K_t} \]

(1-7) \[ \frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta \]

(1-8) \[ u_t = \frac{(1 - \alpha)Y_t}{\lambda_2 \delta h_t^2 - c_t^{1-\sigma}} \]

where \( \lambda \) is the shadow price of schooling investments\(^{11}\).

The balanced growth path can be described as follows:

First, the following equation is derived from equation (1-4) and equation (1-6):

(1-9) \[ \frac{\dot{K}_t}{K_t} + \frac{NC_t}{K_t} = \frac{\rho + \sigma \kappa}{\alpha} \]

where \( \kappa \) is the growth rate of per capita consumption.

On the balanced growth path, the growth rate of the physical capital stock \( \dot{K}_t / K_t \) is constant. Therefore, \( NC_t / K_t \) is constant. This means that the growth rate of \( c_t \) and \( K_t \) is equal. Thus \( \frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \kappa \)^{12}

Here, the growth rate of human capital (h_t) is assumed to be \( \nu \). Then the following equation (1-10) is derived.

---

\(^{11}\) The Lagrange multiplier (\( \lambda \)) is the shadow price of schooling investment; see Dorfman (1969).

\(^{12}\) Here, total population is assumed to be constant throughout the time.
where, \( z_t = 1 - u_t \), \( \zeta = \frac{z_t}{z_t^*} \), \( \frac{A}{A} = 0 \), \( \frac{L}{L} = 0 \)

Then,

\[
\kappa = \zeta + \nu
\]

On the balanced growth path, \( \zeta = 0 \). Therefore,

\[
\nu = \kappa.
\]

Namely, on the balanced growth path of the Lucas model, the growth rates of consumption, the physical capital, human capital and output are equal to be \( \kappa \).

Especially, the important point is that the constant growth rate of the endogenously determined human capital assures the positive aggregate economic growth even in the country with the constant labor supply.

However, this role of the growth engine of the human capital strongly depends on the model specification of the accumulation of the human capital (equation (1-2)).

To clarify this aspect, instead of equation (1-2), the more general equation is specified as:

\[
h_t = h_t^* G(z_t), \quad \text{where}, \quad G'(\cdot) > 0, \quad G(0) = 0
\]

Here, the following equation is derived:

\[
\frac{h_t}{h_t^*} \geq h_t^{\xi - 1} G(z_t) \leq h_t^{\xi - 1} G(1)
\]

From equation (1-14), it is easily understood that the human capital can not be a growth engine if \( \xi < 1 \). Namely, the left-hand side (\( h_t^* / h_t \)) of equation (1-14) converges to 0, because \( h_t^{\xi - 1} \) converges to 0 when \( h_t \) becomes large. So in Lucas model, in order to sustain the growth engine of human capital, the “crucial assumption” that \( \xi \) must be larger than unity is needed to be assured\(^{13}\).

\(^{13}\) In Lucas (1988), he justified this assumption to refer to Rosen (1976). “(This) linearity assumption might appear to be a dead-end because we seem to see diminishing returns in observed, individual patterns of human capital accumulation. ... an alternative explanation for this observation is simply
With taking into consideration of this “crucial assumption” of Lucas model, this paper firstly develops Sadahiro and Shimasawa(1999a,b) with overlapping generations model under the more generally endogenized human capital accumulation equation, and secondly shows some simulation results concerning the impact of the declining population on the technical progress and economic growth.

The paper is organized as follows. Section 2 shows our model structure with specific reference to the determination of the human capital accumulation. Section 3 conducts some simulations under the alternative population growth patterns and some sensitivity analysis with respect to the some parameters of our model. And also, we show the simulation results of the long-run path of the Japanese economy through 2050 under the future population trend which is the medium variants of “Population Projection for Japan (January 1997)” by National Institution of Population and Social Security Research. Section 4 concludes.

2. Model Structure

This model is the overlapping generations model. Each generation maximizes his/her inter-temporal utility function with taking into his/her bequest. The representative firm maximizes its profit under the production function. The model is one country closed model, where not only goods market but also factor markets are perfectly competitive. The government sector is composed of the pension sector and the non-pension sector.

