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Abstract

This paper proposes a new method to estimate parameters of dynamic general equilibrium models under a liquidity trap based on the Monte Carlo particle filter, proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998). This method is a natural extension of Yano (2009). In our method, we estimate the parameters using the time-varying-parameter approach, which is often used to infer invariant parameters practically. Our method is based on Bayesian statistics and nonlinear, non-Gaussian, and non-stationary state space modeling to estimate unknown parameters and states. In most previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, analyzes how stable structural parameters are. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. The second advantage of our method is that we are able to estimate new Keynesian DSGE models under a liquidity trap (Krugman (1998)) because nonlinear, non-Gaussian, and non-stationary state space models allow model switching. In our method, the fit of a DSGE model is evaluated using the log-likelihood of it. Thus, we are able to compare the fits of DSGE models. Moreover, we estimate time-varying trends of macroeconomic data: real output, inflation rate, and real interest rate. Our method is an alternative to detrending methods based on the Hodrick-Prescott filter, the Baxter-King filter, the Christiano-Fitzgerald filter, and other filtering algorithms. In our framework, we emphasize that natural rates of macroeconomic data, time-varying parameters, and unknown states are estimated simultaneously. In empirical analysis, we estimate new Keynesian DSGE models under a liquidity trap using Japanese macroeconomic data which includes the "zero-interest-rate" period (1999-2006). The analysis shows that the growth rate of natural output declines in the late 1990s, but, becomes as high as about 2% in the mid-2000s. The target rate of inflation is too low in the 1990s and the 2000s, and it causes deflation in the Japanese economy.

Keywords: dynamic stochastic general equilibrium models, monetary policy, zero interest rate bound, liquidity trap, Monte Carlo particle filter

JEL Classification Codes: C11, C13, E32

1 Introduction

In recent years, some topics in the Japanese economy are the reasons behind the long-term stagnation in the 1990s and the deflation from the late 1990s to the early 2000s. The 1990s are often called "a lost decade" because the real growth of the Japanese economy suddenly slowed down and the economy experienced a long-term recession at the time. Furthermore, deflation, and growth rate of GDP deflator from 1994 to the early 2000s are observed in the economy. To fight against deflation, the Bank of Japan adopted a zero-interest-rate policy from 1999 to 2006 and a quantitative-easing policy from 2001 to 2006.

The reasons behind the lost decade have been actively debated. Was it is caused by aggregate supply factors (such papers as Hayashi and Prescott (2002), Hayashi (2003) and Miyao (2006)), or aggregate demand factors (such papers as Kuttner and Posen (2001) and Kuttner and Posen (2002)) [2]? Hayashi and Prescott (2002) point out that the slowdown of total factor productivity growth in the 1990s and the reduction of the work-week length cause the long-term recession. Thus, Hayashi (2003) proposes structural reforms of the Japanese economy to escape from long-term stagnation. Krugman (1998), however, emphasizes the importance of monetary factors. He points out that the economy is "trapped" by the non-negativity constraint on short-term nominal interest rates because of deflation, and calls the situation a liquidity trap [3]. To escape from the trap and long-term stagnation, he proposes adopting inflation targeting in the Japanese economy. The two seminal papers beget a great number of papers, for example, McCallum (2000), Svensson (2001), Eggertsson and Woodford (2003), Jung et al. (2005), Baba et al. (2005), Adam and Billi (2006), Braun and Waki (2006), Braun and Shioji (2006), and Eggertsson and Pugsley (2006), and Nakajima (2008). Ugai (2007) is a survey on the zero-interest-rate policy and the quantitative-easing policy of the Bank of Japan, and many related papers are cited therein. The discussion of these papers is based on dynamic stochastic general equilibrium models.

In recent years, new Keynesian, dynamic stochastic general equilibrium models of monetary analysis have been rapidly developing. The early works of Kimball (1995), Roberts (1995), and Yun (1996) beget the subsequent many papers (see McCallum and Nelson (1999), Clarida et al. (1999), Gali (2002), and related literatures which are referred therein) [4]. "Middle-size" new Keynesian models are developed by Christiano et al. (2005) and Smets and Wouters (2003), and their models are often adopted by practitioners in the government and the central bank. The fit performance of their models is discussed by Fout (2005), Trabandt (2006), and Del Negro et al. (2007). However, there exist some problems to analyze the Japanese economy in the 1990s and the 2000s using DSGE models because we don't have standard tools to estimate the natural rates of the economy and parameters of DSGE models under a liquidity trap (the non-negativity constraint on short-term nominal interest rates) [5].

[2]Sekine et al. (2003), Hosono and Sakuragawa (2004), and previous studies point out the importance of the non-performing loan problem in the lost decade. See Caballero et al. (2006) and Miyao (2006) and related papers are cited therein. However, the NPL problem is outside the scope of this paper because our model does not include financial intermediaries. The roles of financial intermediaries and the NPL problem in the decade will be explained in a future study.

[3]Eggertsson (2008) describes that a liquidity trap is defined as a situation in which the short-term nominal interest rate is zero. In this paper, we follow his definition.

[4]See also Walsh (2003), Woodford (2003), Kato (2006), and Gali (2008), and related literatures, which are referred therein.

[5]In practice, the Hodrick and Prescott (1997) filter is often used to estimate the natural output of the Japanese economy.

Bayesian statistics are now becoming a standard tool to estimate DSGE models. Smets and Wouters (2003), Levin et al. (2005), Del Negro et al. (2007), Smets and Wouters (2007), and Hirose and Naganuma (2007) estimate parameters of new Keynesian DSGE models using Markov Chain Monte Carlo methods (MCMC). Fernandez-Villaverde and Rubio-Ramirez (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007a) have shown that the Monte Carlo particle filter (MCPF) and maximizing likelihood can be successfully applied to estimate DSGE models [6]. An and Schorfheide (2007) is an excellent survey on this area [7]. Using MCMC, Iiboshi et al. (2005), Sugo and Ueda (2008), and Ichiue et al. (2008) estimate DSGE models for Japan in the "pre-zero-interest-rate" period (1970[1981]-1995) [8]. They avoid using data from the "zero-interest-rate" period (1999-2006) because it is necessary to estimate the nonlinear Taylor rule (the Taylor rule with the non-negativity constraint on short-term nominal interest rates). However, the periods are a matter of serious concern for long-term stagnation and deflation. Thus, there exists a need to estimate DSGE models for the Japanese economy including the "zero-interest-rate" period (1999-2006).

This paper proposes a new method to estimate parameters of dynamic general equilibrium models under a liquidity trap based on the Monte Carlo particle filter, proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998) [9]. This method is a natural extension of Yano (2009). Our method is based on Bayesian statistics and nonlinear, non-Gaussian, and non-stationary state space modeling (NNNSS) to estimate unknown parameters and states. In our method, we estimate the parameters using the time-varying-parameter approach, which is often used to infer invariant parameters practically. In most previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, analyzes how stable structural parameters are. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. It is a general framework for estimating DSGE models. Additionally, we would like to stress that the novel feature of our method is that we are able to estimate DSGE models under a liquidity trap (Krugman (1998)) because it is based on NNNSS. In the other words, it is able to estimate DSGE models with the nonlinear Taylor rule. In our method, the fit of a DSGE model is evaluated using the log-likelihood of it. Thus, we are able to compare the fits of DSGE models. Moreover, we estimate time-varying trends of macroeconomic data: natural output, a inflation rate, and a real interest rate [10]. To estimate trends of macroeconomic data, the Hodrick-Prescott filter, the Baxter-King

However, it is an open question whether the HP filter and the magic number, which are suggested in Hodrick and Prescott (1997), are appropriate for estimation of Japanese natural output. Urasawa (2008) uses the Baxter and King (1999) filter to provide the stylized facts of the Japanese business cycles. See Christiano and Fitzgerald (2003) also.

[6] Amisano and Tristani (2007) estimates a small DSGE model on euro area data, using the conditional particle filter to compute the model likelihood.

[7] Canova (2007) and Dejong and Dave (2007) are comprehensive introductions to Bayesian macroeconometrics.

[8] To estimate DSGE models for Japan, Fuchi et al. (2005) use GMM and Fujiwara (2007) uses maximum likelihood estimation.

[9] Introductions to Monte Carlo particle filters are Gordon et al. (1993), and Doucet et al., eds (2001), Ristic et al. (2004). Yano (2008b) and Yano and Yoshino (2007) propose time-varying structural vector autoregressions based on the Monte Carlo particle filter and a self-organizing state space model. Time-varying structural vector autoregressions based on Markov chain Monte Carlo methods are proposed by Primiceri (2005) and Canova and Gambetti (2006).

[10] Smets and Wouters (2007) estimate invariant trends of macroeconomic data. Yano (2009) proposes a time-varying estimation method of natural rates based on DSGE models.

filter, the Christiano-Fitzgerald filter, and other filtering algorithms are also often used in practice. Our method is an alternative to these filters, and it is "structural" estimation of time-varying economic trends. In empirical analysis, we estimate new Keynesian DSGE models under a liquidity trap using Japanese macroeconomic data, which includes the "pre-zero-interest-rate" period (1980-1998), the "zero-interest-rate" period (1999-2006), and the "post-zero-interest-rate" period (2007). One restriction on our method, however, exists. We assume that the timings of when the economy is trapped in a liquidity trap and its subsequent escape are given. In other words, these timings are exogenous.

In most previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, analyzes how stable structural parameters are. The time-varying-parameter approach is practically often used in state space modeling to estimate parameters, for example, Kitagawa (1998) and Liu and West (2001). Even if we assume the random walk priors, which are described in section 3, it does not indicate that the deep parameters of DSGE models are "time-varying." Our framework is just a practical one to estimate deep parameters. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. Thus, it is suitable to analyze how stable structural parameters are. The second advantage of our method is that we are able to estimate new Keynesian DSGE models under a liquidity trap (Krugman (1998)) because NNNSS allows model switching.

Our paper is closely related with Fernandez-Villaverde and Rubio-Ramirez (2007b) [11]. However, there exist several large differences between our paper and theirs. The first point is that they focus on the stabilities of "structural" parameters of the Taylor rule. In contrast we estimate any parameters using the TVP approach. The second point is that they use MCPF to estimate the second-order approximation of DSGE models, whereas, we focus on the nonlinearity of the Taylor rule of the economy under a liquidity trap. The third point is that they use maximizing the likelihood of MCPF to estimate parameters, while, we adopt a self-organizing state space model for parameter estimation. Yano (2008a) reports that the variances of the estimates of a self-organizing state space model are smaller than the ones of the maximizing-likelihood approach. The fourth point is that we estimate a time-varying trend of real output, a time-varying inflation target, and a time-varying equilibrium real interest rate.

This paper is structured as follows. In section 2, we describe a new Keynesian DSGE model. In section 3, we explain our method based on the Monte Carlo particle filter and a self-organizing state space model. In section 4, we show the results of our empirical analysis. In section 5, we describe conclusions and discussions.

[11] Justiniano and Primiceri (2008) estimate DSGE models allowing for time variation in the volatility of the structural innovations using MCMC. Bjornland et al. (2008) estimate the time-varying natural rate of interest and output and the implied medium-term inflation target for the US economy based on DSGE models using MCMC and the Kalman filter. Hatano (2004) estimates structural parameters of a overlapping generations model using the Kalman filter.

2 The Model

2.1 Households

In the economy, there is a continuum of households indexed by $j \in (0,1)$. The households consume and provide labor. The utility of the household j is given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \Big[\frac{1}{1-\sigma_C}(C_{j,t} - hC_{j,t-1})^{1-\sigma_c} - \frac{1}{1+\sigma_L} L_{j,t}^{1+\sigma_L}\Big], \tag{1}$$

where E_0^j is the expectation operator, conditional on household j's information at time 0, $C_{j,t}$ is household j's consumption, $L_{j,t}$ is household j's labor hours, t is a time index, and h, χ, and η are constants. The constraint condition of the household j is given by

$$C_{j,t} + \frac{B_{j,t}}{P_t} \leq W_t L_{j,t} + (1+i_{t-1})\frac{B_{j,t-1}}{P_t} + \Pi_{j,t}, \tag{2}$$

where $B_{j,t}$ is household j's domestic bonds, W_t is the average real wage, i_t is the short-term nominal interest rate, and $\Pi_{j,t}$ is the profit of the firm j. In addition to Eq. (2), we assume the households are subject to the no-Ponzi condition.

$$\lim_{T\to\infty} E_0[(\prod_{t=0}^{T} \frac{1}{1+i_t})B_{j,T}] = 0. \tag{3}$$

2.2 Final Good Sector

In the final good sector, a single final good is produced by a perfectly competitive, representative firm. The final good is produced using a continuum of intermediate good, $Y_{j,t}$, indexed by $j \in (0,1)$. The final good, Y_t, is produced using the aggregate technology.

$$Y_t = \Big[\int_0^1 (Y_{j,t})^{\frac{1}{1+\lambda_p}} dj\Big]^{1+\lambda_p}, \tag{4}$$

where $Y_{j,t}$ is the quantity of intermediate good j, λ_p is a parameter. The demand curve for $Y_{j,t}$ is given by

$$Y_{j,t} = \Big(\frac{P_{j,t}}{P_t}\Big)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t, \tag{5}$$

where $P_{j,t}$ is the price of intermediate good j and P_t is the aggregate price of the final good. The aggregate price is given by

$$P_t = \Big[\int_0^1 (P_{j,t})^{-\frac{1}{\lambda_p}} dj\Big]^{-\lambda_p}. \tag{6}$$

2.3 Intermediate Goods Firms

In the intermediate goods sector, monopolistic competitive domestic firms produce intermediate goods which is indexed by $j \in (0,1)$. The firm j's production function is given by

$$Y_{j,t} = Z_t L_{j,t}, \tag{7}$$

$L_{j,t}$ is labor to produce the jth intermediate good. The technology level, Z_t, is given by

$$\log Z_t = (1-\xi_Z)\log \bar{Z} + \xi_Z \log Z_{t-1} + \epsilon_{Z,t}, \tag{8}$$

where $\epsilon_{Z,t} \sim N(0, \sigma^2_{Z,t})$ and $\bar{Z}$ and ξ_Z are constants. The firms' real marginal costs are given by

$$MC_t = \frac{W_t}{Z_t}. \tag{9}$$

In the sticky prices model, proposed by Calvo (1983), a fraction $1 - \xi_p$ of all firms re-optimize their nominal prices while the remaining ξ_p fraction of all firms do not re-optimize their nominal prices. Following Christiano et al. (2005), firms that cannot re-optimize their price index to lagged inflation are as follows.

$$P_{j,t} = \pi_{t-1} P_{j,t-1}, \tag{10}$$

where $\pi_t = P_t / P_{t-1}$. We call this price setting "lagged inflation indexation." The firm j chooses $P_{j,t}$ to maximize

$$\begin{aligned} &E_t \sum_{l=0}^{\infty} (\beta \xi_p)^l \Big[\frac{P_{j,t}}{P_{t+l}} X_{tl} - MC_{t+1} \Big] Y_{j,t+l}, \\ &\text{subject to } Y_{j,t} = \Big(\frac{P_{j,t}}{P_t} \Big)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t, \end{aligned} \tag{11}$$

where X_{tl} is

$$X_{tl} = \begin{cases} \pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+l-1} \text{ for } l \geq 1 \\ 0 \text{ for } l = 0. \end{cases} \tag{12}$$

The aggregate price index of sticky prices and inflation indexation is obtained by

$$P_t = [(1 - \xi_p (\tilde{P}_t)^{\frac{1}{1-\lambda_p}} + \xi_p (\pi_{t-1} P_{t-1})^{\frac{1}{1-\lambda_p}}]^{1-\lambda_p}. \tag{13}$$

2.4 Monetary Policy

The monetary authority is assumed to determine the nominal interest rate according to the nonlinear Taylor rule (the Taylor rule with non-negativity constraint on the short-term nominal interest rate.)

$$i_t = \max \big[r_0, i_{t-1}^{\rho_i} (Y_t^{\phi_Y} \pi_t^{\phi_\pi})^{(1-\rho_i)} e^{\epsilon_t^i} \big], \tag{14}$$

where $r_0 \geq 0$ is the lower bound of the nominal interest rate, ρ_i, ϕ_Y, and ϕ_π are constants, and $\epsilon_{i,t} \sim N(0, \sigma^2_{i,t})$. In ordinary cases, r_0 is zero or nearly equal to zero.

