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Abstract

The DSGE-VAR model is used for estimating the parameters in a VAR model in a Bayesian fashion with the prior selected as if the artificial data were generated from a DSGE model. The more data is generated from the DSGE model, the more information from the DGSE model is incorporated. How important is the information from the DSGE model can be evaluated by changing the number of the data generated from the DSGE model and comparing the fit to the actual data. This article explains the DSGE-VAR model and applies it to the macroeconomic data in Japan. The DSGE model used in this article is a New-Keynesian model with several frictions such as stickiness in price and wage, habit formation in consumption and adjustment cost in investment, developed by Christiano, Eichenbaum and Evans (CEE, 2005). The comparison of the marginal likelihoods of the DSGE-VAR models with different sample sizes of the artificial data generated from the CEE model provides evidence that there is misspecification in the CEE model but the information from the CEE model is still useful, which is consistent with the finding of Del Negro et al. (2006) for the US economy.

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1 Introduction

After the publication of seminal work by Kydland and Prescott (1982), dynamic stochastic general equilibrium (DSGE) models have become popular in macroeconomics. One of the main advantages of DSGE models over macroeconometric models such as a vector autoregressive (VAR) model is that DSGE models can identify various shocks in a theoretically consistent way since they are structured from micro-foundation theories. Hence, DSGE models have attracted the attention of policy makers as well as macroeconomists.

DSGE models have been developed as New-Keynesian DSGE models with some market frictions. Among such New-Keynesian DSGE models, the most successful is the one proposed by Christiano, Eichenbaum and Evans (CEE, 2005). They introduce (1) Calvo-style nominal price and wage for expressing nominal rigidity, (2) habit formation in preference for consumption, (3) adjustment costs in investment and (4) variable capital utilization. As a result, their model has the ability to capture the time-series properties of macroeconomic data equivalent to a VAR model and can successfully explain inertia in inflation and persistence in output observed in the real world, which other macroeconomic models such as a real business cycle (RBC) model cannot explain.

As a method for the empirical analysis of DSGE models, Bayesian inference using Markov chain Monte Carlo (MCMC) techniques has become increasingly popular.[1] This method enables us to compare the DSGE model with non-nested models such as a VAR model using the posterior odds ratio, which is a usual tool for a Bayesian model comparison. Using the posterior odds ratio between the CEE and VAR models, Smets and Wouters (2003) show that the CEE model fits the data as well as the VAR model. As pointed out in Del Negro et al. (2006), this result, however, depends heavily on the choice of the prior distribution of the parameters and the sample periods.

As an alternative tool for evaluating DSGE models, DSGE-VAR model has been developed by Del Negro and Schorfheide (2004) and Del Negro et al. (2006) (see also Del Negro and Schorfheide (2006)). This model is used for estimating the parameters in a VAR model in a Bayesian fashion with the prior selected as if artificial data were generated from a DSGE model. The more data is generated from the DSGE model, the more information from the DGSE model is incorporated. How important is the information from the DSGE model can be evaluated by changing the number of data generated from the DSGE model and comparing the fit to the actual data.

This article explains the DSGE-VAR model and evaluates the DSGE model by applying the DSGE-VAR model to the macroeconomic data in Japan. Some researchers such as Iiboshi et al. (2006) and Sugo and Ueda (2008) have already fitted DSGE models to the macroeconomic data in Japan using the Bayesian method but they have not evaluated DSGE models. As far as I know, no researchers have fitted the DSGE-VAR

[1] See Iiboshi et al. (2006), Ang and Schorfheide (2007) and Fernández-Villaverde (2009), Canova (2007), Dejong and Dave (2007), and Pytlarczyk (2007) for the details.

model to the macroeconomic data in Japan. The DSGE model used in this article is the same as the one used by Iiboshi et al. (2006), which is almost the same as the CEE model. The dataset we use is also the same as the one used by Iiboshi et al. (2006), which consists of major seven macroeconomics quarterly series in Japan: real GDP, consumption, investment, labor input, real wage, inflation and nominal interest rate. As is well known, the zero interest rate bound started at 1999:Q1 in Japan. It is plausible that the macroeconomic behavior at the period of zero interest rate bound would be apart from the ordinary economic situation. Accordingly, the sample period is limited over 1970:Q1 through 1998:Q4, which is prior to the period of zero interest rate bound.

The comparison of the marginal likelihoods of the DSGE-VAR models with different sample sizes of the artificial data generated from the CEE model provides evidence that there is misspecification in the CEE model but the information from the CEE model is still useful, which is consistent with the finding of Del Negro et al. (2006) for the US economy.

The remainder of this article is organized as follows. Section 2 explains the DSGE-VAR model and how it can be applied to the evaluation of DSGE models. Section 3 reviews the CEE model used as a DSGE model. Section 4 describes the data used in the estimation. In Section 5, we evaluate the CEE model for the Japanese economy using the DSGE-VAR model. Section 6 concludes the paper.

2 Bayesian Analysis of DSGE-VAR Model

2.1 DSGE-VAR Model

Consider a pth order VAR model

$$\boldsymbol{y}_t = \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{\Phi}_p \boldsymbol{y}_{t-p} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim NID\left(\boldsymbol{0}, \boldsymbol{\Sigma}_u\right) \tag{1}$$

where $\boldsymbol{y_t}$ denotes a vector of n demeaned macroeconomic variables, and the error term, $\boldsymbol{u}_t$, is assumed to follow a serially independent normal distribution with mean $\boldsymbol{0}$ and covariance matrix $\boldsymbol{\Sigma}_u$.

Suppose that we have the sample of $\boldsymbol{y_t}$ $(t = 1, \ldots, T)$. Then, let $\boldsymbol{Y}$ be the $T \times n$ matrix with rows $\boldsymbol{y}_t'$, i.e., $\boldsymbol{Y} = [\boldsymbol{y}_1', \ldots, \boldsymbol{y}_T']'$. Let $k = np$, $\boldsymbol{X}$ be the $T \times k$ matrix with rows $\boldsymbol{x}_t' = [\boldsymbol{y}_{t-1}', \ldots, \boldsymbol{y}_{t-p}']$, $\boldsymbol{U}$ be the $T \times n$ matrix with rows u_t', and $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_p]'$. Then, the VAR model (1) can be expressed as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\Phi} + \boldsymbol{U} \tag{2}$$

The DSGE-VAR model is used for estimating the parameters $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}_u$ in the above VAR model using a Bayesian method. The prior of $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}_u$ conditional on $\boldsymbol{\theta}$, $f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u | \boldsymbol{\theta})$, is set as if artificial data were generated from the DSGE model with parameter $\boldsymbol{\theta}$. Suppose that $T^* = \lambda T$ artificial observations $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ are generated from the DSGE model with the parameter vector $\boldsymbol{\theta}$. Then, the likelihood function of the VAR model (1) or (2) can be expressed as

$$\begin{aligned} & f(\boldsymbol{Y}^*(\boldsymbol{\theta})|\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u) \\ & \propto \ |\boldsymbol{\Sigma}_u|^{-\lambda T/2} \\ & \quad \times \exp\left[-\frac{1}{2}\mathrm{tr}\left\{\boldsymbol{\Sigma}_u^{-1}(\boldsymbol{Y}^{*'}\boldsymbol{Y}^* - \boldsymbol{\Phi}'\boldsymbol{X}^{*'}\boldsymbol{Y}^* - \boldsymbol{Y}^{*'}\boldsymbol{X}^*\boldsymbol{\Phi} + \boldsymbol{\Phi}'\boldsymbol{X}^{*'}\boldsymbol{X}^*\boldsymbol{\Phi})\right\}\right] \end{aligned} \tag{3}$$

If we really generate $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$ and use them, this likelihood varies stochastically depending on $(\boldsymbol{Y}^*, \boldsymbol{X}^*)$. Hence, instead of generating them, $\boldsymbol{Y}^{*'}\boldsymbol{Y}^*$, $\boldsymbol{X}^{*'}\boldsymbol{Y}^*$, $\boldsymbol{Y}^{*'}\boldsymbol{X}^*$ and $\boldsymbol{X}^{*'}\boldsymbol{X}^*$ are replaced by their expected values. Let $\Gamma^*_{yy}(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{y}_t\boldsymbol{y}_t']$, $\Gamma^*_{xy}(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{x}_t\boldsymbol{y}_t']$ and $\Gamma^*_{yx}(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{y}_t\boldsymbol{x}_t']$. Given the parameter vector $\boldsymbol{\theta}$ in the DSGE model, these population moments can be calculated analyticaly. The method for calculating these moments will be explained in Section 3.5.