2.1 Household Sector

We consider an economy in which every person lives for a fixed number of periods. The each generation enters into labor market at the 20 age (1st period) and retires at the 60 age (40th period) and dies at the 78 age (58th period). His/her utility function is specified as:

\[
U_i = \frac{1}{1-\gamma} \sum_{j=1}^{58} \frac{1}{1+\rho} \left[ \gamma^{-1}(c_{i,j}^{1-\gamma} + \theta z_{i,j}^{1-\gamma} + \beta_j \text{beq}_{i,j}^{1-\gamma}) \right], \gamma > 1, \theta > 0, \beta_{j=58} = 0, \beta_{j=58} > 0
\]

, where the meaning of notations are listed in Annex.

The arguments of the utility function are the inter-temporal consumption (c_{i,j}), the schooling investment (z_{i,j}) and bequest to his/her descendant (beq_{i,j}).

His/her inter-temporal budget equation is described as follows:

that an individual’s lifetime is finite, so that the return to increments falls with time".
Each generation maximizes his/her utility function (equation (1)) under the budget constraint (equation (2)).

With the maximization procedure, the following Euler equations can be solved concerning inter-temporal consumption, time allocation to schooling investment, bequest and the relationship between consumption and schooling investment and physical capital accumulation.

(3) \[ c_{i,j} = \left\{ \frac{1 + r_i (1 - \tau r_i)}{1 + \rho} \right\}^\frac{1}{\tau} \left\{ \frac{1 + \tau c_{t-1}}{1 + \tau c_t} \right\}^\frac{1}{\tau} c_{i,j-1}, \quad C_t = \sum_{j=1}^{58} N_{t,j} c_{i,j} \]

(4) \[ z_{i,j} = \left\{ \frac{1 + r_i (1 - \tau r_i)}{1 + \rho} \right\}^\frac{1}{\tau} \left\{ w_{t-1} (1 - \tau w_{t-1}) \right\}^\frac{1}{\tau} \left\{ \frac{w_t (1 - \tau w_t)}{w_{i,t}} \right\}^\frac{1}{\tau} \left\{ \frac{hc_{i,j-1}}{hc_{i,t}} \right\}^\frac{1}{\tau} z_{i,j-1} \]

(5) \[ beq_i = \beta \left( \frac{1 + \tau c_{t-1}}{1 + \tau c_{t-1}} \right)^\frac{1}{\tau} c_{i,j} \]

(6) \[ z_{i,j} = \theta \left( \frac{1 + \tau c_t}{1 - \tau w_t} \right)^\frac{1}{\tau} w_t \] \[ \left( \frac{1 + \tau c_t}{1 + \tau c_t} \right)^\frac{1}{\tau} c_{i,j} \]

(7) \[ a_{i,j} = a_{i,j-1} \left\{ 1 + r_i (1 - \tau r_i) \right\} \left\{ 1 + \tau c_t \right\} c_{i,j} \left\{ 1 - z_{i,j} - (1 + \tau c_t) c_{i,j} \right\} \]

\[ + (1 - \tau p_{i,j}) \right\} pr_{i,j} - (1 - \tau w_t) pp_{i,j} \]

\[ PA_t = \sum_{j=1}^{58} N_{t,j} a_{i,j} = K_t^S \]

Each generation optimally allocates his/her total time (normalized as unity) into schooling activity \((z_{t,j})\) and labor activity \((1-z_{t,j})\). Therefore, effective labor supply at each period is defined as:

(8) \[ L_{t,j}^S = \sum_{j=1}^{58} N_{t,j} h_{c_{t,j}} (1 - z_{t,j}) \]

2.2 Human Capital Sector
Before specifying the modeling of human capital accumulation, we at first overview the characteristics of the human capital.

The human capital is, needless to say, embodied into each person. Therefore its characteristics is intrinsically rival and exclusive\textsuperscript{14}. This implies that each generation’s embodied human capital would not be transferred to the new generation if some transmission mechanism is not institutionalized, which in turn bring about the decrease of the human capital as time goes on.