2.5 Market Clearing

In the economy, we ignore capital and investment. Thus, in the final market equilibrium, the final good production is equivalent to the households' demand for consumption.

$$Y_t = C_t, \tag{15}$$

where $C_t = \big[\int_0^1 (C_{j,t})^{\frac{1}{1+\lambda_p}} dj \big]^{1+\lambda_p}$ and $Y_{j,t} = C_{j,t}$.

2.6 Linearized Model

We linearize the model described above around the non-stochastic steady state. The linearized model consists of the hybrid new IS curve (HNISC), the hybrid new Keynesian Phillips curve (HNKPC), the nonlinear Taylor rule (NTR), [12] and several equations. HNISC is obtained as follows [13].

$$\hat{Y}_t = \frac{h}{1+h}\hat{Y}_{t-1} + \frac{1}{1+h}E_t\hat{Y}_{t+1} - \frac{1}{\sigma_C(1+h)}E_t\left[\hat{i}_t - \hat{\pi}_{t+1}\right] + \epsilon_{Y,t}. \tag{16}$$

HNKPC is obtained as follows.

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p(1+\beta)}(\hat{W}_t - \hat{Z}_t) + \epsilon_{\pi,t}. \tag{17}$$

The other equation are

$$\hat{W}_t = \sigma_C(\hat{Y}_t - h\hat{Y}_{t-1}) + \sigma_L\hat{L}_t, \tag{18}$$

$$\hat{Y}_t = \hat{Z}_t + \hat{L}_t, \tag{19}$$

and

$$\hat{Z}_t = \xi_Z\hat{Z}_{t-1} + \epsilon_{Z,t}, \tag{20}$$

where $\epsilon_{Y,t} \sim N(0, \sigma^2_{Y,t})$, $\epsilon_{\pi,t} \sim N(0, \sigma^2_{\pi,t})$, and $\epsilon_{Z,t} \sim N(0, \sigma^2_{Z,t})$. The linearized NTR is given by

$$\hat{i}_t = \begin{cases} 0, \ \ \text{(if a liquidity trap)}, \\ \rho_i\hat{i}_{t-1} + (1-\rho_i)(\phi_Y\hat{Y}_t + \phi_\pi\hat{\pi}_t) + \epsilon^i_t, \ \ \text{(if not a liquidity trap)}. \end{cases} \tag{21}$$

2.7 State Space Model

Structural linear rational expectations models are given by

$$\boldsymbol{\Gamma}_0\boldsymbol{x}_t = \boldsymbol{\Gamma}_1\boldsymbol{x}_{t-1} + \boldsymbol{\Psi}\boldsymbol{z}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t + \boldsymbol{C}, t = 1, \ldots, T, \tag{22}$$

where $\boldsymbol{x}_t = [E_t\hat{Y}_{t+1}, E_t\hat{\pi}_{t+1}, \hat{Y}_t, \hat{\pi}_t, \hat{i}_t, \hat{W}_t, \hat{L}_t, \hat{Z}_t]^t$, $\boldsymbol{z}_t = (\epsilon_{Y,t}, \epsilon_{\pi,t}, \epsilon_{i,t}, \epsilon_{z,t})^T \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_t)$ with $\boldsymbol{\Sigma}_t = diag((\sigma_{Y,t})^2, (\sigma_{\pi,t})^2, (\sigma_{i,t})^2, (\sigma_{z,t})^2)$, $\boldsymbol{\Pi} = \boldsymbol{0}$, and $\boldsymbol{C} = \boldsymbol{0}$. Sims (2002) proposes the solution of linear rational expectations models using QZ decomposition [14]. Following Sims (2002), reduced linear rational expectations models are obtained by

$$\boldsymbol{x}_t = \boldsymbol{\Theta}_1\boldsymbol{x}_{t-1} + \boldsymbol{\Theta}_0\boldsymbol{z}_t + \boldsymbol{\Theta}_c, \tag{23}$$

where $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_c$, and $\boldsymbol{\Theta}_0$ are described in Sims (2002) [15]. The measurement equation of the model is

$$\begin{bmatrix} YGR_t \\ INFL_t \\ INT_t \end{bmatrix} = \begin{bmatrix} Y^s \\ \pi^s \\ r^s + \pi^s \end{bmatrix} + \begin{bmatrix} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t\hat{Y}_{t+1} \\ E_t\hat{\pi}_{t+1} \\ \hat{Y}_t \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{W}_t \\ \hat{L}_t \\ \hat{Z}_t \end{bmatrix} + \boldsymbol{v}_t, \tag{24}$$

[12]See Taylor (1993), Henderson and McKibbin (1993), and Clarida et al. (2000).

[13]In this paper, a hat over a variable indicates the percentage deviation from its steady state value.

[14]In empirical analysis, we use Sims's gensys.R and related codes. See http://sims.princeton.edu/yftp/gensys/

[15]We set $\boldsymbol{\Pi}$ to $\boldsymbol{0}$ to rule out the indeterminacy and sunspot equilibria shown in Sims (2002) and Lubik and Schorfheide (2003).

where $\boldsymbol{v}_t = \left(\epsilon^v_{Y,t}, \epsilon^v_{\pi,t}, \epsilon^v_{i,t}\right)^T \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_{v,t})$ with $\boldsymbol{\Sigma}_{v,t} = diag((\sigma^v_{Y,t})^2, (\sigma^v_{\pi,t})^2, (\sigma^v_{i,t})^2)$ [16]. The growth rate of real output, YGR_t, is a log difference of real GDP per capita, rate of inflation, INF_t, is the a log difference of GDP deflator, and nominal interest rate, INT_t, is the uncollateralized overnight call rate. Real growth, inflation, and interest rate are annualized. The symbols, Y^s, π^s, and r^s are the trend of real output, the target rate of inflation, and the trend of real interest rates, respectively.

In our method, we estimate the parameters of Eq. (16)- (24) using the TVP approach. Thus, we define the vector of time-varying parameters as follows.

$$\tilde{\theta}_t = [h_t, \sigma_{C,t}, \xi_{p,t}, \sigma_{L,t}, \xi_{Z,t}, \rho_{i,t}, \phi_{Y,t}, \phi_{\pi,t}, \sigma_{Y,t}, \sigma_{\pi,t}, \sigma_{i,t}, \sigma_{Z,t}, Y^s_t, \pi^s_t, r^s_t, \sigma^v_{Y,t}, \sigma^v_{\pi,t}, \sigma^v_{i,t}], \tag{25}$$

where t is a time index. Note that we suppose that β is given. Structural linear rational expectations models are redefined by

$$\boldsymbol{\Gamma}_{0,t}\boldsymbol{x}_t = \boldsymbol{\Gamma}_{1,t}\boldsymbol{x}_{t-1} + \boldsymbol{\Psi}_t\boldsymbol{z}_t + \boldsymbol{\Pi}_t\boldsymbol{\eta}_t + \boldsymbol{C}_t. \tag{26}$$

Reduced linear rational expectations models are also redefined by

$$\boldsymbol{x}_t = \boldsymbol{\Theta}_{1,t}\boldsymbol{x}_{t-1} + \boldsymbol{\Theta}_{0,t}\boldsymbol{z}_t + \boldsymbol{\Theta}_{c,t}. \tag{27}$$

In previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, analyzes how stable structural parameters are. The time-varying-parameter approach is often used in state space modeling to estimate invariant parameters, for example, Kitagawa (1998) and Liu and West (2001). Even if we assume the random walk priors, which are described in section 3, it does not indicate that the deep parameters are "time-varying." Our framework is just a practical one to estimate deep parameters. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. Thus, it is suitable to analyze how stable structural parameters are. The second advantage of our method is that we are able to estimate new Keynesian DSGE models under the liquidity trap (Krugman (1998)) because NNNSS allow model switching.

3 Estimation Method

To estimate a state vector $\boldsymbol{x}_t$ and a time-varying-parameter vector, $\tilde{\boldsymbol{\theta}}_t$, we adopt the Monte Carlo Particle Filter (MCPF), proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998).

3.1 Nonlinear, Non-Gaussian, and Non-stationary State Space Model

In this subsection, we describe a nonlinear, non-Gaussian, and non-stationary state space model and a self-organizing state space model (MCPF is described in the next subsection).

A nonlinear, non-Gaussian, and non-stationary state space model for the time series $\boldsymbol{Y}_t,\ t = \{1, 2, \cdots, T\}$ is defined as follows.

$$\begin{aligned} \boldsymbol{x}_t &= f_t(\boldsymbol{x}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s), \\ \boldsymbol{Y}_t &= h_t(\boldsymbol{x}_t, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_o), \end{aligned} \tag{28}$$

[16] This equation is a modified version of the measurement equation of An and Schorfheide (2007) and Hirose and Naganuma (2007).

where $\boldsymbol{x}_t$ is an unknown $n_x \times 1$ state vector, $\boldsymbol{v}_t$ is $n_v \times 1$ system noise vector with a density function $q(\boldsymbol{v}|\cdot)$ [17], $\boldsymbol{\epsilon}_t$ is $n_\epsilon \times 1$ observation noise vector with a density function $r(\boldsymbol{\epsilon}|\cdot)$. The function $f_t : \boldsymbol{R}^{n_x} \times \boldsymbol{R}^{n_v} \to \boldsymbol{R}^{n_x}$ is a possibly nonlinear time-varying function and the function $h_t : \boldsymbol{R}^{n_x} \times \boldsymbol{R}^{n_\epsilon} \to \boldsymbol{R}^{n_y}$ is a possibly nonlinear time-varying function. The first equation of (28) is called a system equation and the second equation of (28) is called an observation equation. We would like to emphasize the functions, f_t and h_t, are possibly time dependent. A system equation depends on a possibly unknown $n_s \times 1$ parameter vector, $\boldsymbol{\xi}_s$, and an observation equation depends on a possibly unknown $n_o \times 1$ parameter vector, $\boldsymbol{\xi}_o$. This NNNSS specifies the two following conditional density functions.

$$\begin{aligned} &p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_s), \\ &p(\boldsymbol{Y}_t|\boldsymbol{x}_t, \boldsymbol{\xi}_o). \end{aligned} \tag{29}$$

We define a parameter vector $\boldsymbol{\theta}$ as follows.

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\xi}_s \\ \boldsymbol{\xi}_o \end{bmatrix}. \tag{30}$$

We denote that θ_j is the jth element of $\boldsymbol{\theta}$ and $J(= n_s + n_o)$ is the number of elements of $\boldsymbol{\theta}$. This type of state space model (28) contains a broad class of linear, nonlinear, Gaussian, or non-Gaussian time series models. In state space modeling, estimating the state space vector $\boldsymbol{x}_t$ is the most important problem. For the linear Gaussian state space model, the Kalman filter, which is proposed by Kalman (1960), is the most popular algorithm to estimate the state vector $\boldsymbol{x}_t$. For nonlinear or non-Gaussian state space models, there are many algorithms. For example, the extended Kalman filter (Jazwinski (1970)) is the most popular algorithm; other examples are the Gaussian-sum filter (Alspach and Sorenson (1972)), the dynamic generalized model (West et al. (1985)), and the non-Gaussian filter and smoother (Kitagawa (1987)). In recent years, MCPF for NNNSS has been a popular algorithm because it is easily applicable to various time series models [18].

In econometric analysis, generally, we don't know the parameter vector $\boldsymbol{\theta}$. In our framework, the unknown parameter vectors are $\boldsymbol{\xi}_o$ and $\boldsymbol{\xi}_s$ [19]. In traditional parameter estimation, maximizing the log-likelihood function of $\boldsymbol{\theta}$ is often used. The log-likelihood of $\boldsymbol{\theta}$ in MCPF is proposed by Kitagawa (1996). However, MCPF is problematic to estimate the parameter vector $\boldsymbol{\theta}$ because the likelihood of the filter contains errors from the Monte Carlo method. Thus, you cannot use nonlinear optimizing algorithm like Newton's method [20]. To solve the problem, Kitagawa (1998) proposes a self-organizing state space model. In Kitagawa (1998), an augmented state vector is defined as follows.

$$\boldsymbol{z}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{\theta} \end{bmatrix}. \tag{31}$$

An augmented system equation and an augmented measurement equation are defined as

$$\begin{aligned} \boldsymbol{z}_t &= F_t(\boldsymbol{z}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s), \\ \boldsymbol{Y}_t &= H_t(\boldsymbol{z}_t, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_o), \end{aligned} \tag{32}$$

[17] The system noise vector is independent of past states and current states.

[18] Many applications are shown in Doucet et al., eds (2001).

[19] Details of $\boldsymbol{\xi}_o$ and $\boldsymbol{\xi}_s$ are discussed in the next subsection.

[20] See Yano (2008a).

where

$$F_t(\boldsymbol{z}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s) = \begin{bmatrix} f_t(\boldsymbol{x}_{t-1}, \boldsymbol{v}_t) \\ \boldsymbol{\theta} \end{bmatrix}$$

and

$$H_t(\boldsymbol{z}_t, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_o) = h_t(\boldsymbol{x}_t) + \boldsymbol{\epsilon}_t.$$

This NNNSS is called a self-organizing state space (SOSS) model. Moreover, Kitagawa (1998) suggests to adopt a time-varying parameter approach if we can use a "symmetric" random walk model, $\tilde{\boldsymbol{\theta}}_t = \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{v}_{2,t}$, with $\boldsymbol{v}_{2,t}$ a white noise sequence distributed with a density function $p_2(v_{2,t})$. The nonlinear function F_t is then defined by

$$F_t(\boldsymbol{z}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s) = \begin{bmatrix} f_t(\boldsymbol{x}_{t-1}, \boldsymbol{v}_{1,t}) \\ \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{v}_{2,t} \end{bmatrix},$$

where the system noise vector is $\boldsymbol{v}_t = (\boldsymbol{v}_{1,t}, \boldsymbol{v}_{2,t})^t$. In our method we combine time-varying parameters, $\tilde{\boldsymbol{\theta}}_{t-1}$, and invariant parameters, $\boldsymbol{\theta}$. An augmented vector of time-varying/invariant parameters is defined $\boldsymbol{\Theta_{t-1}} = (\tilde{\boldsymbol{\theta}}_t, \boldsymbol{\theta})^t$. The augmented state vector is redefined as follows.

$$\boldsymbol{z}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{\Theta_t} \end{bmatrix}. \tag{33}$$

Thus, our nonlinear, non-Gaussian, and non-stationary function F_t is redefined by

$$F_t(\boldsymbol{z}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s) = \begin{bmatrix} f_t(\boldsymbol{x}_{t-1}, \boldsymbol{v}_{1,t}) \\ \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{v}_{2,t} \\ \boldsymbol{\theta} \end{bmatrix}. \tag{34}$$

Note that the measurement equation is the second equation of (32). In our method, we stress that states, time-varying parameters, and invariant parameters are estimated simultaneously. Therefore, our problem is how to estimate $\boldsymbol{z}_t$.