We further assume the improper prior $f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u) \propto |\boldsymbol{\Sigma}_u|^{-(n+1)/2}$. Then, (3) can be converted to the following prior.

$$\begin{aligned} & f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u|\boldsymbol{\theta}) \\ & = \ c^{-1}(\boldsymbol{\theta})|\boldsymbol{\Sigma}_u|^{-\frac{1}{2}(\lambda T+n+1)} \\ & \quad \times \exp\left[-\frac{1}{2}\mathrm{tr}\left\{\lambda T\boldsymbol{\Sigma}_u^{-1}(\Gamma^*_{yy}(\boldsymbol{\theta}) - \boldsymbol{\Phi}'\Gamma^*_{xy}(\boldsymbol{\theta}) - \Gamma^*_{yx}(\boldsymbol{\theta})\boldsymbol{\Phi} + \boldsymbol{\Phi}'\Gamma^*_{xx}(\boldsymbol{\theta})\boldsymbol{\Phi})\right\}\right] \end{aligned} \tag{4}$$

where $c^{-1}(\boldsymbol{\theta})$ is a normalizing constant to ensure that the prior density (4) integrates to one.

The prior distribution (4) is of the well-known Inverted Wishart-Normal form

$$\boldsymbol{\Sigma}_u|\boldsymbol{\theta} \sim \mathrm{IW}(\lambda T\boldsymbol{\Sigma}^*_u(\boldsymbol{\theta}), \lambda T - k, n) \tag{5}$$

$$\boldsymbol{\Phi}|\boldsymbol{\Sigma}_u, \boldsymbol{\theta} \sim \mathrm{N}(\boldsymbol{\Phi}^*(\boldsymbol{\theta}), \boldsymbol{\Sigma}_u \otimes (\lambda T\Gamma^*_{xx}(\boldsymbol{\theta}))^{-1}) \tag{6}$$

where

$$\boldsymbol{\Phi}^*(\boldsymbol{\theta}) = \Gamma^{*-1}_{xx}(\boldsymbol{\theta})\Gamma^*_{xy}(\boldsymbol{\theta}) \tag{7}$$

$$\boldsymbol{\Sigma}^*_u(\boldsymbol{\theta}) = \Gamma^*_{yy}(\boldsymbol{\theta}) - \Gamma^*_{yx}(\boldsymbol{\theta})\Gamma^{*-1}_{xx}(\boldsymbol{\theta})\Gamma^*_{xy}(\boldsymbol{\theta}) \tag{8}$$

If $\lambda \geq k + n$ and $\Gamma^*_{xx}(\boldsymbol{\theta})$ is invertible, the prior denisty (4) is proper and nondegenerate, so that the normalizing constant can be defined as

$$\begin{aligned} c(\boldsymbol{\theta}) \ &= \ (2\pi)^{nk/2}|\lambda T\Gamma^*_{xx}(\boldsymbol{\theta})|^{-n/2}|\lambda T\boldsymbol{\Sigma}^*_u(\boldsymbol{\theta})|^{-(\lambda T-k)/2} \\ & \quad \times 2^{-n(\lambda T-k)/2}\pi^{n(n-1)/4}\prod_{i=1}^{n}\Gamma[(\lambda T - k + 1 - i)/2] \end{aligned} \tag{9}$$

where $\Gamma[\,\cdot\,]$ denotes the gamma function.

2.2 Posterior distribution for DSGE-VAR model

The posterior distribution is written

$$f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u, \boldsymbol{\theta}|\boldsymbol{Y}) = f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u|\boldsymbol{Y}, \boldsymbol{\theta}) f(\boldsymbol{\theta}|\boldsymbol{Y}) \tag{10}$$

With the prior (5) and (6), $f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u|\boldsymbol{Y}, \boldsymbol{\theta})$ is also of the Inverted Wishart-Normal form

$$\boldsymbol{\Sigma}_u|\boldsymbol{Y}, \boldsymbol{\theta} \sim \mathrm{IW}((\lambda+1)T\tilde{\boldsymbol{\Sigma}}_u(\boldsymbol{\theta}), (\lambda+1)T-k, n) \tag{11}$$

$$\boldsymbol{\Phi}|\boldsymbol{Y}, \boldsymbol{\Sigma}_u, \boldsymbol{\theta} \sim \mathrm{N}(\tilde{\boldsymbol{\Phi}}(\boldsymbol{\theta}), \boldsymbol{\Sigma}_u \otimes (\lambda T \Gamma^*_{xx}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{X})^{-1}) \tag{12}$$

where

$$\tilde{\boldsymbol{\Phi}}(\boldsymbol{\theta}) = (\lambda T \Gamma^*_{xx}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{X})^{-1}(\lambda T \Gamma^*_{xy}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{Y}) \tag{13}$$

$$\begin{aligned}\tilde{\boldsymbol{\Sigma}}_u = & \frac{1}{(\lambda+1)T}\Big\{\lambda T \Gamma^*_{yy}(\boldsymbol{\theta}) + \boldsymbol{Y}'\boldsymbol{Y} \\ & -(\lambda T \Gamma^*_{yx}(\boldsymbol{\theta}) + \boldsymbol{Y}'\boldsymbol{X})(\lambda T \Gamma^*_{xx}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{X})^{-1}(\lambda T \Gamma^*_{xy}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{Y})\Big\}\end{aligned} \tag{14}$$

The likelihood function of $\boldsymbol{\theta}$ is given by

$$\begin{aligned} & f(\boldsymbol{Y}|\boldsymbol{\theta}) \\ = & \frac{f(\boldsymbol{Y}|\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u) f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u|\boldsymbol{\theta})}{f(\boldsymbol{\Phi}, \boldsymbol{\Sigma}_u|\boldsymbol{Y})} \\ = & \frac{|\lambda T \Gamma^*_{xx}(\boldsymbol{\theta}) + \boldsymbol{X}'\boldsymbol{X}|^{-n/2} |(\lambda+1)T\tilde{\boldsymbol{\Sigma}}_u(\boldsymbol{\theta})|^{-\{(\lambda+1)T-k\}/2}}{|\lambda T \Gamma^*_{xx}(\boldsymbol{\theta})|^{-n/2} |\lambda T \boldsymbol{\Sigma}^*_u(\boldsymbol{\theta})|^{-\{\lambda T-k\}/2}} \\ & \times \frac{(2\pi)^{-nT/2} 2^{n\{(\lambda+1)T-k\}/2} \prod_{i=1}^{n} \Gamma[((\lambda+1)T-k+1-i)/2]}{2^{n(\lambda T-k)/2} \prod_{i=1}^{n} \Gamma[(\lambda T-k+1-i)/2]} \end{aligned} \tag{15}$$

The marginal posterior density of $\boldsymbol{\theta}$ can be expressed as

$$f(\boldsymbol{\theta}|\boldsymbol{Y}) \propto f(\boldsymbol{Y}|\boldsymbol{\theta}) f(\boldsymbol{\theta}) \tag{16}$$

where the likelihood function $f(\boldsymbol{Y}|\boldsymbol{\theta})$ is given by equation (15) and the prior density $f(\boldsymbol{\theta})$ will be explained below.