In this aspect, “schooling” system is to be well designed to make the each generation’s embodied human capital socialized and transferred to the new generation. Therefore, some devises for modeling of this sort of schooling and transferring system are indispensable\textsuperscript{15}.

As explained above, each generation optimally allocates a given time into the producing activity and schooling activity for promoting his/her human capital (labor productivity). Generally speaking, the more he/she invests his/her given time into schooling, the less he/she attains utility in a short run, because he/she gets the relative smaller wage than he/she would be otherwise (decrease of labor supply $\rightarrow$ decrease wage income $\rightarrow$ the decrease of consumption $\rightarrow$ decrease of utility level). However, as time goes on, his/her utility would be higher due to the higher wage income induced by the accumulated human capital and high labor productivity. Therefore, by balancing these short-run effect and long-run effect, he/she inter-temporally decides the optimal schooling investment.

In this model, the accumulation of human capital is formulated in line with Rebelo (1991) as\textsuperscript{16}:

\begin{equation}
hc_{i,j+1} = (1 - \delta_{hc})hc_{i,j} + B (m_k k_t^{\gamma} (hc_{i,j} z_{i,j})^{1-\gamma} - B_{hc} z_{i,j} - B),
\end{equation}

where $k_t$ is the physical capital to labor ratio ($K_t / L_t$). $B$ is the parameter of accumulation efficiency of the human capital, which is the parameter reflecting the

\textsuperscript{14} This characteristics of human capital is in opposite position with the idea-based growth model, which has the characteristics of non-rival and non-exclusiveness.

\textsuperscript{15} Here, the externality of human capital is ruled out. According to Mankiw, Romer and Weil (1992), the human capital has the characteristics of externality in the fact that the higher the average educational level is (the more people has the higher level of human capital), the more the room for improving the labor productivity is remained.

\textsuperscript{16} Fougeré=Merette(1999) formulated the human capital accumulation equation as

\begin{equation}
\text{hc}_{i,j+1} = (1 - \delta_{hc})\text{hc}_{i,j} + B\text{u}^{t}_{i,j},
\end{equation}

and conducted some simulations on the impact of the aging phenomenon on the economic growth. However, their modeling of the human capital is not endogenous growth model. Because, in their equation, human capital is determined without any interaction with economic variables, human capital is exogenous variable and, therefore, is not endogenous growth model.
characteristics of schooling system. $m$ is the portion of physical capital stock for the production of the human capital stock. $\phi$ is elasticity.

Equation (9) expresses that some portion of physical capital is needed for the accumulation of the human capital in addition to the schooling investment. And the human capital stock increase most efficiently in the case that all available time is allocated to schooling investment ($z=1$). Conversely, the human capital decrease by the order of depreciation ($\delta_{hc}$) in the case that he/she invest nothing into schooling activity ($z=0$).

Here, under the equation (9), we can avoid above mentioned “crucial assumption” of Lucas model. Namely, the increase rate of the human capital is derived from equation (9) as:

$$
\Delta h_{c, i, j} / h_{c, i, j} = B \left( m \cdot k / h_{c, i, j} \right)^{\phi} (z_{i, j})^{1-\phi}
$$

Therefore, on the balanced growth path, because $k_{t} / h_{c, i, j}$ and $z_{i, j}$ converge to constant (non-zero) value, $\Delta h_{c, i, j+1} / h_{c, i, j}$ also converges to constant (non-zero) value, which guarantees the positive economic growth induced by growth engine of the human capital\(^{17}\).

The initial human capital level of the new comer’s generation is assumed to be given a certain percentage of previous generation’s accumulated human capital.

$$
h_{c, i, 1} = \pi_{hc} \sum_{j=1}^{40} h_{c, i-1, j}
$$

Aggregate human capital is defined as:

$$
HC_{t} = \sum_{j=1}^{40} h_{c, i, j} N_{t, j}
$$

2.3 Firm Sector

The input-output structure is described by the Cobb-Douglass production function with constant return to scale. The firm decides the demand for physical capital and effective labor so as to maximize its profit with the given factor prices of wage and rent which are determined in perfect competitive markets. Here, the

\(^{17}\) In aggregation terms, $\Delta h_{C} / h_{C} = \Delta k_{t} / k_{t} = \text{constant on the balanced growth path}$. See equation (25).
externality of human capital is ruled out.