3.2 Monte Carlo Particle Filter

The Monte Carlo particle filter is a variant of sequential Monte Carlo algorithms. In MCPF, expectations of a posterior distribution are approximated using "particles" that have weights.

$$E[p(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t})] \simeq \frac{1}{\sum_{m=1}^{M} w_t^m} \sum_{m=1}^{M} w_t^m \delta(\boldsymbol{z}_t - \boldsymbol{z}_t^m), \tag{35}$$

where w_t^m is the weight of a particle $\boldsymbol{z}_t^m$, M is the number of particles, and δ is the Dirac's delta function [21]. Weights w_t^m $m = \{1, 2, \cdots, M\}$ are defined as follows.

$$w_t^m = r(\psi(\boldsymbol{y}_t, \boldsymbol{z}_t^m))\left|\frac{\partial \psi}{\partial \boldsymbol{y}_t}\right|, \tag{36}$$

[21] The Dirac delta function is defined as

$$\delta(x) = 0, \text{ if } x \neq 0,$$
$$\int_{\infty}^{\infty} \delta(x)dx = 1.$$

where ψ is the inverse function of the function h [22]. The right hand side of Eq. (36) is the likelihood function of an NNNSS model. In the standard algorithm of MCPF, the particles x_t^m are resampled with sampling probabilities proportional to $w_t^1, \cdots, w_t^M$. Resampling algorithms are discussed in Kitagawa (1996). After resampling, we have $w_t^m = 1/M$. Therefore, Eq. (35) is rewritten as

$$E[p(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t})] \simeq \frac{1}{M}\sum_{m=1}^{M}\delta(\boldsymbol{z}_t - \hat{\boldsymbol{z}}_t^m), \tag{37}$$

where $\hat{\boldsymbol{z}}_t^m$ are particles after resampling. Particles $\boldsymbol{x}_t^m$ $m = \{1, 2, \cdots, M\}$ are sampled from a system equation:

$$\boldsymbol{z}_t^m \sim p(\boldsymbol{z}_t|\boldsymbol{z}_{t-1}^m, \boldsymbol{\xi}_s). \tag{38}$$

Kitagawa (1996) shows that the log-likelihood of $\boldsymbol{\theta}$ is approximated by

$$l(\boldsymbol{\theta}) \simeq \sum_{t=1}^{T}\log(\sum_{m=1}^{M} w_t^m) - T\log M, \tag{39}$$

where T is the number of observations. Using Eq. (39), we can compare the fits of DSGE models. In self-organizing state space modeling, the augmented state vector is estimated using MCPF. Thus, states and parameters are estimated simultaneously without maximizing the log-likelihood of Eq. (32) because the parameter vector $\boldsymbol{\theta}$ in Eq. (32) is approximated by particles and it is estimated as the state vector in Eq. (31) [23].

On a self-organizing state space model, however, Hürseler and Künsch (2001) points out a problem: determination of initial distributions of parameters for a self-organizing state space model. The estimated parameters of a self-organizing state space model comprise a subset of the initial distributions of parameters. We must know the posterior distributions of parameters to estimate parameters adequately. However, the posterior distributions of the parameters are generally unknown. Parameter estimation fails if we do not know their appropriate initial distributions. Yano (2008a) proposes a method to seek initial distributions of parameters for a self-organizing state space model using the simplex Nelder-Mead algorithm to solve the problem. In this paper, we use uniform distributions for initial distributions of time-varying parameters because most time-varying parameters are restricted to be more than zero and less than unity.

3.3 Time-varying Parameters

In this paper, we assume the "symmetric" random walk prior (the Litterman prior) to estimate time-varying parameters (see Doan et al. (1984)). The random walk prior is given by

$$\tilde{\boldsymbol{\theta}}_t = \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{v}_{2,t}, \tag{40}$$

where $\boldsymbol{v}_{2,t} \sim q(\boldsymbol{v}_{2,t}|\Sigma_{\boldsymbol{\xi}_s})$, $q(\boldsymbol{v}_{2,t}|\Sigma_{\boldsymbol{\xi}_s})$ is a Gaussian distribution, and $\Sigma_{\boldsymbol{\xi}_s}$ is a diagonal matrix. In general, the diagonal components, $\{\xi_{1,s}, \xi_{2,s}, \cdots, \xi_{L,s}\}$, of $\Sigma_{\boldsymbol{\xi}_s}$ are different. In this paper, however, to reduce computational complexity, we assume as follows.

$$\xi_{1,s} = \xi_{2,s} = \cdots = \xi_{L,s} = |\xi_s|. \tag{41}$$

[22] See Kitagawa (1996).

[23] The justification of an SOSS model is described in Kitagawa (1998).

The time evolution of a coefficient is then given by

$$\tilde{\theta}_{i,t} = \tilde{\theta}_{i,t-1} + |\xi_s| v_{2,i,t}, \tag{42}$$

where $v_{2,i,t} \sim N(0,1)$. Note that $\sigma_{L,t}$, $\xi_{Z,t}$, $\phi_{Y,t}$, $\phi_{\pi,t}$, $\sigma_{Y,t}$, $\sigma_{\pi,t}$, $\sigma_{i,t}$, $\sigma_{Z,t}$, $\sigma^v_{Y,t}$, $\sigma^v_{\pi,t}$, $\sigma^v_{i,t}$ are restricted to be positive and h_t, $\sigma_{C,t}$, $\xi_{p,t}$, and $\rho_{i,t}$ are restricted to be more than zero and less than unity. The particles that violate these restrictions are numerically discarded before resampling.

3.4 Algorithm

In our method, we adopt not a smoothing algorithm but a filtering algorithm because the rational expectations hypothesis is consistent with the latter. If we use a smoothing algorithm to estimate time-varying parameters, the estimates of them include the information at times $t+1, t+2, \cdots$ which is not known at time t. Our method to estimate time-varying parameters of DSGE models is summarized as follows:

1. In time t, generate $\boldsymbol{z}_t$ based on the results at time $t-1$.
2. Using particles, the linear rational expectations system is solved to obtain the state transition equation Eq. (23).
3. If a particle implies indeterminacy (or non-existence of a stable rational expectations solution), then the weight of the particle, $\boldsymbol{w}^m_t$, is set to zero.
4. If a unique stable solution exists, then the weight of a particle is calculated using Eq. (36).
5. Resampling particles with sampling probabilities proportional to $w^1_t, \cdots, w^M_t$.
6. Replace t with $t+1$.
7. Go to 1.

4 Empirical Analysis

We use data from 1981:Q1 up to 2007:Q4 [24]. We assume the Japanese economy was trapped in a liquidity trap (the non-negativity constraint on nominal short-term interest rates) at 1999:Q1. Moreover, we suppose the economy escapes from the trap at 2006:Q4 because the quantitative-easing policy and the zero-interest-rate policy of the BOJ are ended at 2006:Q1 and 2006:Q3, respectively. The simulation settings used in empirical analysis are described in appendix B.

Figure 1 shows the annualized estimates of Y^s_t, π^s_t, and r^s_t [25]. The black lines in all figures are means of particles, and the green and red lines are 95% confidence intervals, which are calculated using 100 bootstrap samples of particles. From the mid-1980s to the early 1990, Y^s_t is from about 2% to 5%, and the periods are called the "bubble economy." From the mid-1990s to the early 2000s, Y^s_t is relatively small,

[24] We remove data from 1980:Q1 to 1980:Q4 to avoid the influences of the second oil shock. The details of the data are described in appendix A.

[25] We remove the results from 1981:Q1 to 1984:Q4 to avoid the influences of poor prior distributions.

and the periods are called "a lost decade." To estimate trends of macroeconomic data, the Hodrick and Prescott (1997) filter is often used. In recent years, the Baxter and King (1999) filter and the Christiano and Fitzgerald (2003) filter are also often used. Our method is an alternative to these filters, and it is "structural" estimation of time-varying economic trends. From the mid-1980s to the mid-1990, π_t^s is positive, and it is from 1% to 2%. From the early 1990 to present, π_t^s is negative. The results shows the target rate of the inflation of the BOJ is changed in the early 1990s, and the target in the 1990s and 2000s is too low. From the 2006, the BOJ announces "understanding of the price stability," and it states a stable inflation rate is from 0% to 2%, which is measured by consumer price index, excluding food. This low target rate makes π_t^s negative because it is well known that CPIs have upward bias. From the mid-1980s to the early 1990s, r_t^s is above 2%, and from the early 1990s to present, it is below 1%. The r_t^s is an estimate of equilibrium real rate [26]. Krugman (1998) states ERR of the Japanese economy in the late 1990s is negative. However, our estimate of ERR is not negative but quite low in 1997 and 1998. It strongly suggests that the BOJ, which adopted quite low interest rate policy at the time, needed positive inflation rates to stimulate the economy in the late 1990s. Note that the target rate of inflation, π_t^s, in the 1990s is negative.

[Figure 1 about here.]

Figure 2 shows the estimates of the endogenous variables. The expectational output gap, $E_t\hat{Y}_t$, and the output gap, $\hat{Y}_t$, have fundamentally similar shapes. They indicate that the favorable economic situation ends at early 1990s, and serious recessions happen in the early 1990s, 1997-1998, 2000-2001. The large negative spike in 1989 shows that introducing the consumption tax reduced GDP. We would like to emphasize the large negative spike has little influence on the estimates of parameters. The results indicate that our time-varying-parameter approach is relatively robust to outliers. The inflation expectation, $E_t\hat{\pi}_t$, and the inflation rate, $\hat{\pi}_t$, show that in the mid-1980s and the 1990s negative deviation from the target rate of inflation happen. In particular, the negative deviation in the 1990s is very long, and it indicates the long-term recession of the economy. Interest rate, $\hat{i}_t$, shows the deviation from the equilibrium real interest rate, and it presents the fact that the BOJ made expansionary monetary policy in the late 1980s and the early 1990s. From 1999, the $\hat{i}_t$ is zero because the Japanese economy is under a liquidity trap. The symbols, $\hat{W}_t$, and, $\hat{L}_t$, have similar shapes to $E_t\hat{Y}_t$ and $\hat{Y}_t$. The symbol, $\hat{Z}_t$, shows the negative technology shocks that happened in the early 1990s, the late 1990s, and the early 2000s, and they correspond to the recessions from 1985 to 2007.

[Figure 2 about here.]

Fig. 3 shows the estimates of time-varying parameters. These estimates indicate that some "structural" parameters are time-varying. The results indicate that habit persistence, h, the Calvo parameter, ξ_p, and the coefficient of AR(1) technology process, ξ_z, are relatively stable. The parameter, σ_C, is above 1 in most periods, and it indicates that the utility function of consumption is concave and not the log-type utility function. The parameter, σ_L, is increasing in the all periods, and it indicates that the distortion of labor markets from 1985 to 2006 cannot be captured by our model.

[26] Laubach and Williams (2003) and Trehan and Wu (2007) estimate time-varying equilibrium real rate using a simple, backward-looking model of the U.S. economy.

[Figure 3 about here.]

Figure 4 shows the estimates of time-varying parameters of NTR. The inertia term, ρ_i, is from 0.6 to 0.7 in most periods. It indicates that the BOJ makes the nominal short-term interest rate smooth. The coefficient of the output gap, ϕ_Y, is from 1.0 to 1.6 in most periods, and it shows that the BOJ's reaction of output gap is stable from 1985 to present. The coefficient of the inflation rate, ϕ_π, increases from 0.5 to 3.0, and this result shows that the BOJ is increasing the focus on the inflation rate from 1985 to 2007.

[Figure 4 about here.]

The stochastic volatilities, $\sigma_{Y,t}, \sigma_{\pi,t}, \sigma_{i,t}, \sigma_{Z,t}$, are shown in Figure 5. The stochastic volatilities, $\sigma^v_{Y,t}, \sigma^v_{\pi,t}, \sigma^v_{i,t}$, are shown in Figure 6.

[Figure 5 about here.]

[Figure 6 about here.]

In conventional DSGE modeling with invariant parameters, impulse response functions are often used for policy analysis. IRFs show the responses of variables to a shock, for example, a monetary shock, a technology shock, or a demand shock. In our method, however, IRFs are responses of variables, which are expectational conditional on information known at time t, to a shock because we estimate time-varying parameters of DSGE and a parameter at time t may change at time $t+1$. Expectational impulse response functions to a contractionary monetary shock at 1985:Q1, 1990:Q1, 1995:Q1, and 2007:Q4 are shown in Figure 7, 8, 9, and 10, respectively. These figures show that the the responses of inflation to a monetary shock are different. In 1985:Q1 a contractionary monetary shock caused a positive inflation rate, and it is inconsistent with monetary theory. This odd response is originated from the small value of ϕ_π, which indicates that the BOJ understates the inflation rate in the 1980s. The increasing of ϕ_π makes the responses of inflation normalized from 1990s to 2007. The responses of $\hat{Y}_t$, $\hat{w}_t$, and $\hat{L}_t$ are consistent with monetary theory.

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

EIRFs to technology shock at 1985:Q1, 1990:Q1, 1995:Q1, 2000:Q1, and 2005:Q1, are shown in Figure 11, 12, 13, 14, and 15, respectively. The responses in figures in the "pre-zero-interest-rate" period are more volatile than ones in figures in the the "zero-interest-rate" period because the stabilizing effect of monetary policy is lost under the liquidity trap.

[Figure 11 about here.]

[Figure 12 about here.]

[Figure 13 about here.]

[Figure 14 about here.]

[Figure 15 about here.]

EIRFs to demand shock at 1990:Q1 and 2000:Q1 are shown in Figure 16 and 17. The responses in Figure 17 are more volatile rather than 16 because the stabilizing effect of monetary policy is lost under the liquidity trap. These results are consistent with Section 4.2, Woodford (2003). Following Eggertsson and Woodford (2003), this problem is able to be solved by adopting the flexible $2\% - 3\%$ targeted rate of inflation based on GDP deflator.

[Figure 16 about here.]

[Figure 17 about here.]

EIRFs to supply shock at 1990:Q1 and 2000:Q1 are shown in Figure 18 and 19. The responses in Figure 19 are more volatile rather than 18 because the stabilizing effect of monetary policy is lost under the liquidity trap. These results are consistent with Section 4.2, Woodford (2003).

[Figure 18 about here.]

[Figure 19 about here.]