If $\boldsymbol{\theta}$ can be sampled from (15), it is straightforward to sample $\boldsymbol{\Phi}$ and $\boldsymbol{\Sigma}_u$ from (11) and (12). Following the previous literature, we use the random-walk Metropolis-Hastings algorithm to sample from the density (16).

Random-walk Metropolis-Hastings algorithm

(1) Set $n = 1$.

(2) Sample the proposal $\boldsymbol{\theta}_n^{(\text{proposal})}$ from the random-walk model:

$$\boldsymbol{\theta}_n^{(\text{proposal})} = \boldsymbol{\theta}_{n-1} + \boldsymbol{\nu_t}, \quad \boldsymbol{\nu_t} \sim \text{i.i.d.N}(0, c\boldsymbol{H}),$$

where c is a scalar called the adjustment coefficient, whose choice will be explained below, and $\boldsymbol{H}$ is usually set equal to $-l''^{-1}(\hat{\boldsymbol{\theta}})$, where $\hat{\boldsymbol{\theta}}$ is the mode of $l(\boldsymbol{\theta}) = \ln f(\boldsymbol{\theta}|\boldsymbol{Y})$ and $l''^{-1}(\hat{\boldsymbol{\theta}})$ is the inverse of the second derivative of $l(\boldsymbol{\theta})$ at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$.

(3) Using the sampled draw $\boldsymbol{\theta}_n^{(\text{proposal})}$, calculate the acceptance probability q as follows.

$$q = \min\left[\frac{f(\boldsymbol{\theta}_n^{(\text{proposal})}|\text{data})}{f(\boldsymbol{\theta}_{n-1}|\text{data})}, 1\right]$$

(4) Accept $\boldsymbol{\theta}_n^{(\text{proposal})}$ with probability q and reject it with probability $1 - q$. Set $\boldsymbol{\theta}_n = \boldsymbol{\theta}_n^{(\text{proposal})}$ when accepted and $\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1}$ when rejected.

(5) If $n < N$, set $n = n + 1$ and return to (2). If $n = N$, end.

We must be careful for $\boldsymbol{\theta}_n^{(\text{proposal})}$ not to deviate from $\boldsymbol{\theta}_{n-1}$ so much because the acceptance probability q may be low when those deviate far from each other. This may be achieved by making c low, but $\boldsymbol{\theta}_n^{(\text{proposal})}$ may be sampled only from the narrow range if c is too low. It is a common practice to choose c such that the acceptance probability is around 25%.[2]

2.3 Marginal Likelihood

In the DSGE-VAR model, the prior distribution of the parameters in the VAR model is set using the artificial observations generated from the DSGE model. The more data is generated from the DSGE model, the more information from the DSGE model is incorporated. Thus, we can analyze how important is the information obtained from the DSGE model by changing λ and comparing the fit to the actual data.

In a Bayesian framework, the model selection is usually conducted using the posterior odds ratio. The posterior odds ratio in favor of model i, M_i, to model j, M_j, is given by

$$\text{POR} = \frac{f(M_i|\boldsymbol{Y})}{f(M_j|\boldsymbol{Y})} = \frac{f(\boldsymbol{Y}|M_i)}{f(\boldsymbol{Y}|M_j)}\frac{f(M_i)}{f(M_j)}, \tag{17}$$

where $f(\boldsymbol{Y}|M_i)/f(\boldsymbol{Y}|M_j)$ and $f(M_i)/f(M_j)$ are called Bayes factor and prior odds ratio respectively.

If this posterior odds ratio is greater (less) than one, M_i (M_j) will be selected. The prior odds ratio is usually set to be one, so that the posterior odds ratio is equal to the Bayes factor. The Bayes factor is the ratio of marginal likelihoods $f(\boldsymbol{Y}|M_i)$ and $f(\boldsymbol{Y}|M_j)$, so that the model with a higher marginal likelihood will be selected. Following the previous literature, we use the modified harmonic mean estimator proposed by Geweke (1999) to calculate the marginal likelihood (see Appendix A for the details of this estimator).

[2] Roberts and Rosenthal (2007), Roberts and Rosenthal (2008) and Rosenthal (2008) propose an efficient random-walk Metropolis-Hastings algorithm. where c is chosen automatically. Chib and Ramamurthy (2008) propose a more efficient method by combining Gibbs sampler with independent random-walk Metropolis-Hastings algorithm.

3 The CEE Model

DSGE models have been developed as New-Keynesian DSGE models with some market frictions. Among such New-Keynesian DSGE models, the most successful is the one proposed by Christiano, Eichenbaum and Evans (CEE, 2005). They introduce (1) Calvo-style nominal price and wage for expressing nominal rigidity, (2) habit formation in preference for consumption, (3) adjustment costs in investment and (4) variable capital utilization. In this article, we use this model as a DSGE model. In this section, we explain this model briefly.

3.1 The Household/Investor Sector

3.1.1 Preference and Budget Constraint

Each continuum of households is indexed by $h \in (0,1)$ and assumed to possess an identical preference toward consumption and leisure. In particular, each household seeks to maximize the following utility function.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t(h) - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{l_t(h)^{1+\sigma_L}}{1+\sigma_L} \right] \tag{18}$$

where $c_t(h)$ stands for the real aggregate consumption of household h, H_t stands for the external habit formation which is exogenously given to household h, and $l_t(h)$ stands for the labor supplies of household h. Each household h supplies a differentiated type of labor and, thus, decides to choose the amount of labor supply $l_t(h)$ monopolistically in the labor market. Parameter β stands for the discount rate, σ_c stands for the inverse of the long-run intertemporal elasticity of substitution, and σ_L stands for labor supply elasticity. It is simply assumed that $c_t(h)$ and $l_t(h)$ are additive-separable from each other. Now, the household h faces the following budget constraint for each period.

$$\begin{aligned} & B_t(h) + P_t\left[c_t(h) + inv_t(h) + a(u_t(h))K_t(h)\right] \\ & = R_{t-1}B_{t-1}(h) + W_t(h)l_t(h) + R_t^k \tilde{K}_t(h) + Div_t(h) \end{aligned} \tag{19}$$

where $B_t(h)$ stands for nominal bond holding by household h, P_t stands for the price index of real aggregate consumption goods which is common to all households, $inv_t(h)$ stands for physical investment by household h, R_{t-1} stands for the gross nominal interest rate from period $t-1$ to period t, $W_t(h)$ stands for the nominal wage rate uniquely associated to the household h's differentiated labor supply, and $Div_t(h)$ stands for the nominal dividend income from the firm that the household h owns. It should be noted that this dividend income is already maximized by the firm and therefore it is exogenous to the household's optimization problem.

The household not only acts as a consumer/labor supplier, but also possesses a characteristic of an investor. In other words, the household h lends out the capital,

$K_t(h)$, to the firm and earns rental rate R_t^k from each effective capital, $\tilde{K}_t(h)$. Effective capital is defined as the product of actual capital holdings and capital utilization rate.

$$\tilde{K}_t(h) = u_t(h)K_t(h) \tag{20}$$

The household can increase the rental income by increasing the capital utilization rate. However, in so doing, the household need to pay the cost of capital utilization given by $a(u_t(h))K_t(h)$. Here, the utilization cost function $a(u_t(h))$ is assumed to be increasing and convex function, i.e., $a'(u) > 0$ and $a''(u) > 0$. Further, the utilization cost is assumed to be zero when capital utilization is at the steady state, i.e., $a(u^{ss}) = 0$ where $u^{ss} = 1$.