\( Y_t = AK_t^\alpha L_t^{1-\alpha} \)  
\( K_t^\delta = INV_t + (1-\delta)K_{t-1} \)  
\( r_t (\equiv \frac{\partial Y_t}{\partial K_t}) = \alpha AK_t^{\alpha-1}L_t^{-\alpha} - \delta, \quad w_t (\equiv \frac{\partial Y_t}{\partial L_t}) = (1-\alpha)AK_t^\alpha L_t^{-\alpha} \)

2.4 Government Sector

The government sector is composed of pension sector and non-pension sector.

a) Pension Sector

The revenue items of the pension sector are pension contribution, interest revenue from the accumulated pension fund and subsidy from non-pension government sector. The expenditure item is pension benefit.

\( pp_{i,j} = \xi w_t h_{i,j} (1 - z_{i,j}) \), \( PP_t = \sum_{j=1}^{ref} pp_{i,j} N_{t,j} \)
\( pr_{i,j} = fix_t + \zeta vpr_t \), \( PR_t = \sum_{j=ref+1}^{58} pr_{i,j} N_{t,j} \)
\( PP_t + GPS_t + r_t (1-\tau r)FUND_t-1 = PR_t + FUND_t - FUND_t-1 \)

b) non-pension government sector

While revenue items are wage tax, consumption tax, capital tax, tax on bequest, expenditure items are government consumption, government investment, subsidy to pension sector etc.

Here, we assume the balanced budget principle. Therefore, consumption tax rate \( (\tau C_t) \) is endogenously determined so as to maintain the balanced budget.

\( TAX_t = \tau w_t L_{et} - PP_t + \tau C_t + \tau bBEQ_t + \tau w_p PR_t \)
\( GPS_t = \psi \sum_{j=ref+1}^{58} N_{t,j} fix_t \)
\( TAX_t = CG_t + INVG_t + GPS_t \)

2.5 Market Equilibrium and Dynamic Property

In order to close the model structure, the following three market equilibrium conditions must be hold.

The first and second condition is the equilibrium in the factor markets. Namely, factor prices are endogenously determined so as to maintain the market equilibrium
The third condition is the equilibrium in the goods market.

\[ Y_t = C_t + G_t + INV_t \]

In the model simulation, private investment \((INV)\) is determined by using this equilibrium condition.

Finally, on the balanced growth path, the following dynamic properties will hold:

\[ \Delta Y_t / Y_t = \Delta H C_t / H C_t = \Delta h c_t / h c_t = \Delta E t / E t = \kappa + \eta \]

\[ \Delta (Y_t / L_e) / (Y_t / L_e) = 0 \]

\[ \Delta (Y_t / L_t) / (Y_t / L_t) = \Delta h c_t / h c_t = \Delta k_t / k_t = \kappa, \]

where \(h c_t = H C_t / N_t, k_t = K_t / L_t\).

3. Model Property and Simulation Results

In this section, firstly, in order to clarify our model property, we conduct some simulations under the two alternative population growth rates. Secondly, we will focus on the sensitivity analysis with respect to the key parameters. Thirdly, we will highlight the long-run trend of Japanese economy under the assumption of medium variants of "Population Estimates for Japan by National Institution of Population and Social Security Research (January, 1997).

3.1 Simulation Results under the Two Alternative Population Growth Rates

Here, we will assume two hypothetical population growth rates after 1950. One population growth rate is -0.5% per annum (Case 1). The other is +0.5% per annum (Case 2). The assumed values of the key parameters are listed in Appendix. The simulation results of main variables are described as follows.