In practice, the Hodrick and Prescott (1997) filter is often used to estimate the natural output of the Japanese economy. However, whether the HP filter and the magic number, which are suggested in Hodrick and Prescott (1997), are appropriate for estimation of Japanese natural output is an open question. Urasawa (2008) uses the Baxter and King (1999) filter to provide the stylized facts of Japanese business cycles. Our method is an alternative to these filters, and it is "structural" estimation of time-varying economic trends. In Figure 20, we compare our annualized estimates of output gap with estimates of the HP filter and the CF filter. In the upper panel of Figure 20, we show our estimate (the black line) and the estimate of the HP filter (the blue line). From 1985 to the mid-1990s the black line is different from the blue one. The blue line indicates that the output gap is negative in the late 1980s and positive in the early 1990s. In the late 1980s, Japanese economy was in the "bubble" economy, and in the early 1990s, was in the "Heisei" recession. The output gap based on the HP filter is not consistent with these facts, and the one based on our method is consistent with them. Before the mid-1990s, our method is better than the HP filter. The black line coincides with the blue one from the mid-1990s to the 2000s. In the lower panel of Figure 20, we show our estimate (the black line) and the estimate of the CF filter (the green line). The green line is much smoother rather than the black line, and the black one coincides with the green one from the late 1990s to the 2000s. We conclude that our estimate of output gap relatively coincides with the estimates, which are calculated by the HP/CF filters, although our method is totally different from the filters.

[Figure 20 about here.]

Using the log-likelihood of a model, Eq. (39), we compare DSGE models: the model in section 2, the DSGE model without inflation idexation, and the DSGE model without habit formation. The log-likelihoods of models and the estimates of $|\xi_s|$ are shown in Table 1. These results indicate that our model in section 2 is better than the other models. They also show that the inflation inertia and the habit persistence have crucial roles in empirical analysis based on DSGE models, and they are consistent with An and Schorfheide (2007), Hirose and Naganuma (2007), and related studies.

[Table 1 about here.]

5 Conclusion and Discussion

This paper proposes a new method to estimate parameters of dynamic general equilibrium models under a liquidity trap based on the Monte Carlo particle filter and a self-organizing state space model. This method is a natural extension of Yano (2009). Our method analyzes how stable structural parameters are. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. The novel feature of our method is that we are able to estimate parameters of new Keynesian DSGE models under a liquidity trap (Krugman (1998)), because nonlinear, non-Gaussian, and non-stationary state space models allow model switching. Moreover, we estimate time-varying trends of macroeconomic data: real output, inflation rate, and real interest rate. To estimate trends of macroeconomic data, the Hodrick-Prescott filter, proposed by Hodrick and Prescott (1997), is often used. In recent years, the Baxter-King filter, proposed by Baxter and King (1999), and the Christiano-Fitzgerald filter, proposed by Christiano and Fitzgerald (2003) are also often used. Our method is an alternative to these filters, and it is a "structural" estimation of time-varying economic trends. We conclude that our estimate of output gap relatively coincides with the estimates, which are calculated by the HP/CF filters, although our method is totally different from the filters. In empirical analysis, we estimate new Keynesian DSGE models under a liquidity trap using Japanese macroeconomic data, which include the "zero-interest-rate" period (1999-2006). The analysis shows that the growth rate of natural output declines in the late 1990s but becomes as high as about 2% in the mid-2000s. The target rate of inflation is too low in the 1990s and the 2000s, and it causes deflation in the Japanese economy. In the the "zero-interest-rate" period, the impulse responses to technology shocks, aggregate demand shocks, and aggregate supply shocks are more volatile than the other period because the stabilizing effect of monetary policy is lost under the liquidity trap. These results are consistent with Section 4.2, Woodford (2003). Following Eggertsson and Woodford (2003), this problem can be solved by adopting the flexible $2\% - 3\%$ targeted rate of inflation based on GDP deflator.

In a new study, we are estimating new Keynesian, small open economy DSGE models, new Keynesian DSGE models with liquidity-constraint households, Christiano et al. (2005), and second-order approximation of DSGE models. Furthermore, our method can be easily extended to estimate state-dependent-pricing models with random menu costs, proposed by Dotsey et al. (1999) [27]. In policy analysis of DSGE models, impulse response functions are often used. In our framework, the effectiveness of the traditional

[27] Gertler and Leahy (2006) and Bakhshi et al. (2007) derive a Phillips curve equation from a DSGE model with state-dependent pricing.

way is ambiguous because parameters in DSGE models are time-varying. If we calculate impulse response function at time t, the results of them may be meaningless because the parameters may have changed at time $t+1$. Canova and Gambetti (2006) proposes the use of generalized impulse response functions in time-varying structural vector autoregressions. However, in time-varying analysis of DSGE models, it is an open question. We assume that the timings of when the economy is trapped in a liquidity trap and its subsequent escaped from it are given. The endogenous timings are our future work.

A Data Source

We use quarterly macroeconomic data on the Japanese economy from 1981:Q1 to 2007:Q4.

- Uncollateralized overnight call rate, averaged over three months (Bank of Japan): uncollateralized overnight call rate, monthly average (July 1985-December 2007) and collateralized overnight call rate, monthly average (January 1981 - July 1985) are linked at July 1985.
  http://www.boj.or.jp/en/theme/research/stat/market/index.htm

- Seasonally-adjusted real/nominal GDP (Cabinet Office): quarterly estimates of GDP, chained, (1994:Q1-2006:Q3, Reference-year = 2000) and quarterly estimates of GDP, fixed-based, (1981:Q1-1994:Q1, Base-year = 1995) are linked at 1994:Q1.
  http://www.esri.cao.go.jp/en/sna/menu.html
  http://www.esri.cao.go.jp/en/sna/qe081-2/gdemenuea.html
  http://www.esri.cao.go.jp/en/sna/qe052-2/gdemenuebr.html

- Seasonally-adjusted GDP deflator (Cabinet Office): the deflator is calculated from seasonally-adjusted real/nominal GDP.

- Seasonally-adjusted labor force population, averaged over three months (Ministry of Internal Affairs and Communications): January 1981 - December 2007
  http://www.stat.go.jp/english/data/roudou/lngindex.htm

B Simulation Setting

The discount factor, β, is calibrated to be 0.99. We use uniform distributions for initial prior distributions of states, time-varying parameters, and parameters: $uniform(-1,1)$ for states, $uniform(0,1)$ for time-varying parameters, and $uniform(0,0.2)$ for parameters. The number of particle is 10,000 at time t. Thus, we generate 270,000 random variables at time t. In Eq. (22), we set $\boldsymbol{C}$ to zero.

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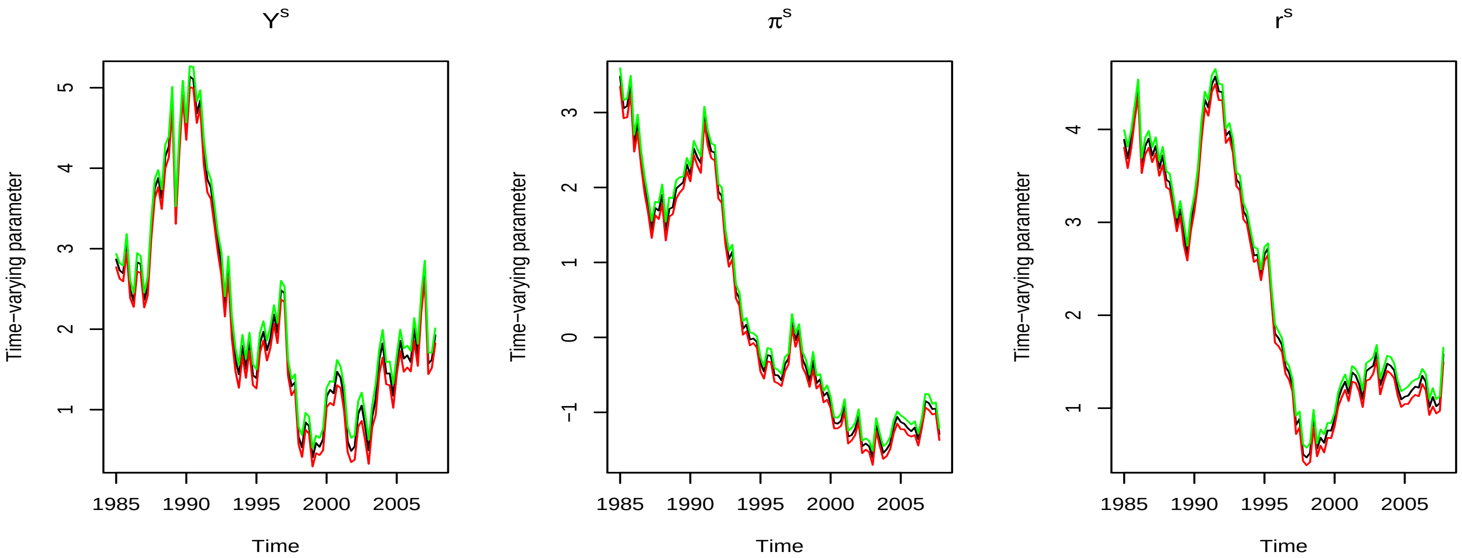

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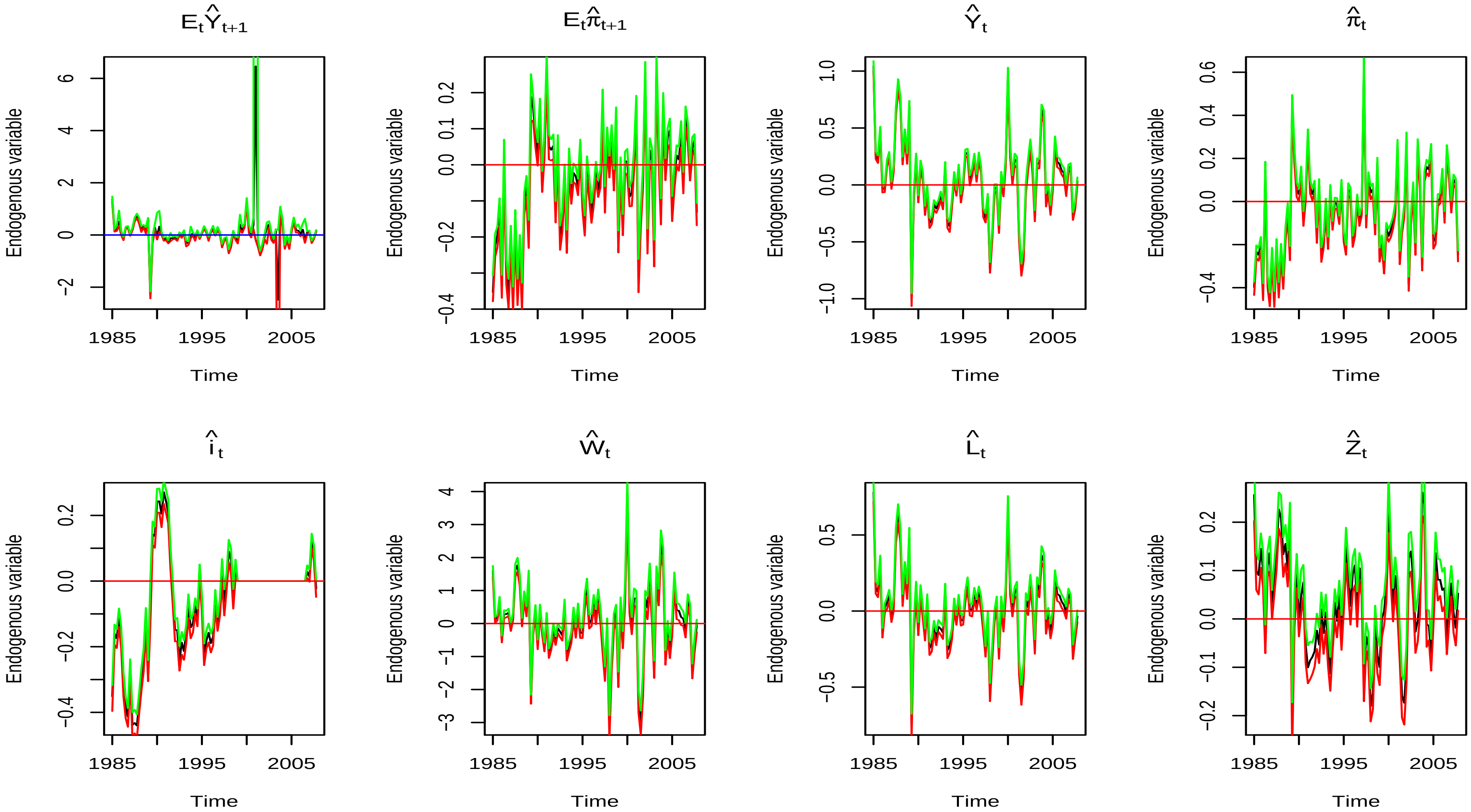

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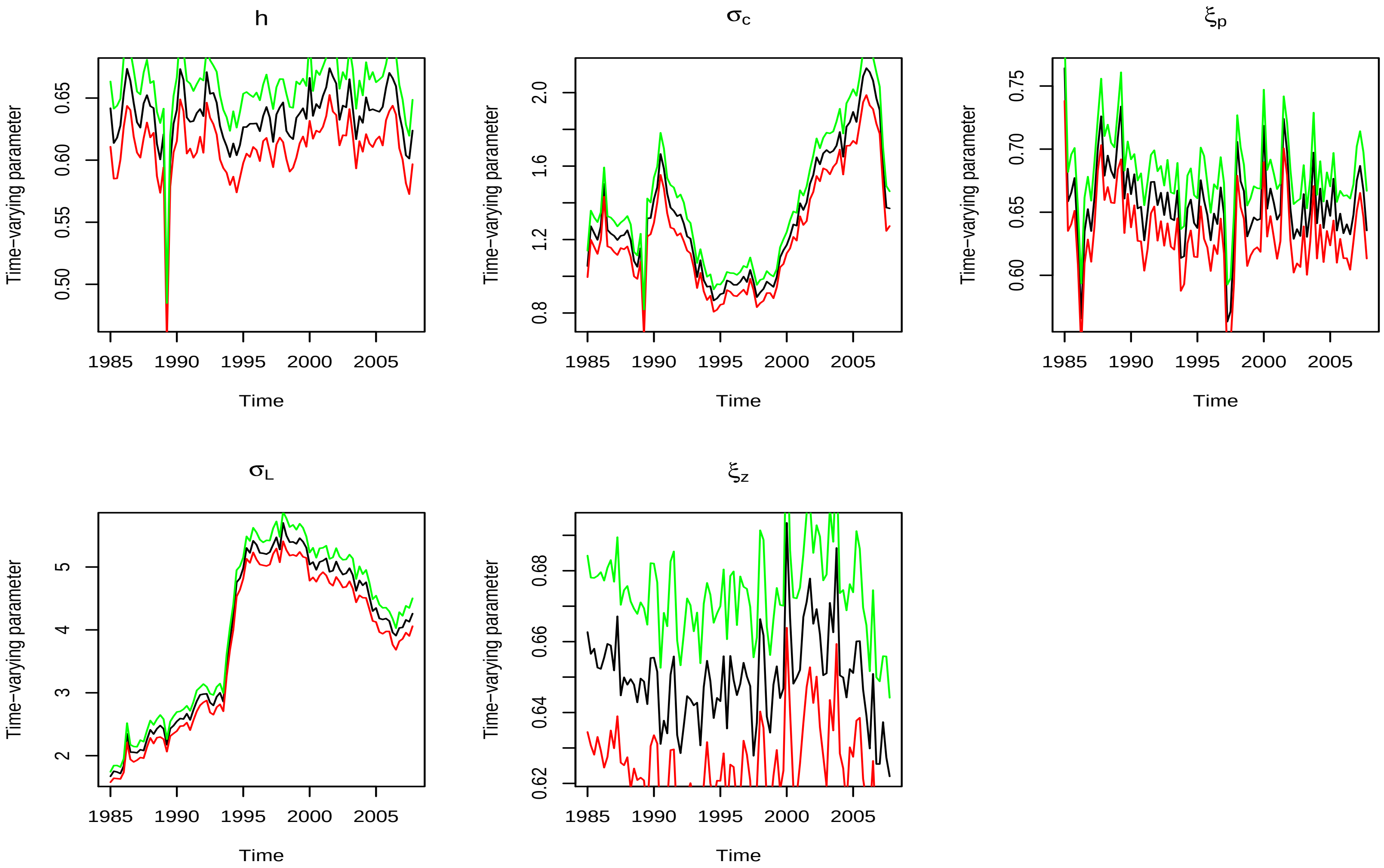