In sum, the LHS of the budget constraint (19) represents the total expenditure (the sum of bond investment, consumption expenditure, physical investment, and capital utilization cost) of household h at period t and the RHS of the budget constraint represents the total revenue (the sum of bond carried over from period $t-1$, labor income, rental income, and dividend income) of the household.

Transforming the nominal budget constraint (19) into the real budget constraint, we obtain the following constraint.

$$\begin{aligned} & b_t(h) + c_t(h) + inv_t(h) + a(u_t(h))K_t(h) \\ & = \frac{R_{t-1}}{\Pi_t} b_{t-1}(h) + w_t(h)l_t(h) + r_t^k u_t(h)K_t(h) + div_t(h) \end{aligned} \tag{21}$$

where $b_t(h) = B_t(h)/P_t$ stands for real bond holdings, $\Pi_t = P_t/P_{t-1}$ stands for inflation rate from period $t-1$ to t, $w_t(h) = W_t(h)/P_t$ stands for real wage, $r_t^k = R_t^k/P_t$ stands for real rental rate, and $div_t(h) = Div_t(h)/P_t$ stands for real dividend.

3.1.2 Capital Accumulation and Capital Adjustment Cost

The household accumulates the capital stock according to the following capital accumulation equation.

$$K_{t+1}(h) = (1-\delta)K_t(h) + \left[1 - S\left(\frac{inv_t(h)}{inv_{t-1}(h)}\right)\right] inv_t(h) \tag{22}$$

where δ stands for depreciation rate of capital and function $S(\cdot)$ stands for adjustment cost for capital defined as a quadratic function as follows.

$$S\left(\frac{inv_t(h)}{inv_{t-1}(h)}\right) = \frac{1}{2\varphi}\left(\frac{inv_t(h)}{inv_{t-1}(h)} - 1\right)^2 \tag{23}$$

As can be seen from the above specification, the larger the deviation of the current physical investment from the previous period becomes, so does the adjustment cost. The existence of this capital adjustment cost creates a motivation for the household to smooth out the physical investment over time. It should be noted that adjustment cost is zero in steady state, i.e., $S(1) = 0$ and $S'(1) = 0$. Further, notice that $\varphi = 1/S''(1)$ in steady state.

3.1.3 Euler Conditions of the Household/Investor Sector

In what follows, we assume that each household h is facing the same initial condition and that the complete state contingent commodity market. Thus, we omit the notation of h except for wage and labor supply. Given the budget constraint (21) and capital accumulation equation (22), the dynamic optimization problem for the household h can be formulated as follows.

$$
\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \left[\frac{(c_t - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{l_t(h)^{1+\sigma_L}}{1+\sigma_L} \right] \\
& + \lambda_t \left[\frac{R_{t-1}}{\Pi_t} b_{t-1} + w_t(h) l_t(h) + r_t^k u_t K_t + div_t - b_t - c_t - inv_t - a(u_t) K_t \right] \\
& + q_t \left[(1-\tau) K_t + \left[1 - S\left(\frac{inv_t}{inv_{t-1}} \right) \right] inv_t - K_{t+1} \right] \Bigg\}
\end{aligned} \tag{24}
$$

where λ_t stands for the Lagrange multiplier attached to the budget constraint at period t and q_t stands for the Lagrange multiplier attached to the capital accumulation equation. The household h seeks to maximize the utility over time by choosing the current consumption, bond holdings, magnitude of capital utilization, physical investment, and capital holdings. The decision regarding the amount of labor supply requires a special treatment due to the assumption of monopolistic competition in the labor market and will be analyzed separately.

The following symmetric first order conditions associated with each control variable c_t, b_t, u_t, inv_t, and K_t will hold for any household h.

$$\text{Consumption: } \lambda_t = (c_t - H_t)^{-\sigma_c} \tag{25}$$

$$\text{Bond holdings: } \lambda_t = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \right] \tag{26}$$

$$\text{Capital utilization: } r_t^k = a'(u_t) \tag{27}$$

$$
\begin{aligned}
\text{Physical investment: } \lambda_t = {} & q_t \left[1 - S\left(\frac{inv_t}{inv_{t-1}} \right) - S'\left(\frac{inv_t}{inv_{t-1}} \right) \frac{inv_t}{inv_{t-1}} \right] \\
& + \beta E_t q_{t+1} \left[S'\left(\frac{inv_{t+1}}{inv_t} \right) \left(\frac{inv_{t+1}}{inv_t} \right)^2 \right]
\end{aligned} \tag{28}
$$

$$\text{Capital holdings: } q_t = \beta E_t \left[q_{t+1}(1-\tau) + \lambda_{t+1} \left(r_{t+1}^k u_{t+1} - a(u_{t+1}) \right) \right] \tag{29}$$

3.1.4 Wage Setting and Labor Supply Behavior

Recall that we assume that the household, j, is monopoly supplier of a differentiated labor service, $l_t(h)$, $h \in (0,1)$. It sells this service to a representative, competitive firm that transforms it into an aggregate labor input l_t, using the following Dixit-Stiglitz-type aggregator function.

$$l_t = \left[\int_0^1 l_t(h)^{1/(1+\lambda_w)} \; dh \right]^{1+\lambda_w} \tag{30}$$

where λ_w is interpreted as the wage markup. The demand curve for $l_t(h)$ is given by

$$l_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-(1+\lambda_w)/\lambda_w} l_t, \quad \text{for } 1 \leq \lambda_w < \infty \tag{31}$$

where W_t is the aggregate nominal wage rate, that is, the nominal price of l_t. It is straightforward to show that W_t is related to $W_t(h)$ via the relationship such as

$$W_t = \left[\int_0^1 W_t(h)^{-1/\lambda_w} \; dh\right]^{-\lambda_w} \tag{32}$$

The household takes l_t and W_t as given.

Households set their wage rate according to a variant of the mechanism used to model price setting by firms. In each period, a household faces a constant fraction, $1 - \xi_w$, of being able to reoptimize its normal wage. The ability to reoptimize is independent across households and time. If a household cannot reoptimize its wage at time t, it sets $W_t(h)$ according to

$$W_t(h) = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}(h) \tag{33}$$

where γ_w is the degree of wage indexation. When $\gamma_w = 0$, there is no indexation and the wage that cannot be reoptimized retain constant. When $\gamma_w = 1$, there is perfect indexation to past inflation.

Finally, from the definition of the aggregate wage index, the law of motion of the aggregate wage index can be shown as

$$W_t^{-1/\lambda_w} = \xi_w \left[W_{t-1}\left(\frac{W_{t-1}}{W_{t-2}}\right)^{\gamma_w}\right]^{-1/\lambda_w} + (1 - \xi_w)\tilde{w}_t^{-1/\lambda_w} \tag{34}$$

where $\tilde{w}$ is the reoptimized wage which is obtained by the result of the maximization problem as below.

$$\frac{\tilde{w}_t}{P_t} E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \left(\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+i}/P_{t+i-1}}\right) \frac{l_{t+i}(h)\; U_{t+i}^c}{1+\lambda_w} = E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \; l_{t+i}(h)\; U_{t+i}^l \tag{35}$$

where U_{t+i}^l is the marginal disutility of labor and U_{t+i}^c is the marginal utility of consumption. The above equation implies that the nominal wage at time t of household h that is allowed to change its wage is set so that the present value of the marginal return to working a markup over the present value of marginal cost (the subjective cost of working). When wages are perfectly flexible ($\xi_w = 0$), the real wage will be a markup (equal to $1+\lambda_w$) over the current ratio of the marginal disutility of labor to the marginal utility of an additional unit of consumption.