(Macroeconomic Growth Rate)

One percentage point difference of the population growth rate between two Cases is straightly reflected into the differences of the growth rates of the labor supply. For example, while the growth rate of the labor supply in Case 1 is 0.5% in 2000-2025, it is -0.6% in Case 2.

This difference in labor supply among two Cases contributes to the difference in macroeconomic growth rates. Here, when labor supply growth rate decreases by 1% point, GDP growth rate decreases by 1% point, accompanied by 1% point decrease of
physical capital stock. This characteristic is derived from the constant return to scale of production function (Chart 1-1, Chart 1-2, Chart 2, Chart 3-1, Chart 3-2).

(Contribution of Production Factors and Aggregate Human Capital Stock)

The growth rates of the efficiency of labor supply (ex-post total factor productivity, TFP\(^{18}\)) are around more than 3% in both Cases. Here, the noticeable point is that positive GDP growth rate is guaranteed even in the phase of declining population society (Case 2), because the growth rate of TFP (3% per annum) is higher than the absolute value of labor supply growth rate (0.5%).

This simulation results mean that the role of the growth engine of the human capital becomes more significant in the declining population case. For example, while the contribution rate of the TFP is about 60% in Case 1, it is about 80% in Case 2 (Chart 1-1, Chart 1-2, Chart 2, Chart 3-1, Chart 3-2).

(Labor Supply and TFP)

From Chart 4, it is easily confirmed that the correlation between labor supply and TFP growth rate is negative; the more labor supply decreases (increases), the more TFP increases (decreases). In this model, 1% decrease of labor supply increases nearly 2 - 3% point increase of TFP growth rate.

(Human Capital by Generation)

As for the human capital level of each generation, the following three points can be mentioned. First, the level of the human capital of any generations becomes higher as he/she becomes older (Chart 5).

Second, the level of the human capital of younger and future generations (e.g. cohort 2000) is higher than that of older generations (e.g. cohort 1960). This characteristic comes from the specification of equation (10). For example, the human capital level of cohort 2000 is about 3.8 times higher than that of cohort 1960.

Third, while the human capital level of older generations (e.g. cohort 1960 and cohort 2000) becomes higher (lower) in line with the higher (lower) population growth rate throughout his/her life time, younger generation’s one (e.g. cohort 2040) is inversely related with population growth rate. Chart 6 shows the relative level of

\(^{18}\) The increase rate of the efficiency of labor supply (ex-post TFP) is defined from equation (12) as follows; 

\[
\frac{\Delta TFP}{TFP} = \frac{1}{1 - \alpha} \left( \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} \right) \frac{\Delta L}{L} \]
human capital of each generation between Case 1 and Case 2 when he/she is age 30 and age 50. From this Chart, it is clear that the younger and future generations after cohort 2020 accumulate higher (lower) human capital level in the case of lower (higher) population growth. Where this characteristic comes from? Chart 7 gives us some hints. According this Chart, physical capital to labor ratio \(k\) by each generation in Case 2 is higher than that of Case 1. But the difference of this capital-labor ratio of younger generations (e.g. cohort 2040) between two Cases is larger than that of older generations (e.g. cohort 1960). Therefore, this larger capital-labor ratio brings about higher human capital, which comes from the property of the equation (9).

As for the growth rate of human capital by generation, two points can be mentioned. First, while older generations (e.g. 1960) lower their accumulation speed of human capital when they become old, the younger generations do not lower their human capital accumulation speed. The second is that human capital accumulation speed in Case 2 becomes slower than that in Case 1 when he/she becomes older for any generations (Chart 8).

(Time Allocation to Schooling Investment)

As for the aggregate time allocation into schooling investment, it is higher in Case 2 than that in Case 1 (Chart 9). This implies that in the case of lower population growth, larger aggregate time allocation to schooling investment induces higher human capital and effective labor supply, which in turn offsets the negative impact of the lower population to GDP growth.

As for the time allocation to schooling investment by generation, three points can be mentioned. First, the time allocation to schooling investment of younger generations becomes larger than that of older generations in both Cases (Chart 10). Second, he/she allocates more time to schooling investment as he/she gets old. Third, in any generations, time to schooling investment in Case 2 is larger than that in Case 2.