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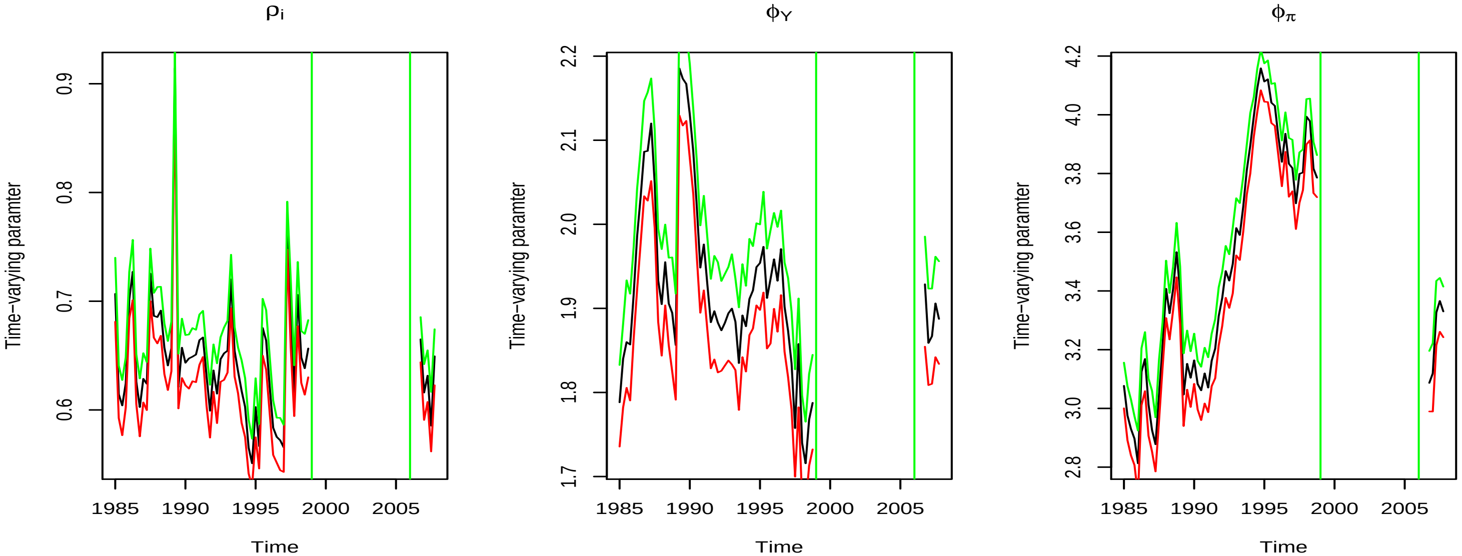

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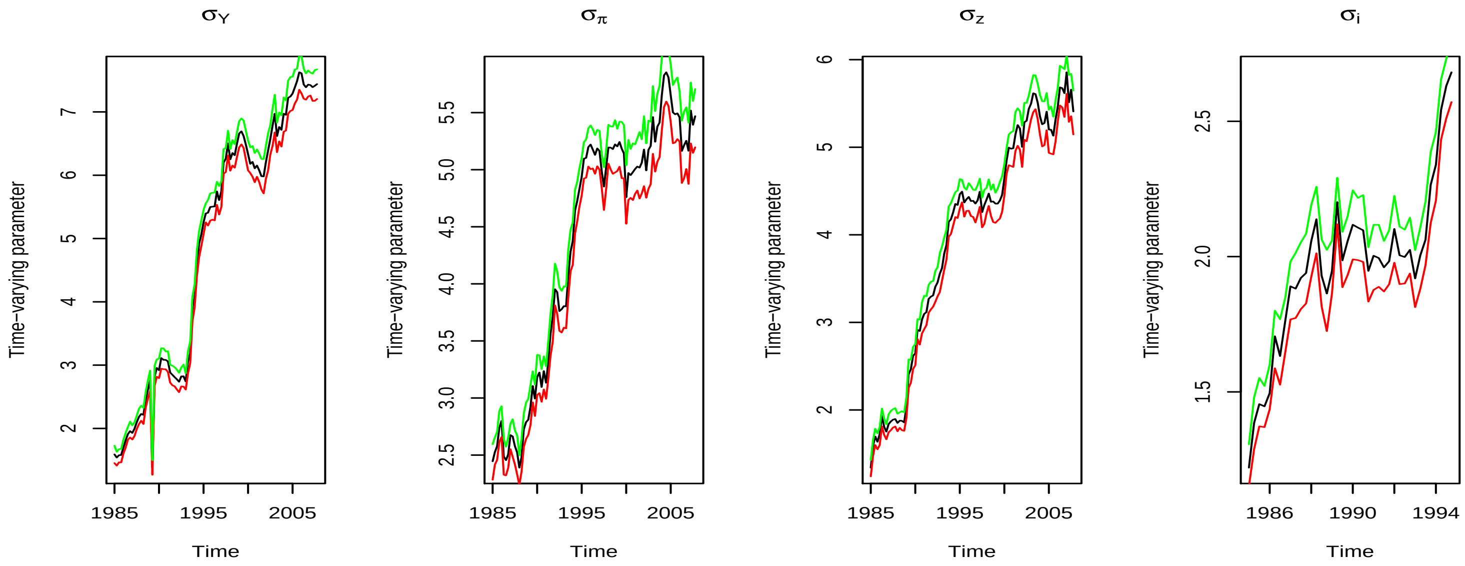

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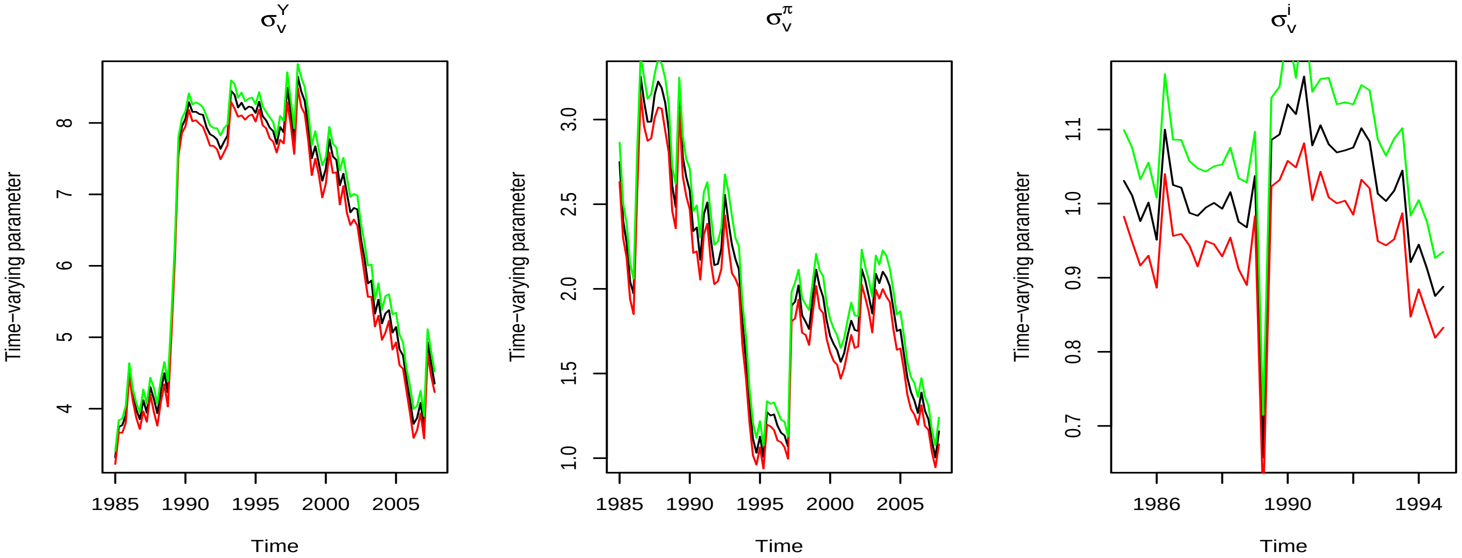

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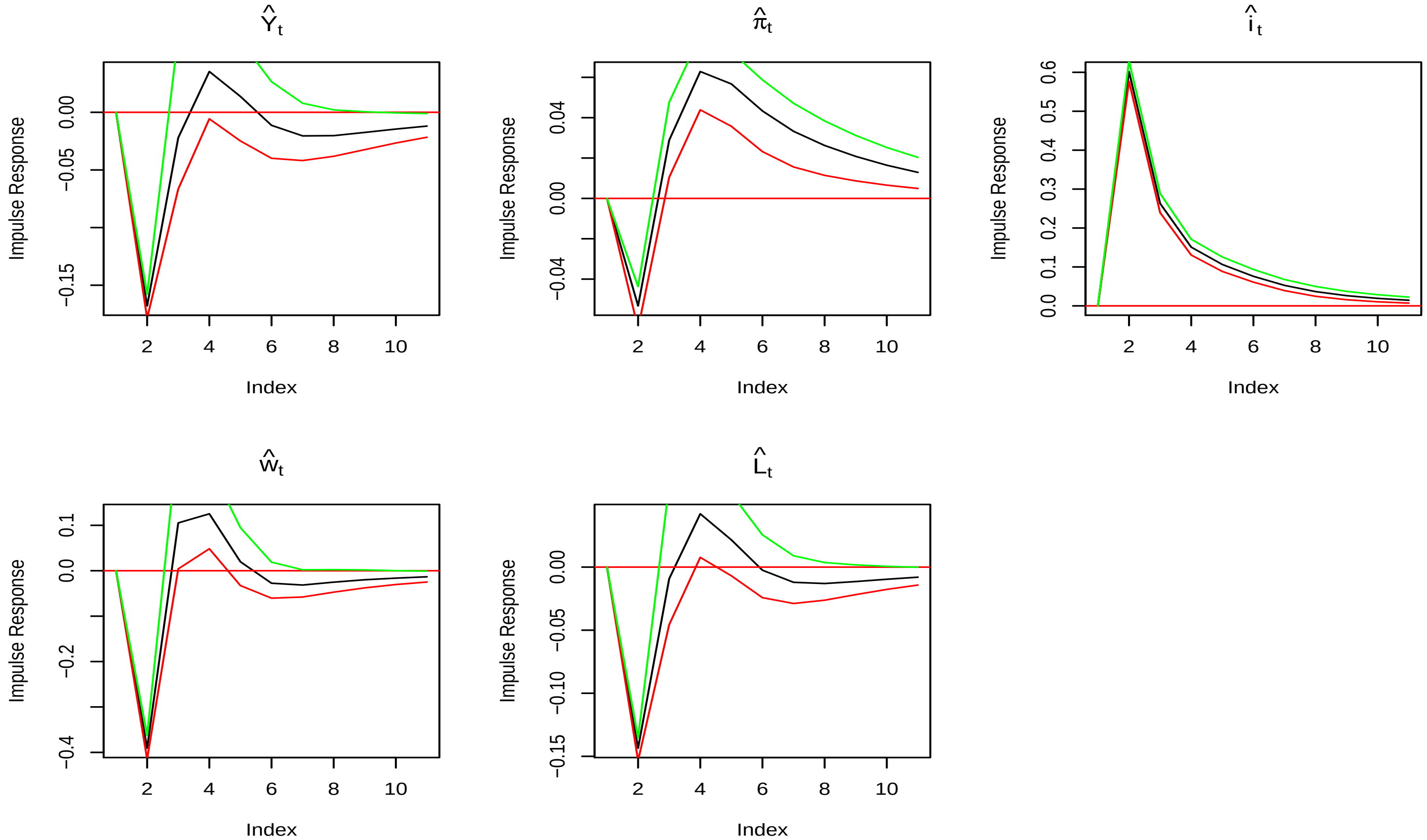

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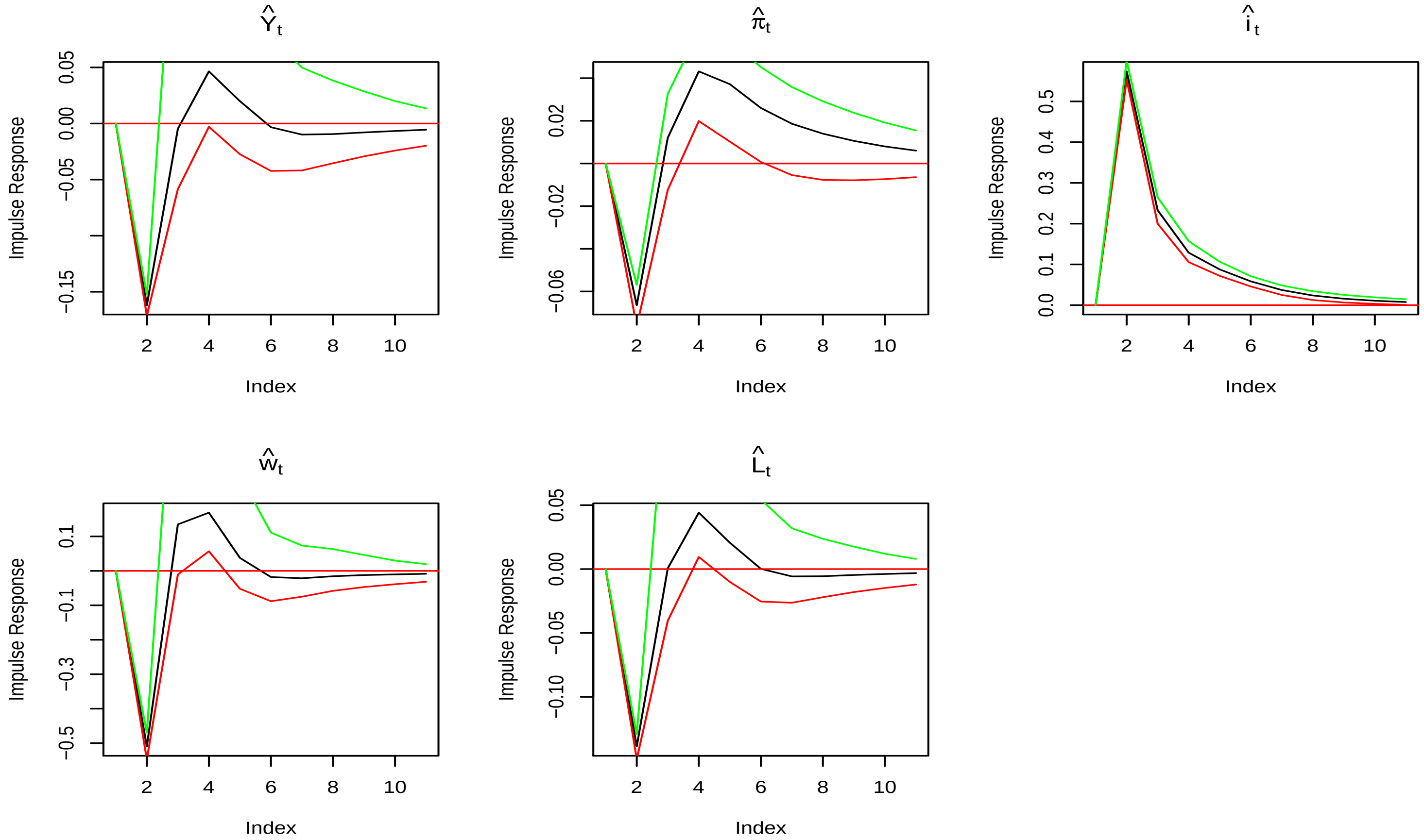