3.2 The Firm Sector

3.2.1 Production Technology and Cost Minimization

In modeling the firms behavior, we basically follow the Calvo (1983) type treatment. There are n monopolistically competitive firms each producing and selling disaggregated good, $y_{j,t}$, in the intermediate goods market. The production function for each monopolistic firms is identically defined as

$$\text{Production function: } y_{j,t} = A_t \, \tilde{K}_{j,t}^{\alpha} \, l_{j,t}^{1-\alpha} - \Phi \tag{36}$$

where A_t is the economy-wide technology shock affecting the productivity of all firms, $\tilde{K}_{j,t}$ stands for the borrowing of effective capital by the firm j, $l_{j,t}$ stands for the aggregate labor force employed by the firm j at period t, and Φ stands for the fixed cost. Notice that due to the existence of fixed cost inside the production function (36), a firm's production technology is no longer constant return-to-scale, but it will be increasing return-to-scale technology.

By the assumption of perfectly competitive rental market for capital and since a firm behaves to be a price-taker in the labor market, the firm j takes the rental price r_t^k and real wage index w_t as given. Provided the target output level $y_{j,t}$, the cost minimization problem for the firm j can be expressed as follows.

$$\text{Cost function: } \min_{\tilde{K}_{j,t}, l_{j,t}} \; w_t l_{j,t} + r_t^k \tilde{K}_{j,t} + mc_{j,t} \left(y_{j,t} - A_t \, \tilde{K}_{j,t}^{\alpha} \, l_{j,t}^{1-\alpha} + \Phi \right) \tag{37}$$

where Lagrange multiplier, $mc_{j,t}$, has an interpretation of the marginal cost of producing $y_{j,t}$. Solving the above cost minimization, the first order condition becomes

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha} \frac{\tilde{K}_{j,t}}{l_{j,t}} \tag{38}$$

where the LHS of equation (38) stands for the opportunity cost between the capital input and labor input and the RHS stands for the marginal rate of technical substitution between two factors. Further, the firm j's marginal cost can be expressed as

$$\text{Marginal cost: } mc_{j,t} = \frac{1}{A_t} \left(\alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \right) w_t^{1-\alpha} (r_t^k)^{\alpha} \tag{39}$$

As can be seen from equation (39), the specification of marginal cost does not depend on subscript j, implying that the marginal cost is symmetric across firms. This is because of the identical specification of the production function and price-taking behavior of firms in the capital market and aggregate labor market. Since marginal cost is symmetric across firms, we simply suppress subscript j in what follows.

3.2.2 Optimal Pricing Rule under Sticky Price

We investigate the optimal setting behavior of the firm j who behaves monopolistically in the intermediate goods market for $y_{j,t}$. Before investigating the optimal pricing rule, we need to specify the demand function for intermediate goods $y_{j,t}$. Following the literature, we specify the intermediate good demand function to be a standard one as follows.

$$\text{Demand function for } y_{j,t}\text{: } y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-(1+\lambda_p)/\lambda_p} y_t \tag{40}$$

where y_t stands for final goods, P_t stands for aggregate price index of final goods y_t, and λ_p is a parameter governing the price elasticity of demand and stands for the firm's markup over the marginal cost. Under the Calvo (1983) type sticky price setting, for any given period t, fraction ξ_p of the entire firms in the economy will not be able to revise their price $p_{j,t}$, whereas fraction $(1 - \xi_p)$ will have a chance to revise their price. For any given chance to revise the price $p_{j,t}$, the individual firm is faced with the following profit maximization problem.

$$\text{Profit function: } \max_{P_{j,t}} E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \left[\left(\frac{P_{j,t}}{P_{t+i}}\right)^{-1/\lambda_p} - mc_{t+i} \left(\frac{P_{j,t}}{P_{t+i}}\right)^{-(1+\lambda_p)/\lambda_p} \right] y_{t+i} \tag{41}$$

The first order condition for the above profit maximization problem yields

$$E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \, y_{j,t+i} \left[\frac{P_{j,t}}{P_{t+i}} - (1 + \lambda_p) \, mc_{t+i} \right] = 0 \tag{42}$$

Rearranging further yields the following optimal pricing rule for firm j.

$$\frac{P_{j,t}}{P_t} = (1 + \lambda_p) E_t \sum_{i=0}^{\infty} f_{t+i} mc_{t+i} \tag{43}$$

where

$$f_{t+i} = \frac{\beta^i \xi_p^i (P_{t+i}/P_t)^{(1+\lambda_p)/\lambda_p} y_{j,t+i}}{\sum_{i=0}^{\infty} \beta^i \xi_p^i (P_{t+i}/P_t)^{1/\lambda_p} y_{j,t+i}} \tag{44}$$

As can be seen from equation (43), the firm will set its price equal to the weighted average of the stream of future marginal costs marked up by the factor $(1 + \lambda_p)$. Notice that, in the case of flexible price setting, the firm will set the price over the current marginal cost marked up by the factor $(1 + \lambda_p)$, whereas, in the case of sticky price setting such as here, the firm who has a chance to revise its price at period t will set the price taking into account the current and future stream of expected marginal costs. If the degree of price stickiness is high (i.e., ξ_p is high), then the firm will take into account the future marginal costs far into the future when setting the price. On the other hand, if the degree of price stickiness is low (i.e., ξ_p is low), then the firm will be relatively shortsighted when considering the future marginal costs. As for the extreme case, when all firms have a chance to revise their prices every period (i.e., $\xi_p = 0$), the pricing rule will reduce to flexible equilibrium pricing rule.

In order to keep the exposition simple, the above pricing rule was derived under the assumption that firms that did not receive the "price-change signal" to keep their prices unchanged from last period. CEE introduces a partial indexation to inflation. In other words, firms that did not have a chance to reoptimize their price will partially index their price to lagged inflation as follows.

$$\text{Price indexation to inflation: } \tilde{p}_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} \tilde{p}_{j,t-1} \tag{45}$$

where parameter γ_p controls the magnitude of indexation to the past inflation. Under this price indexation, the optimal pricing rule will be modified as follows.

$$E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \, y_{j,t+i} \left[\frac{\tilde{p}_{j,t}}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}}\right)^{\gamma_p} - (1+\lambda_p)\, mc_{t+i} \right] = 0, \tag{46}$$

where $\tilde{p}_{j,t}$ stands for the optimal price chosen by the optimizing firm at period t. It should be noted that this price $\tilde{p}_{j,t}$ will be automatically adjusted next period according to the indexation specified in equation (45), even if the firm does not receive a "price-changing signal". Taking a close look at equation (46) and comparing it with the pricing rule without indexation, we notice the presence of modifying term, $(P_{t-1+i}/P_{t-1})^{\gamma_p}$. Assuming the trend of positive inflation, the presence of this modifying term will render the optimal price $\tilde{p}_{j,t}$ to be lower compared to the case where there is no price indexation as in equation (42), i.e., $\tilde{p}_{j,t} < P_{j,t}$. Thanks to the automatic price adjustment mechanism even for a period without a "price-changing signal," a firm is protected from a loss caused by an inflation and, thus, does not need to charge an "inflation premium" when setting a price at period t. If there is no automatic price adjustment mechanism, then a firm need to take into account for the risk of future inflation and, therefore, need to charge "inflation premium" when setting the price at period t. This is the reason why the optimal price with inflation index will be lower than the case without inflation index.