(Relationship between Physical Capital Stock and Human Capital Stock)

As explained Section 2, growth rates of physical capital and human capital will

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19 The time allocation into schooling investment in macro basis is derived as follows;

\[
Y_t = AK_t^{\alpha} (u_t, h_t, L_t) ^{1-\alpha} \leftrightarrow z_t = 1 - \left( \frac{Y_t}{AK_t^{\alpha}} \right) ^{1-\alpha} \frac{1}{h_t L_t}, \quad \text{where } u_t = 1 - z_t
\]

20 In Lucas (1988), ultimate reason of growth rate divergence is rooted in this simulated results.
converge to the constant level as time goes on (Chart 11). And this level is small in the case of lower population growth (Chart 11). This is because high population growth rate brings about high growth of physical capital, which in turn brings about high growth of physical capital.

(Wage to Rental Ratio)

As for the wage to rental ratio, it is higher in Case 2. This is because in Case 2 labor becomes scarce, which in turn promotes the substitution from labor to capital through the relatively higher wage and lower rental (Chart 12).

This change of relative factor price configuration is reflected into the ratio of physical capital stock and human capital stock (Chart 13); This ratio is higher in Case 2 than in Case 1.

(Economic Welfare by Generation\textsuperscript{21})

As for the economic welfare of each generation, while the older generation (cohort before 1920) gets higher life time utility than the newer generation (cohort after 1920) in Case 1, younger generations get higher life time utility in Case 2. (Chart 14). This is because younger generations accumulate their human capital stock.

3.2 Sensitivity Analysis

In this section we will conduct the sensitivity analysis with respect to the key parameters and examine the robustness of the above mentioned model property (especially economic growth rates).

Here we will focus on the sensitivity simulation with respect to efficiency of human capital accumulation (B), the degree of bequest motive (β), time preference (ρ), the inverse of inter-temporal elasticity of substitution in consumption. The assumed values of these key parameters are shown in Table 15.

The first is the comparison between Base case and Case 1 (B). From this comparison, we can find that the magnitude of B significantly affects the macroeconomic growth rate through the channel of human capital accumulation. Namely, when the magnitude of B becomes half (0.28 → 0.14), the macroeconomic growth rate decreases by 1% point (2.0% → 1.1% in 2025-2050) accompanied by the

\textsuperscript{21} Chart 14 shows the each generation's life time utility level in Case 2 (EV\textsubscript{case 2} ) normalized by Case 1's life time utility level (EV\textsubscript{case 1}); namely, it is \( \frac{EV\textsubscript{case 2}}{EV\textsubscript{case 1}} \), where EV is each generation's life time utility level.
lower growth rate of human capital (3.1% $\rightarrow$ 2.2%) and physical capital (2.2% $\rightarrow$ 1.3%) (Chart 16, Chart 17, Chart 18, Chart 19).

Next is the impact of higher $\beta$ and lower $\rho$. The noticeable point is that the changes of these parameters do not affect to the macroeconomic variables of GDP and human capital.

Finally, we can confirm the small change of $\gamma$ has a big impact on aggregate variables (Case 4). The larger inter-temporal elasticity of substitution is ($\gamma=1.2 \rightarrow 1.1$), the smaller TFP and GDP growth rates become.

This very sensitive result comes from this following mechanism. Namely, when the inter-temporal elasticity of substitution in utility function becomes larger ($\gamma=1.2 \rightarrow 1.1$), the individuals put more weight in the present consumption than future consumption, which gives him/her an incentive to consume (save) more (less), which promotes to allocate his/her time into less (more) schooling (producing) activity, which, in turn, decreases the growth rate of physical capital and human capital. This mechanism contributes to the lower economic growth rate.

3.3 Long-Run Japanese Economy under the Declining Population Growth Rate

Final simulation is the long-run trend of Japanese economy under the assumption of medium variants of “Population Estimate for Japan” by National Institute of Population and Social Research (January 1997). In this simulation, we use Base case as for the key parameters of the model.