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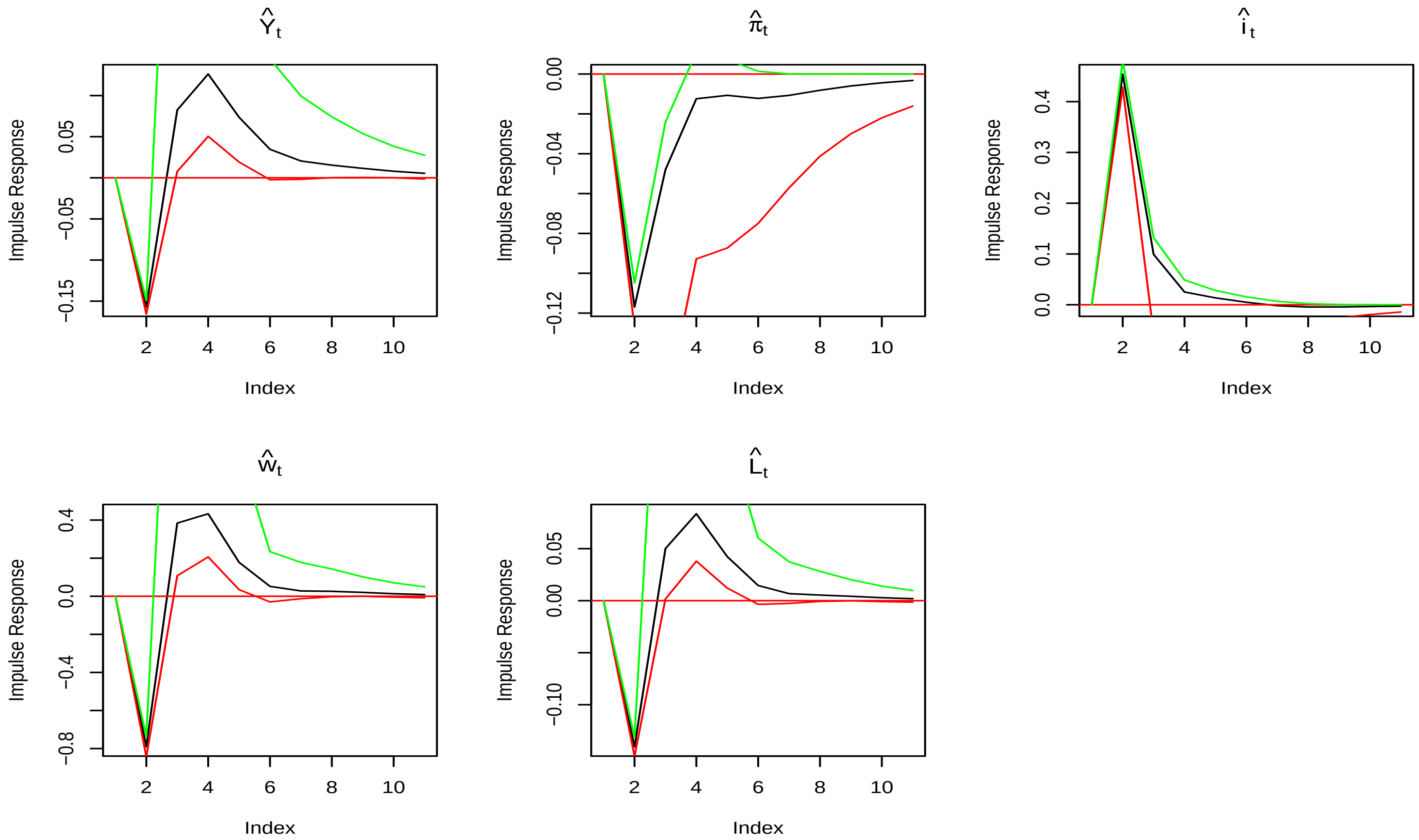

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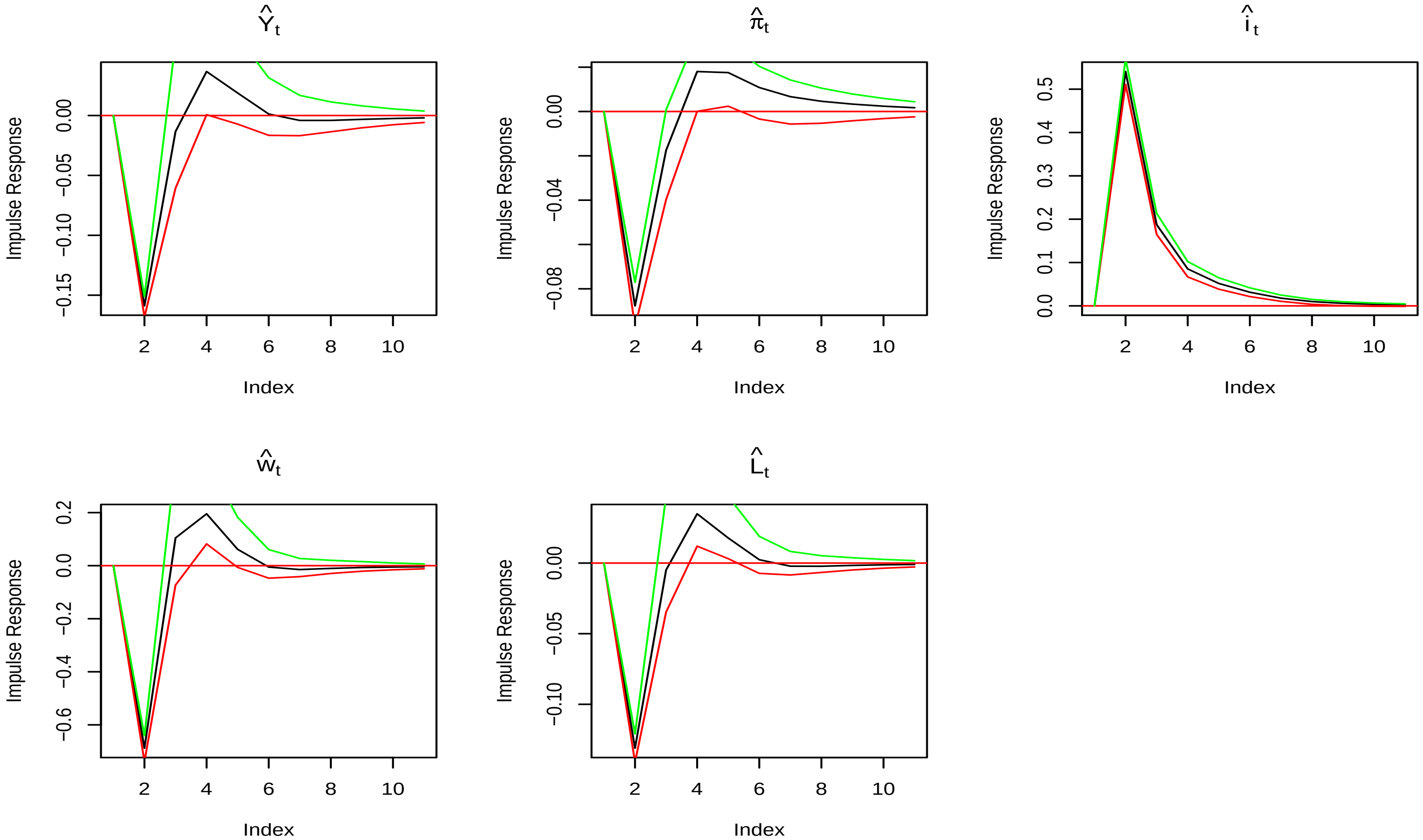

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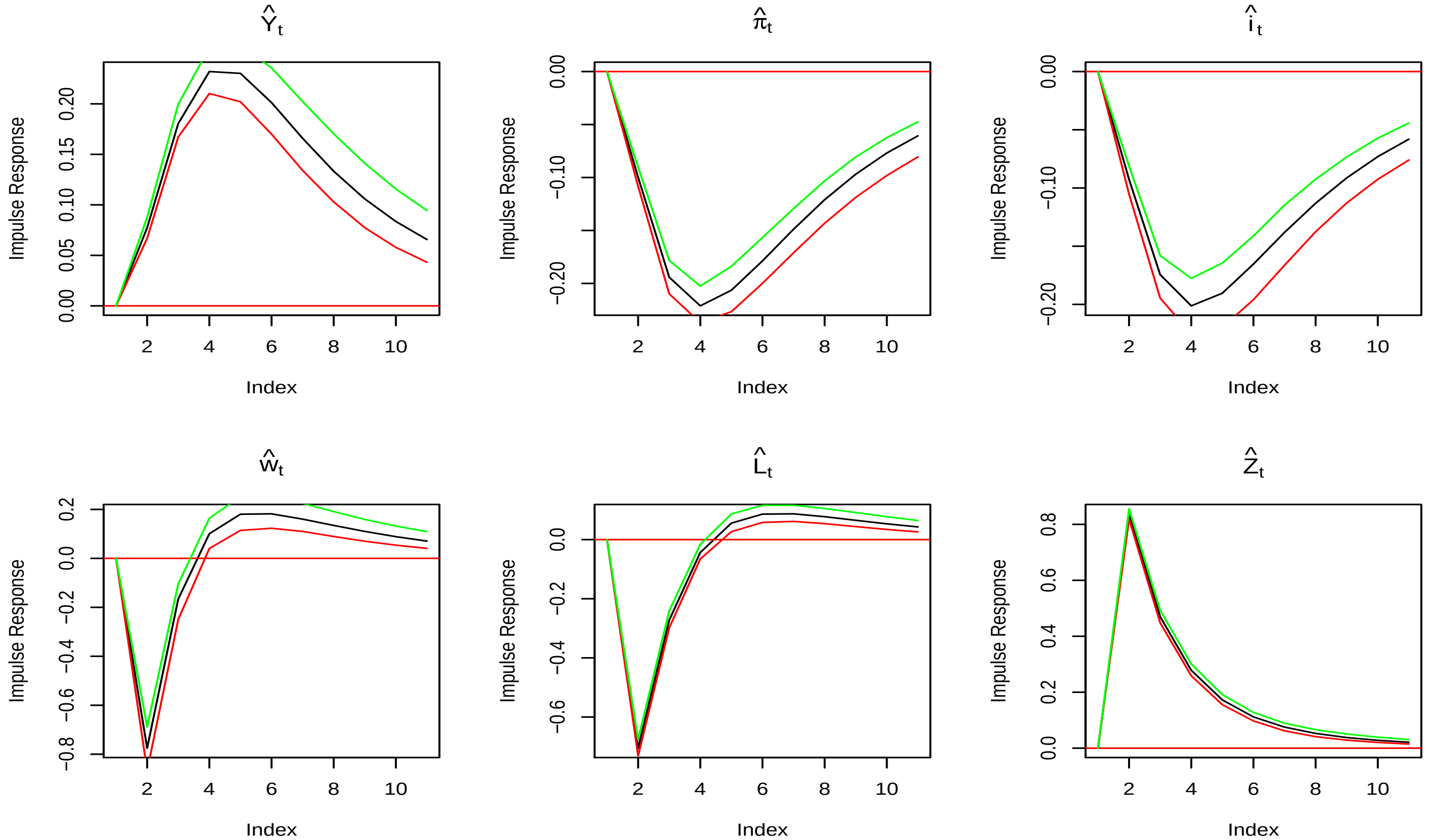

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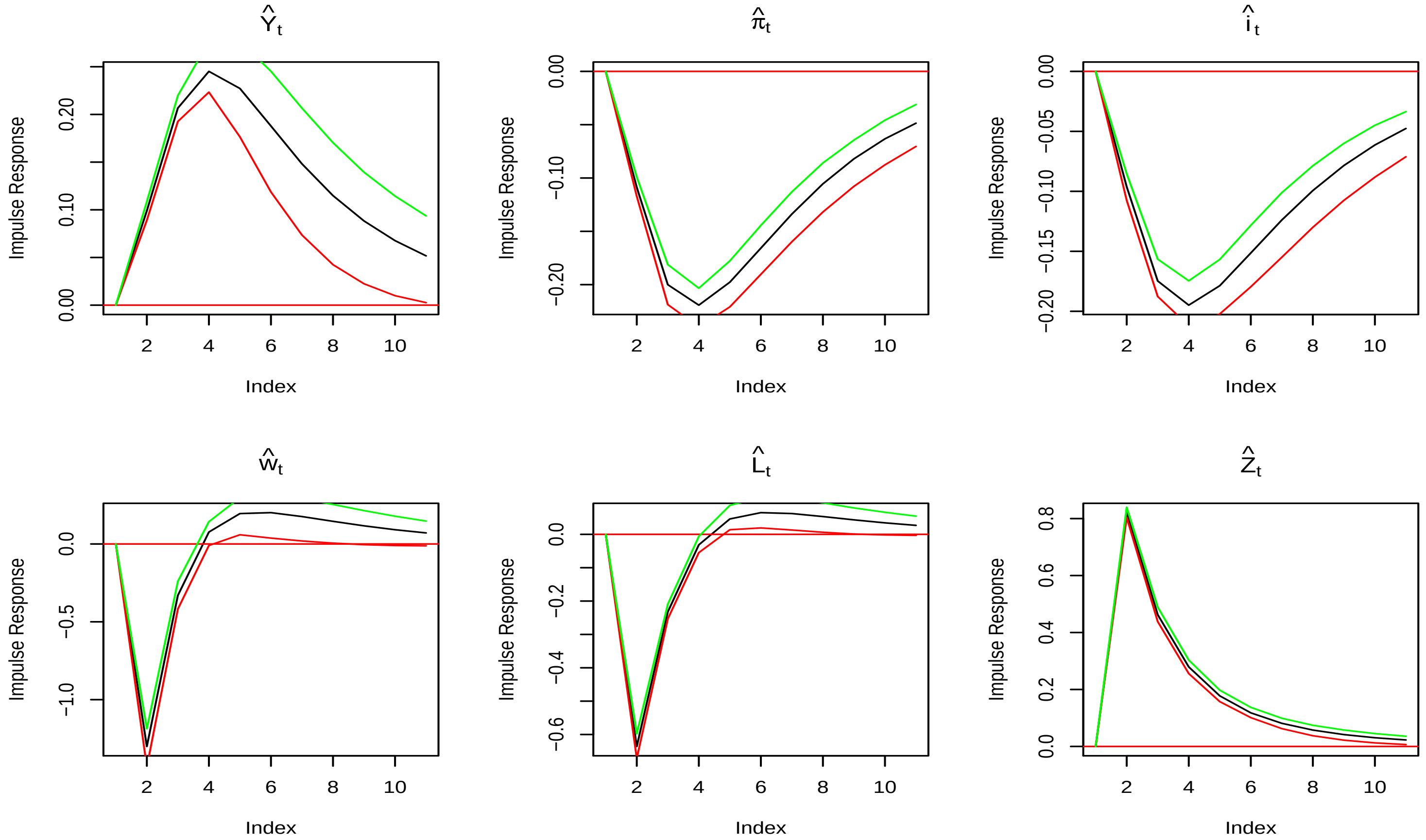