Finally, from the definition of the aggregate price index, the law of motion of the aggregate price index can be shown as follows.

$$P_t^{-1/\lambda_p} = \xi_p \left[P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} \right]^{-1/\lambda_p} + (1-\xi_p)\tilde{p}_{j,t}^{-1/\lambda_p} \tag{47}$$

3.3 Market Clearing Condition

We impose the market clearing condition for the final goods market. We require the supply of final goods to be equal to the demand of final goods for consumption, investment, capital utilization, and government expenditure. Thus, the market clearing condition can be expressed as follows.

$$y_t = c_t + inv_t + a(u_t)K_{t-1} + g_t \tag{48}$$

3.4 Log-Linearization of the Model

To solve the rational expectations models analytically, we log-linearize the above model around the steady state. The hat above a variable denotes the log derivation from steady state: i.e. $\hat{x} = \ln x - \ln x^{ss}$ where x^{ss} is steady state.

3.4.1 Equilibrium Conditions from Housing/Investor Sector

(1) Consumption Euler equation:

$$\hat{c}_t = \frac{h}{1+h}\hat{c}_{t-1} + \frac{1}{1+h}E_t\hat{c}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(\hat{R}_t - E_t\hat{\Pi}_{t+1}) + \frac{1-h}{(1+h)\sigma_c}(u_t^c - E_t u_{t+1}^c) \quad (49)$$

where an additional assumption that the external habit stock H_t is proportional to aggregate past consumption: $H_t = h\ c_{t-1}$, is introduced where h denotes the habit persistence parameter. When $h = 0$, equation (49) reduces to the traditional forward-looking consumption equation. With external habit formation, consumption depends on a weighted average of past and expected future consumption. Note that in this case the interest elasticity of consumption depends not only on the intertemporal elasticity of substitution (σ_c), but also on habit persistence parameter. A high degree of habit persistence will tend to reduce the impact of the real rate on consumption for a given elasticity of substitution. A persistent shock u_t^c is AR(1) process with coefficient ρ^c, and, therefore, the expected value $E_t u_{t+1}^c$ can be rewritten as $\rho^c u_t^c$.

(2) Investment Euler equation

$$\widehat{inv}_t = \frac{1}{1+\beta}\widehat{inv}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{inv}_{t+1} + \frac{\varphi}{1+\beta}\hat{q}_t + \frac{\beta}{1+\beta}(E_t u_{t+1}^{inv} - u_t^{inv}) \quad (50)$$

where we set $\varphi = 1/\bar{S}''$ and the inverse, $1/\varphi$, implies the elasticity of investment on the price of capital. Modeling the capital adjustment cost as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks. A positive shock to the adjustment cost function, u_t^{inv}, temporarily reduces investment. The expected value $E_t u_{t+1}^{inv}$ is set as $\rho^{inv} u_t^{inv}$ using AR(1) coefficient ρ^{inv}.

(3) Asset Pricing Euler equation:

$$\hat{q}_t = -(\hat{R}_t - E_t\hat{\Pi}_{t+1}) + \frac{1-\tau}{1-\tau+\overline{r}^k}E_t\hat{q}_{t+1} + \frac{\overline{r}^k}{1-\tau+\overline{r}^k}E_t\hat{r}_{t+1}^k + \varepsilon_t^q \quad (51)$$

where $\beta = 1/(1-\tau-\bar{r}_k)$, τ is the depreciation rate, and $\bar{r}_k$ is steady-state rental rate. The current value of the capital stock, Tobin's q, depends negatively on the ex ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a white noise shock to the required rate of return on equity investment, ε_t^q, is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium.

(4) Real wage law of motion:

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\Pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\Pi}_t + \frac{\gamma_w}{1+\beta}\hat{\Pi}_{t-1} - \frac{\lambda_w(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)(\lambda_w+(1+\lambda_w)\sigma_L)\xi_w}\left[\hat{w}_t - \sigma_L\hat{l}_t - \frac{\sigma_c}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) - u_t^L - \varepsilon_t^w\right] \tag{52}$$

The real wage, $\hat{w}_t$, is a function of expected and past real wages and the expected, current, and past inflation rate where the relative weight depends on the degree of indexation of the nonoptimized wages, γ_w. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. The last term implies a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labor market. The size of this effect will be greater, the smaller the degree of wage rigidity, the lower the demand elasticity for labor and the lower the inverse elasticity of labor supply, σ_L. The shock to labor supply, u_t^L, follows the AR(1) process, while the shock to real wage, ε_t^w, is assumed to obey i.i.d-normal.

(5) Capital Accumulation equation:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\widehat{inv}_{t-1} \tag{53}$$

Capital, K_t, is decreased by the depreciation of capital and increased by investment. Note that τ is a double meaning: the depreciation rate of capital and the ratio of investment to capital so that the former is used in the first term of RHS and the latter is used in the second term.

3.4.2 Equilibrium Conditions from Firm Sector

(6) Cost minimization condition:

$$\hat{l}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{K}_{t-1} \tag{54}$$

where $\psi = \psi'(1)/\psi''(1)$ is the inverse of elasticity of the capital utilization cost function. For a given installed capital stock, labor demand depends negatively on the real wage, $\hat{w}_t$, and positively on the rental rate of capital, $\hat{r}_t^k$.

(7) Production Function:

$$\hat{y}_t = \phi\hat{u}_t^a + \phi\alpha\hat{K}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{l}_t \tag{55}$$

where ϕ is one plus the share of the fixed cost in production, and $\hat{u}_t^a$ is productivity shock. This equation is derived from the production function (13).

(8) Inflation law of motion:

$$\hat{\Pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \hat{\Pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\Pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p} \left[\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p \right] \tag{56}$$

Inflation, $\hat{\Pi}_t$, depends on past and expected future inflation and the current marginal cost, which itself is a function of the rental rate on capital $\hat{r}_t$, the real wage $\hat{w}_t$, and productivity shock u_t^a. When $\gamma_p = 0$, this equation reduces to the standard purely forward-looking Phillips curve. In the other words, the degree of indexation, γ_p, determines how backward looking the inflation process is. The elasticity of inflation with respect to changes in the marginal cost, $\left[\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p\right]$, depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$) and the i.i.d-normal price-markup shock, ε_t^p, is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

(9) Employment equation:

$$\hat{e}_t = \beta \hat{e}_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(\hat{l}_t - \hat{e}_t) \tag{57}$$

where e_t is employment, l_t is labor input, and ξ_e is a constant probability at which firms are able to adjust employment to its desired total labor input. This equation reflects the fact that the employment is likely to respond more slowly than the labor input.

3.4.3 Miscellaneous Equilibrium Conditions

(10) Market clearing conditions:

$$\hat{y}_t = (1 - \tau k_y - g_y)\hat{c}_t + \tau k_y \widehat{inv}_t + \overline{r}_t^k \psi k_y \hat{r}_t^k + g_y u_t^g \tag{58}$$

where k_y is the steady state capital-output ratio, and g_y is the steady state government spending-output ratio. We assume that the government spending shock follows a first-order autoregressive process with an i.i.d-normal error term: $u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g$.

(11) Monetary policy rule:

$$\hat{R}_t = \rho_m \hat{R}_{t-1} + (1-\rho_m)\left[\mu_\pi \hat{\Pi}_{t-1} + \mu_y \hat{y}_t \right] + \varepsilon_t^m \tag{59}$$

The monetary authorities follow a generalized Taylor rule by gradually responding to deviations of lagged inflation from a zero-percent inflation objective and the lagged output gap, $\hat{y}_t$. The parameter ρ_m captures the degree of interest rate smoothing. Also we assume monetary policy shock, ε_t^m, follows a white noise process.