The main message here is that Japanese economy can sustain the positive economy growth even under the negative growth rate of labor supply. This is mainly due to the positive growth rate of human capital which more than offsets the negative growth rate of labor supply (Chart 20, Chart 20-1, Chart 20-2). This is confirmed from the fact that TFP shows the largest contribution to the positive economic growth (Chart 21, Chart 22).

4. Conclusion

Many developed countries including Japan are now facing the unprecedented challenges of declining population in the early 21st century. This note has touched upon the impact of declining population on the macro-economy.

In the field of theoretical economics, some issues on impact of declining population have been developed with the endogenous growth models.

Especially, endogenous human capital was modelled in Lucas (1988). However,
simulation analysis of the declining population on the economy has not yet fully developed. So this paper has extended the overlapping generations model with endogenous the accumulation process of human capital, and conducted some simulation analysis concerning the impact of declining population on Japanese economy.

The main results of this paper are as follows;

First, the importance of continuous human capital investment was confirmed in order to assure the sustainable economic growth. The endogenously determined growth rate of human capital more than offsets the negative population growth rates, which, in turn, assures the positive aggregate economic growth rate as well as per positive capital growth rate.

Second, the individual has an incentive to allocate his/her time into schooling investment for accumulating his/her human capital in the phase of declining population growth.

Third, there is the negative correlation between the growth rate of labor supply and human capital.

From these simulation results, we can conclude that, in order to attain the sustainable economic growth, more policy priority should be put on the schooling (educational) activity and training activity of the individuals, which in turn increases the level of human capital.

Finally, some remaining issues are as follows;

First, this paper has focus on the human capital as an engine of economic growth. However, the growth engine is not confined to human capital only. Another growth engine including R&D investments should be focused. So our OLG model should be developed to endogenize R&D as well as human capital.

Second, in addition to schooling investment for improving human capital, other channels including “learning by doing”, “on the job training”, trade and income distribution etc should be endogenously determined in the model.

Finally, in our simulation, population (birth rate) was exogenous variable. So endogenously determined population is indispensable for the future research.
[References]
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<Notation>

\( \alpha \) : capital income share \\
\( \beta \) : degree of bequest motive \\
\( \gamma \) : inverse of inter-temporal elasticity of substitution \\
\( \delta_{hc} \) : human capital depreciation rate \\
\( \theta \) : degree of educational investment motive \\
\( \rho \) : time preference \\
\( \pi_{hc} \) : efficiency of human capital transmission \\
\( \tau_b \) : tax rate on inheritance \\
\( \tau_c \) : tax rate on consumption \\
\( \tau_p \) : income tax on pension benefit \\
\( \tau_r \) : tax rate on interest income \\
\( \tau_w \) : tax rate on labor income \\
\( A \) : scale parameter of production function \\
\( \a_{i,j} \) : physical wealth asset of generation \( i \) at age \( j \) \\
\( B \) : efficiency of human capital accumulation \\
\( \text{beq} \) : bequest \\
\( c_{i,j} \) : consumption of generation \( i \) at age \( j \)
hc_{i,j} : human capital stock
I_i : inheritance of generation i
i : number of i the generation
j : age
K : physical capital stock
k : physical capital to labor ratio
L_e : effective labor
m : portion of physical capital stock for producing human capital stock
N : number of generations
n : population growth rate
pp_{i,j} : pension burden of generation i at age j
pr_{i,j} : pension benefit of generation i at age j
PDV : discount factor
r : rental rate
t : year(=i+j-1)
U : utility function
Y : real GDP
w : wage rate
z_{i,j} : time allocated to schooling investment of generation i at age j
### Chart 1-1  Growth Rates of GDP Components (%, per annum)

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<tr>
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### Chart 1-2  Contribution of Production Factors (%, per annum)

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<td>-0.4</td>
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<tr>
<td>TFP</td>
<td>2.4</td>
<td>2.1</td>
<td>2.4</td>
<td>2.1</td>
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</table>
Chart 3-2  Contribution of GDP Components (Case 2)