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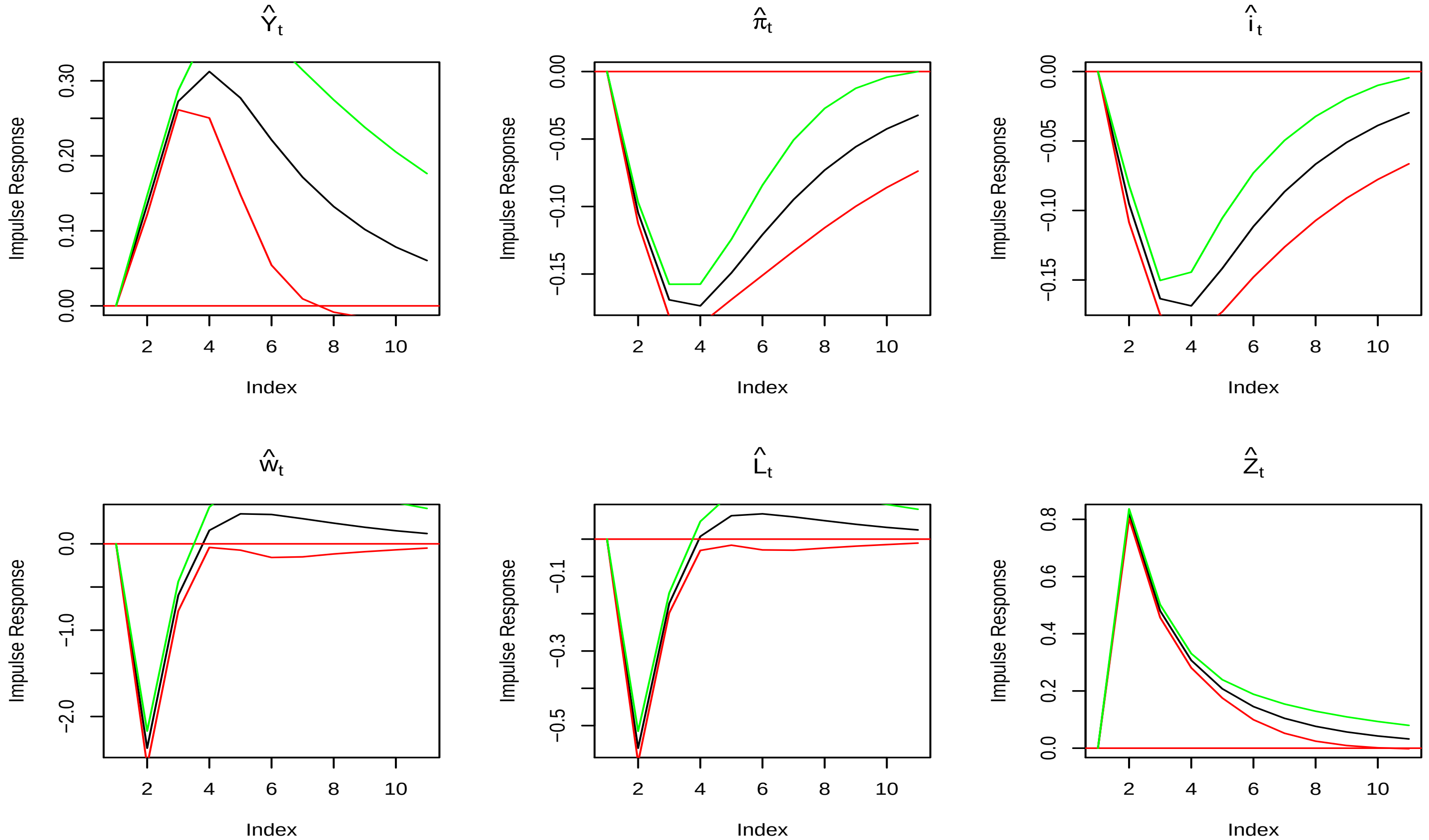

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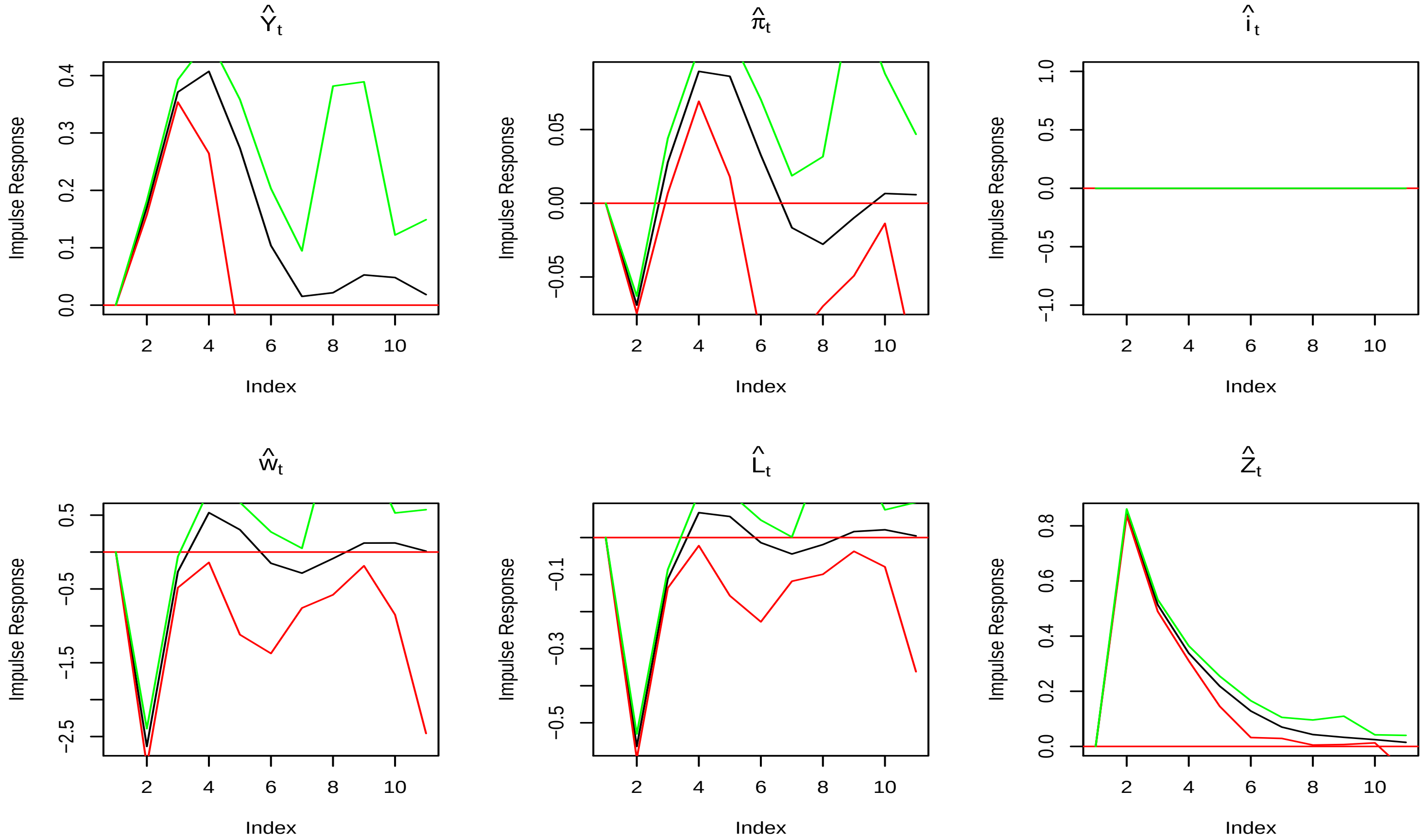

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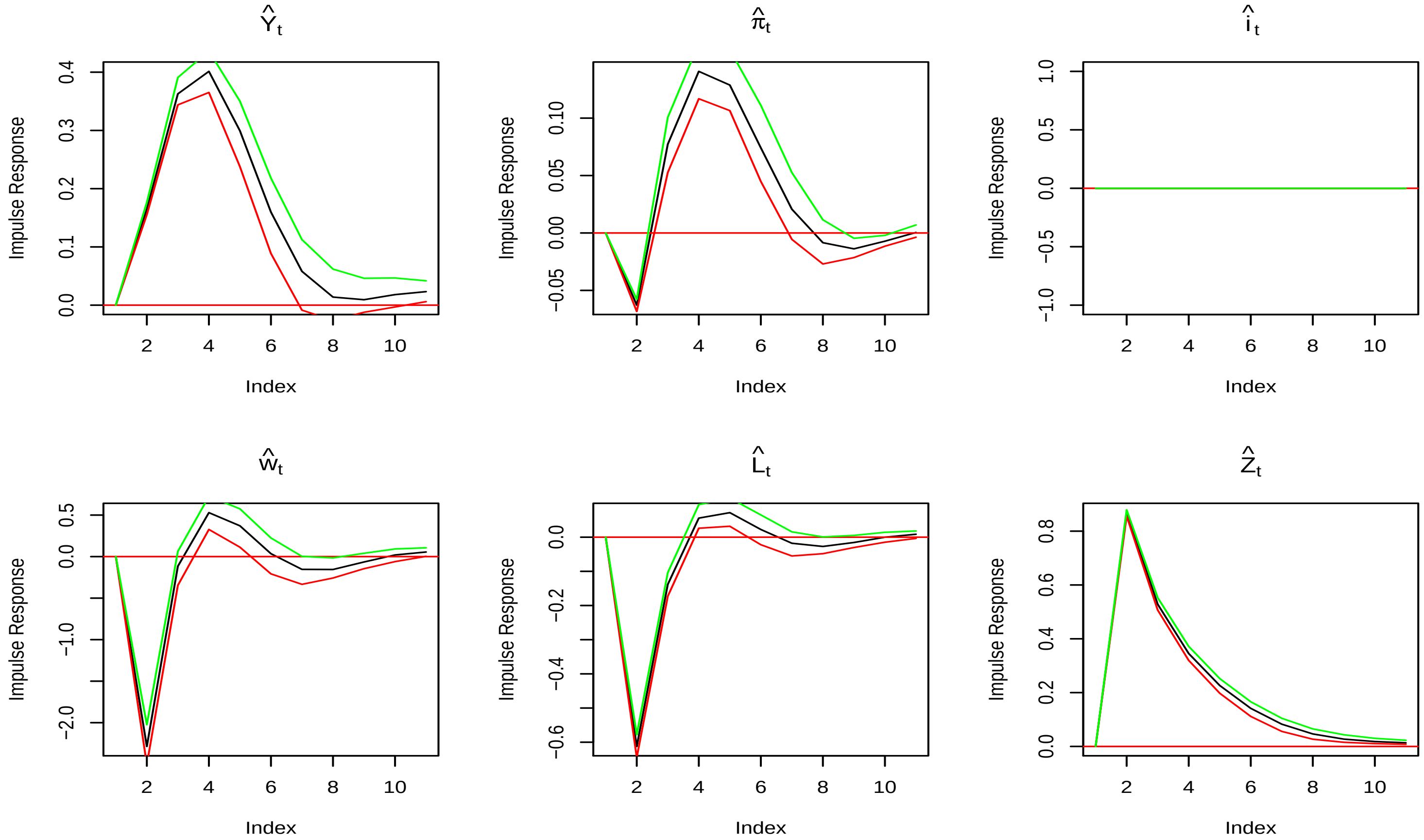

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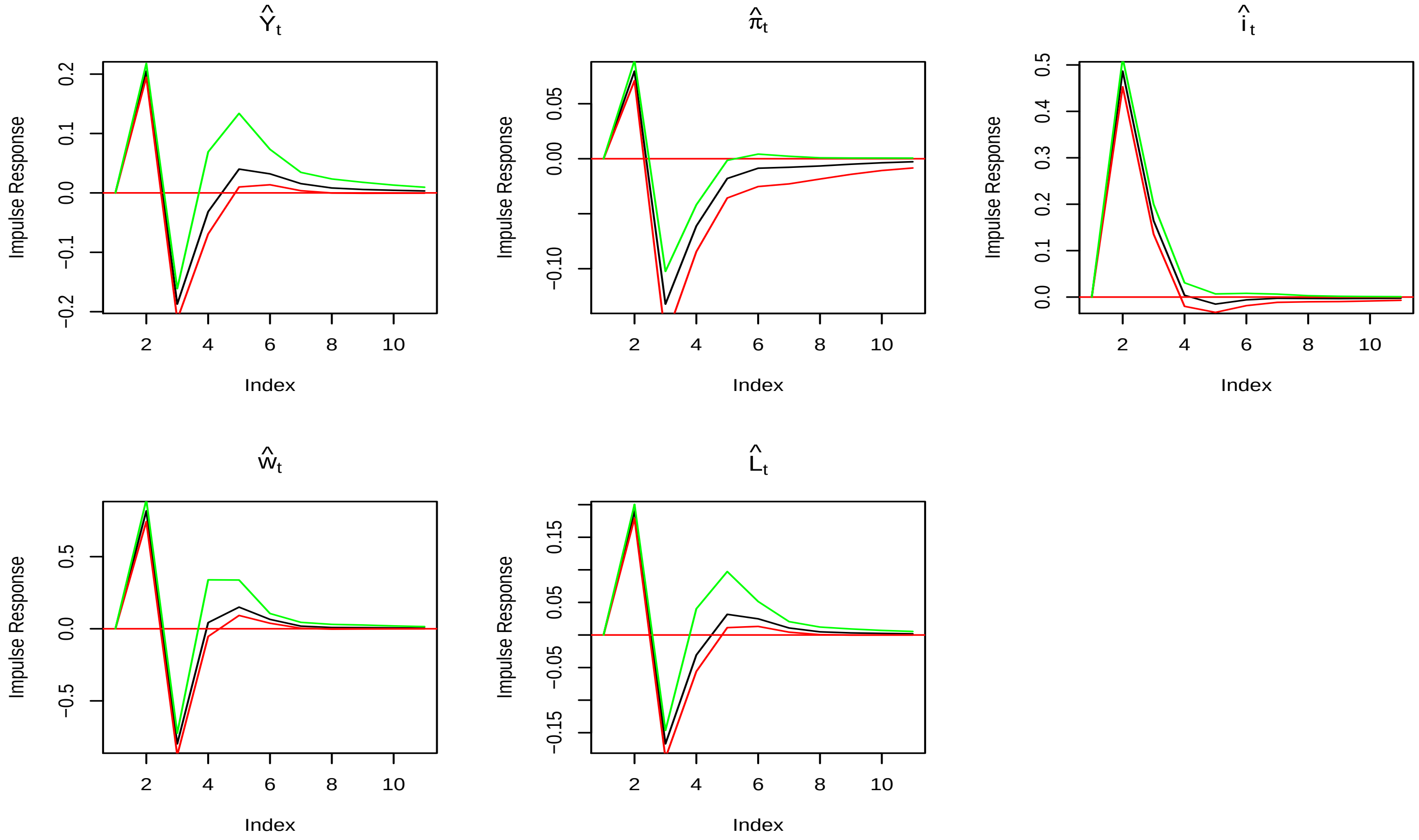