3.4.4 Persistent Shocks and Forecast Errors

The five persistent shocks built in above equations are characterized by the first-order autoregressive process with an i.i.d-normal error term as follows.

(12) preference shock:

$$u_t^c = \rho^c u_{t-1}^c + \varepsilon_t^c$$

(13) investment shock:

$$u_t^{inv} = \rho^{inv} u_{t-1}^{inv} + \varepsilon_t^{inv}$$

(14) labor shock:

$$u_t^L = \rho^L u_{t-1}^L + \varepsilon_t^L$$

(15) productivity shock:

$$u_t^a = \rho^z u_{t-1}^a + \varepsilon_t^a$$

(16) government spending shock:

$$u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g$$

And there are six forecast errors in the model as below.

(17) Inflation forecast error:

$$\eta_t^\pi = \hat{\pi}_t - E_{t-1}\hat{\pi}_t$$

(18) Real wage forecast error:

$$\eta_t^w = \hat{w}_t - E_{t-1}\hat{w}_t$$

(19) Equity premium forecast error:

$$\eta_t^q = \hat{q}_t - E_{t-1}\hat{q}_t$$

(20) Investment forecast error:

$$\eta_t^{inv} = \widehat{inv}_t - E_{t-1}\widehat{inv}_t$$

(21) Consumption forecast error:

$$\eta_t^c = \hat{c}_t - E_{t-1}\hat{c}_t$$

(22) Rental Rate forecast error:

$$\eta_t^{rk} = \hat{r}_t^k - E_{t-1}\hat{r}_t^k$$

3.4.5 System of the Log-Linearized Model

The system of the log-linearized model around the steady state are integrated as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t} \tag{60}$$

where $\boldsymbol{s_t}$ is a vector of endogenous variables: $\boldsymbol{s_t} = [y_t, \pi_t, w_t, k_t, inv_t, c_t, R_t, r_t^k, L_t,$ $E_t\pi_{t+1} E_t w_{t+1}, E_t w_{t+1}, E_t q_{t+1}, E_t inv_{t+1}, E_t c_{t+1}, E_t r_{t+1}^k, u_t^c, u_t^{inv}, u_t^L, u_t^a, u_t^g]'$, and $\boldsymbol{\varepsilon_t}$ is a vector of exogenous shocks: $\boldsymbol{\varepsilon_t} = [\varepsilon_t^c, \varepsilon_t^{inv}, \varepsilon_t^q, \varepsilon_t^L, \varepsilon_t^w, \varepsilon_t^a, \varepsilon_t^p, \varepsilon_t^g, \varepsilon_t^m]'$. $\boldsymbol{\eta_t}$ is a vector of forecast errors: $\boldsymbol{\eta_t} = [\eta_t^\pi, \eta_t^w, \eta_t^q, \eta_t^{inv}, \eta_t^c, \eta_t^{rk}]'$. $\boldsymbol{\Gamma_0}$, $\boldsymbol{\Gamma_1}$, $\boldsymbol{\Psi}$, and $\boldsymbol{\Pi}$ are the matrices of parameters. See Appendix B in which these matrices are described in detail.

3.5 State-Space Form

Using the Sims (2002) method, we can solve equation (60) as

$$\boldsymbol{s_t} = \boldsymbol{\Theta_1 s_{t-1}} + \boldsymbol{\Theta_0 \varepsilon_t} \tag{61}$$

(see Appendix C for the detail). The vector of observation variables $\boldsymbol{y_t}$ is represented by

$$\boldsymbol{y_t} = \boldsymbol{A s_t} \tag{62}$$

where $\boldsymbol{A}$ is a $n \times k$ matrix expressing relations between observed variables $\boldsymbol{y_t}$ and unobserved variables $\boldsymbol{s_t}$.

Equations (61) and (62) constitute a linear state-space form. Given the parameter $\boldsymbol{\theta}$ in the DGSE model, we can obtain the population moments $\Gamma_{yy}^*(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{y}_t \boldsymbol{y}_t']$, $\Gamma_{xy}^*(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{x}_t \boldsymbol{y}_t']$ and $\Gamma_{yx}^*(\boldsymbol{\theta}) = E_{\boldsymbol{\theta}}[\boldsymbol{y}_t \boldsymbol{x}_t']$ in equation (15) using the following equations.

$$E_{\boldsymbol{\theta}}[\boldsymbol{y}_t \boldsymbol{y}_t^{'}] = \boldsymbol{Z}(\boldsymbol{\theta}) \boldsymbol{\Omega}_{ss} \boldsymbol{Z}^{'}(\boldsymbol{\theta}) \tag{63}$$

$$E_{\boldsymbol{\theta}}[\boldsymbol{y}_t \boldsymbol{y}_{t-h}^{'}] = \boldsymbol{Z}(\boldsymbol{\theta}) \boldsymbol{\Theta_1}^h \boldsymbol{\Omega}_{ss} \boldsymbol{Z}^{'} \tag{64}$$

where $\boldsymbol{\Omega}_{ss} = E_{\boldsymbol{\theta}}[\boldsymbol{s}_t \boldsymbol{s}_t^{'}]$, which is obtained as

$$\text{vec}(\boldsymbol{\Omega}_{ss}) = [\boldsymbol{I} - \boldsymbol{\Theta_1} \otimes \boldsymbol{\Theta_1}]^{-1} \text{vec}\left(\boldsymbol{\Theta_0 \Theta_0}'\right) \tag{65}$$

4 Data

In order to evaluate the CEE model for the Japanese economy using the DSGE-VAR model, we use seven quarterly macroeconomic series in Japan: output, consumption, investment, labor input, real wage, inflation, nominal rate as data. The sample period is limited over the period 1970:Q1 - 1998:Q4 because of excluding the period of zero interest rate bound from 1999:Q1, in which the law of equilibrium motions of macroeconomies is plausible to be apart away from the ordinary economic dynamics.

The details of seven series are as follows. Output series is real GDP per labor force, seasonally adjusted (unit is 1 million yen at 1990). Consumption series is real consumption per labor force, s.a., (unit same as above). Investment series is real investment per labor force, s.a., (unit same as above). Labor input series are derived from work hour index times total employment divided by labor population. Real wage series is real wage index calculated from nominal wage index divided by GDP deflator. Inflation series is GDP deflator inflation rate (quarterly, annual rate, decadal demeaned). Nominal rate series is the uncollateralized call rate (annual rate, decadal demeaned). The four real series, output, consumption, investment, labor, and real wage, are transformed to their logarithms and then detrended using the Hodrick and Prescott (HP) filter. Nominal rate and inflation are detrended using HP filter without log-transformation. After those procedures, all above data for estimating the model are obtained by being demeaned.

There is an issue of how to filter inflation rate. For the Japanese case, we simply assumed that 70's inflation target, 80's inflation target and 90's inflation target were different. Thus, by constructing decadal dummies for 70's and 80's, we demeaned the inflation rate accordingly. Also, we have demeaned the call rate using same decadal dummy coefficients. We know this is a controversial treatment, but we didn't know any better way to deal with it.

The capital stock and the rental rate on capital are dealt with as unobserved variables based on the manner of Smets and Wouters (2003).

In our DSGE model following the earlier studies such as Smets and Wouters (2003) who studied the euro area, Onatski and Williams (2004) and Levin, Onatski, Williams and Williams (2005) who studied the U.S. area, some parameters need to be calibrated. We chose most of the calibrated parameters following Hayashi and Prescott (2002). We set the discount factor, β, equal to 0.99, and the depreciation rate, τ, equal to 0.25, and the share of capital, α, equal to 0.3. The ratio of steady-state government spending to total output, g_y, is assumed to be 0.2, while the steady-state capital output ratio, k_y, is assumed to be 2.2. In addition, we also need to fix the parameter capturing the markup in wage setting, λ_w, as this parameter is not identified. We set λ_w equal to 0.05. The steady-state rental rate on capital (or the value of capital) is derived such as $\bar{r}^k = \frac{1}{\beta} - (1 - \tau)$.