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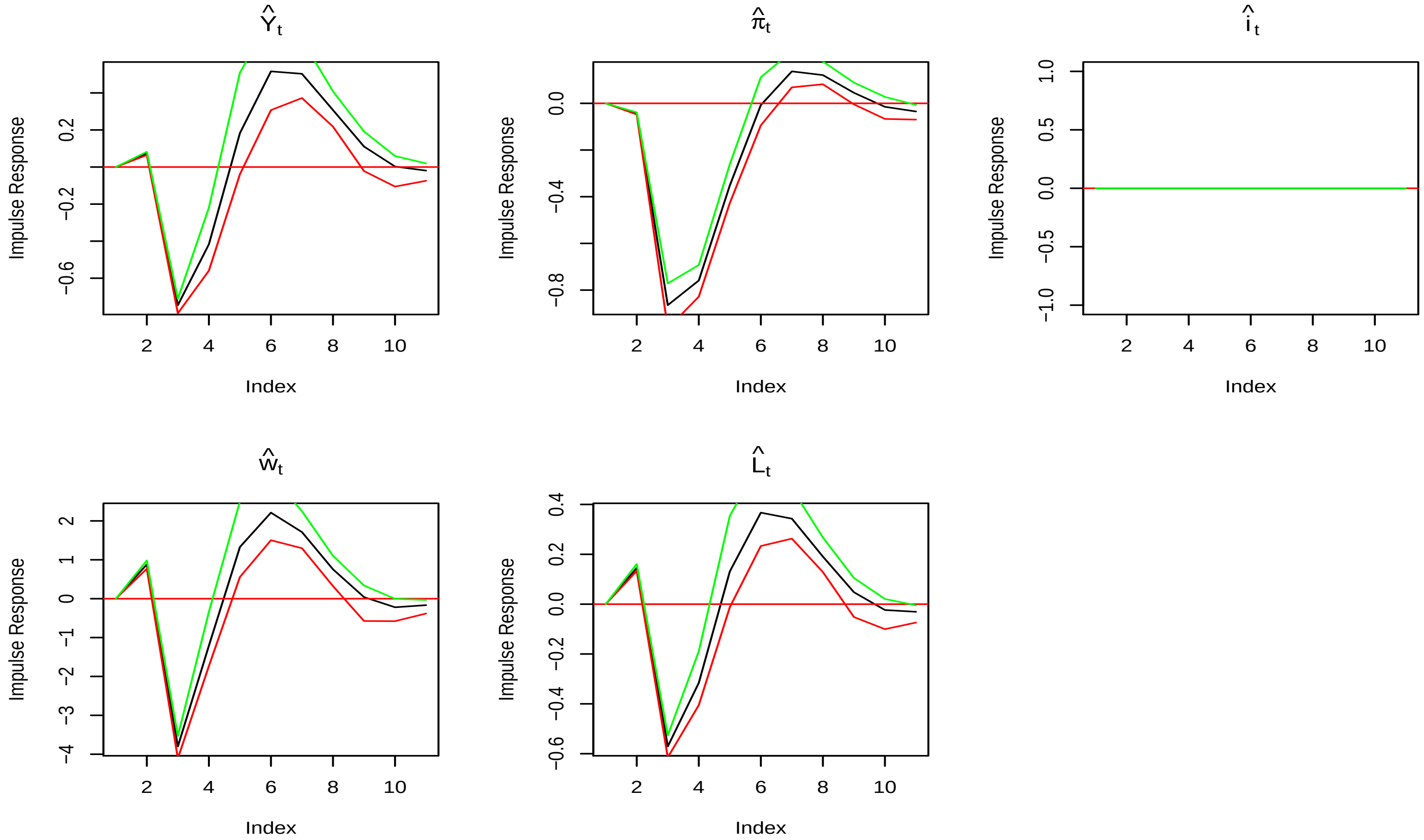

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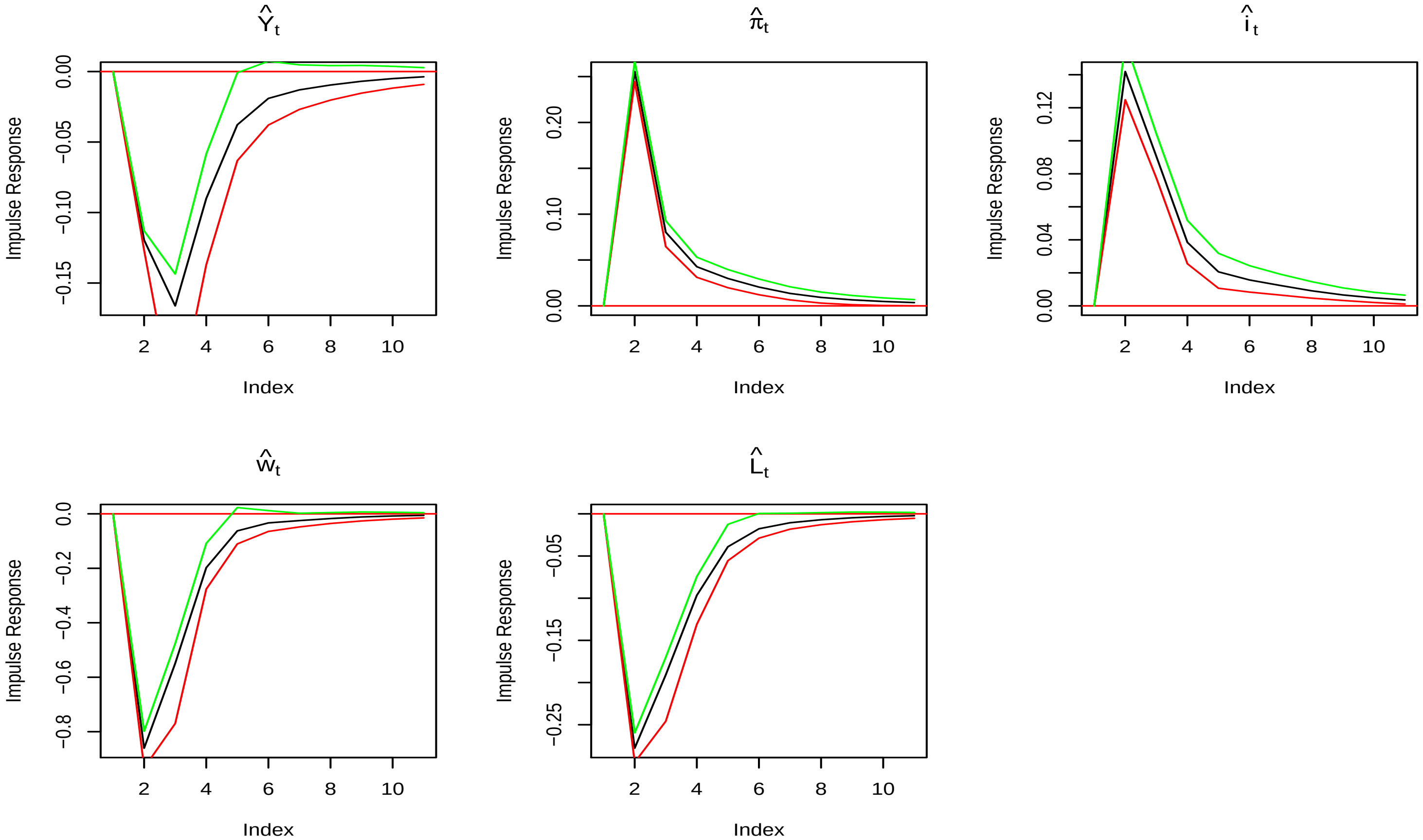

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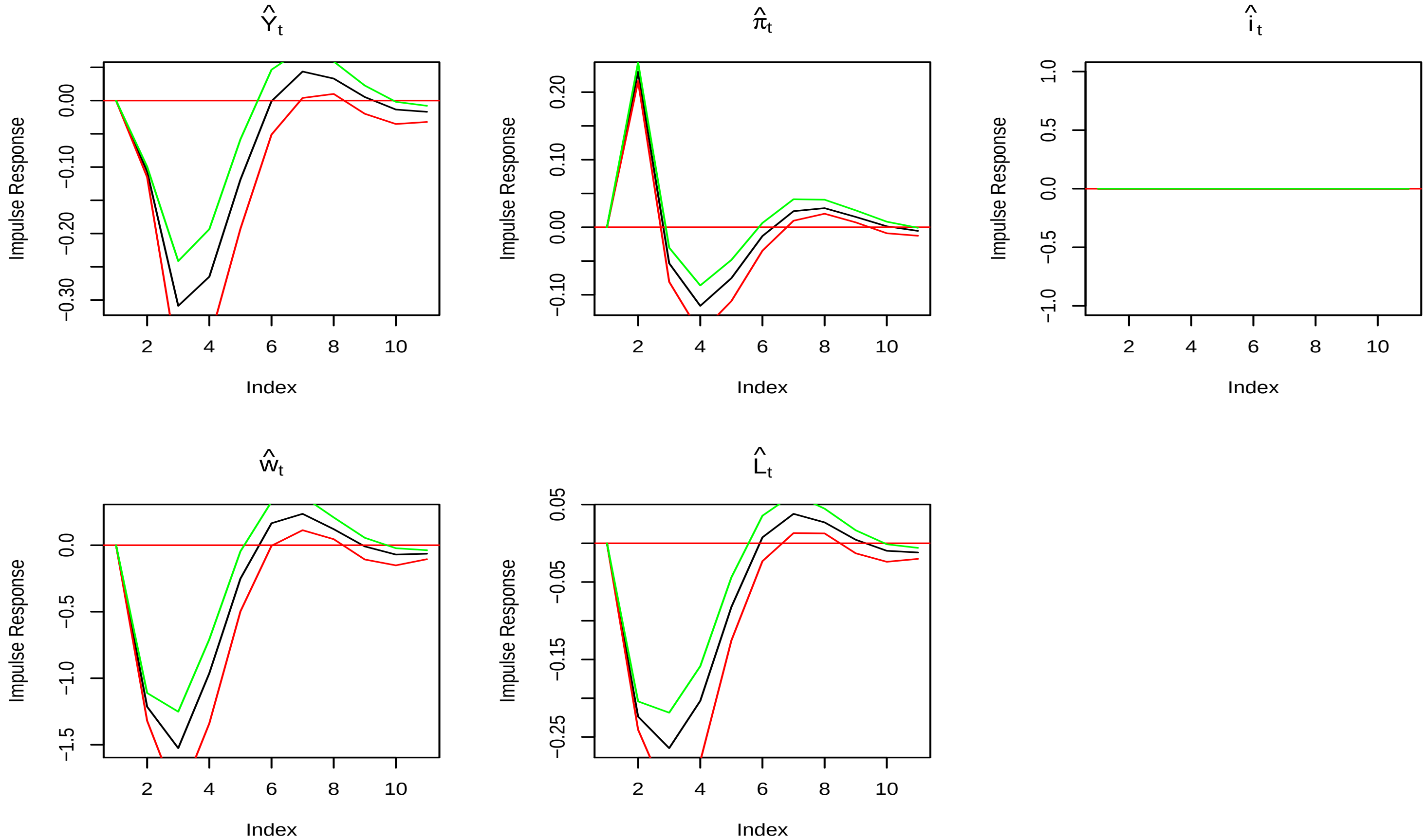

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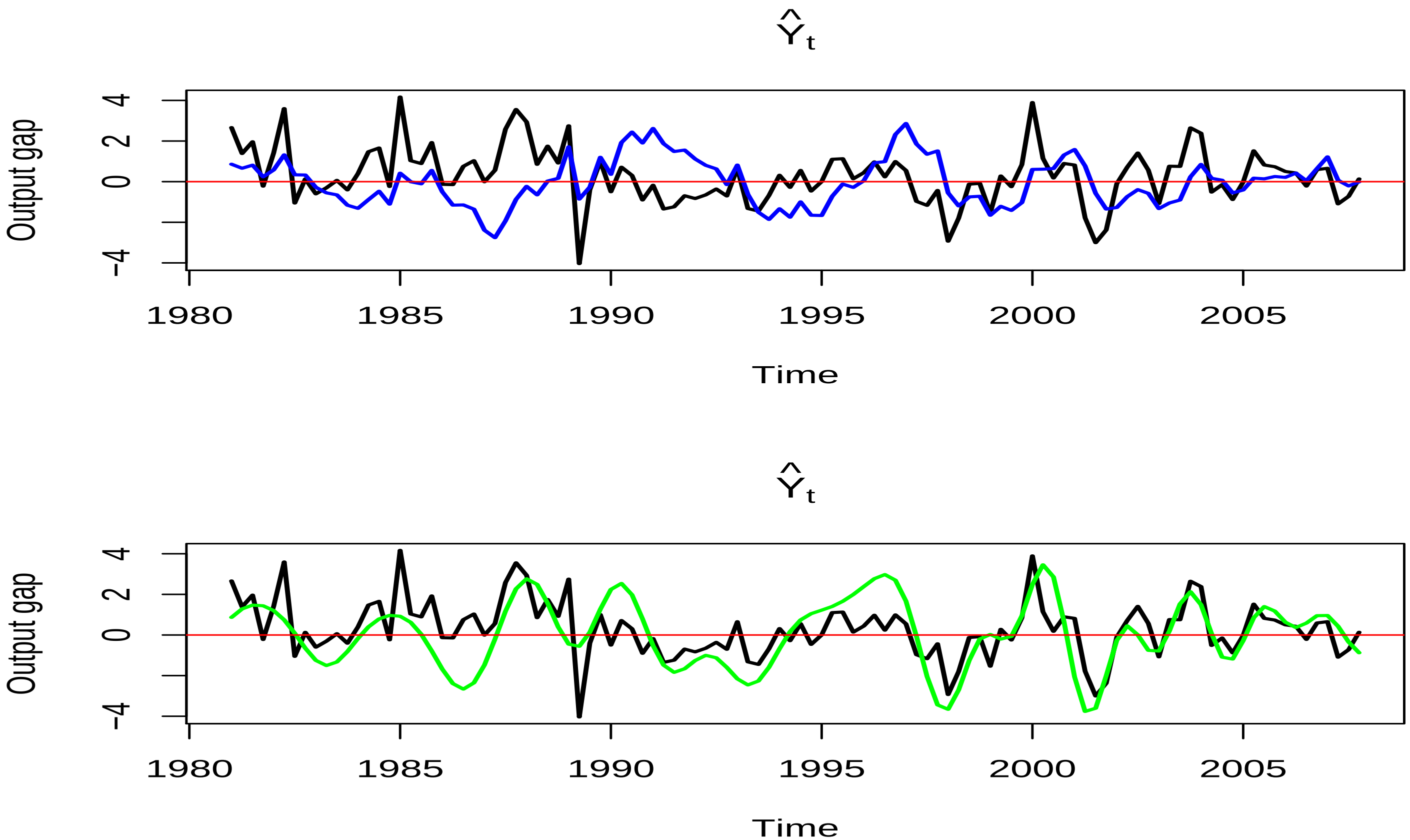

Figure 20: Output gap: Comparing filtering methods

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Table 1: Log-likelihood of model

Model	Log-likelihood	$\lvert\xi_s\rvert$
Standard Model	-695.80	0.4837
Model without inflation indexation	-698.57	0.3476
Model without habit formation	-699.92	0.3885