The prior distributions of the other 27 estimated parameters following the manner of Smets and Wouters (2003) are given in Table 1. All the variances of the shocks are assumed to be distributed as an inverted Gamma distribution with a degree of freedom equal two. This distribution guarantees a positive standard deviation with a rather large domain. The distribution of the autoregressive parameters in the six persistent shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1. The beta distribution covers the range between zero and one, but a rather tight standard error was used in order to have a clear separation between the persistent shocks and temporary shocks. The technology, utility, and price-setting parameters were assumed to be either Normal distributed or Beta distributed (for the parameters were restricted to the $0-1$ range).

[Insert Table 1.]

Appendix B summarizes the CEE model and preliminary settings.

5 Evaluation of CEE Model using the DSGE-VAR Model

The lag-length of VAR model is set 4. As long as $\lambda \geq (k+n)/T$, the prior is proper. Unless the prior is proper, we cannnot calculate the marginal likelihood. Hence, the values of λ analyzed are 0.31 (the smallest value for which we have a proper prior), $0.5, 0.75, 1, 1.25, 1.5, 2, 5,$ and ∞. For each λ, we conduct the MCMC simulation with 150,000 iterations. The first 50,000 draws are discarded and then the next 100,000 are recorded.

Figure 1 shows the logarithm of the marginal likelihood of DSGE-VAR(λ) for different values of λ. We rescale the x-axis according to $x = \lambda/(1+\lambda)$. The log-marginal likelihood has an inverted U-shape. It increases as λ moves from 0.31 to 1 and decreases as λ moves from 0.75 to 1. On one hand, the substantial drop in marginal likelihood between $\lambda = .075$ ($x = 0.43$) and ∞ ($x = 1$) provides strong evidence of misspecification for the CEE model. On the other hand, the rise in marginal likelihood between $\lambda = 0.31$ ($x = 0.24$) and 0.75 ($x = 0.43$) provides evidence that the information from the CEE model is still useful.

[Insert Figure 1.]

Next, we did the same exercise assuming that $\theta = 0$, i.e., no habit formation. The result is depicted in Figure 2. The marginal likelihood with habit formation is always larger than that without habit formation, indicating that the habit formation is important in the Japanese economy.

[Insert Figure 2.]

6 Conclusion

This paper analyzes the fit of the CEE (2005) model, which is the most successful among New-Keynesian DSGE models in explaining the behavior of macroeconomic variables in the U.S. and euro area, for the Japanese economy over 1970:Q1 through 1998:Q4, which is prior to the period of zero interest rate bound. Using the DSGE-VAR model, we find that there is misspecification in the CEE model but the information from the CEE model is useful.

This paper is the first to apply the DSGE-VAR model to the Japanese economy, and there remain some issues that should be pursued. First, we should calculate the impulse response functions of the DSGE-VAR model and compare it with those of the DSGE model. Second, we should analyze the forecasting performance of the DSGE-VAR model.

Appendix A: Modified harmonic mean estimator of marginal likelihood

There are several methods for calculating the marginal likelihood. The most popular method in the DSGE literature is the modified harmonic mean estimator proposed by Geweke (1999). In what follows, we express the marginal likelihood as $f(\boldsymbol{Y})$ by omitting the condition M_i or M_j. Suppose that $g(\boldsymbol{\theta})$ is a probability density function. Then, the following equality holds.

$$\begin{aligned} E\left[\frac{g(\boldsymbol{\theta})}{f(\boldsymbol{Y}|\boldsymbol{\theta})f(\boldsymbol{\theta})}\right] &= \int \frac{g(\boldsymbol{\theta})}{f(\boldsymbol{Y}|\boldsymbol{\theta})f(\boldsymbol{\theta})} f(\boldsymbol{\theta}|\boldsymbol{Y}) d\boldsymbol{\theta} \\ &= \int \frac{g(\boldsymbol{\theta})}{f(\boldsymbol{Y}|\boldsymbol{\theta})f(\boldsymbol{\theta})} \frac{f(\boldsymbol{Y}|\boldsymbol{\theta})f(\boldsymbol{\theta})}{f(\boldsymbol{Y})} d\boldsymbol{\theta} \\ &= \int \frac{g(\boldsymbol{\theta})}{f(\boldsymbol{Y})} d\boldsymbol{\theta} \\ &= \frac{1}{f(\boldsymbol{Y})} \int g(\boldsymbol{\theta}) d\boldsymbol{\theta} \\ &= \frac{1}{f(\boldsymbol{Y})} \end{aligned} \tag{A1}$$

Therefore, we can estimate the marginal likelihood as

$$f(\boldsymbol{Y}) = \frac{1}{E\left[\frac{g(\boldsymbol{\theta})}{f(\boldsymbol{Y}|\boldsymbol{\theta})f(\boldsymbol{\theta})}\right]} \approx \left[\frac{1}{N}\sum_{i=1}^{N} \frac{g(\boldsymbol{\theta}_i)}{f(\boldsymbol{Y}|\boldsymbol{\theta}_i)f(\boldsymbol{\theta}_i)}\right]^{-1} \tag{A2}$$

Newton and Raftray (1994) propose to set $g(\boldsymbol{\theta}) = f(\boldsymbol{\theta})$. Then, equation (A2) will be:

$$f(\boldsymbol{Y}) \approx \left[\frac{1}{N}\sum_{i=1}^{N} \frac{1}{f(\boldsymbol{Y}|\boldsymbol{\theta}_i)}\right]^{-1} \tag{A3}$$

This is called the marmonic mean estimator of the marginal likelihood. However, this method is unstable because if $f(\boldsymbol{Y}|\boldsymbol{\theta}_i)$ is close to 0, then its reciprocal may be overflowed.

To avoide this problem, Geweke (1999) proposes to set $g(\boldsymbol{\theta})$ as the truncated normal:

$$\begin{aligned} g(\boldsymbol{\theta}) &= \tau^{-1}(2\pi)^{-k/2}|\boldsymbol{\Sigma}|^{-1/2} \exp\left[-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})\right] \\ &\quad \times I\left[(\boldsymbol{\theta}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu}) \le F^{-1}_{\chi^2_k}(\tau)\right] \end{aligned} \tag{A4}$$

where k is the dimension of $\boldsymbol{\theta}$ and $F^{-1}_{\chi^2_k}(\tau)$ is the inverse cdf of a χ^2 distribution with k degrees of freedom. $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are set equal to the sample mean and covariance matrix computed from the posterior draws $\boldsymbol{\theta}$. $I[\cdot]$ is the indicator function that takes one if the inequality in the bracket is satisfied and zero otherwise.

This modified harmonic mean estimator sets $g(\boldsymbol{\theta}_i)/[f(\boldsymbol{Y}|\boldsymbol{\theta}_i)f(\boldsymbol{\theta}_i)]$ in equation (A2) equal to 0 if the extreme value of $\boldsymbol{\theta}$ is sampled. We set $\tau = 0.95$.

Appendix B: Summary of the CEE Model and Preliminary Settings

Model Description (Log-linearized version)

Consumer/Investor's Equilibrium Conditions

1. Consumption Euler equation:

$$\hat{c}_t = \frac{\theta}{1+\theta}\hat{c}_{t-1} + \frac{1}{1+\theta}E_t\hat{c}_{t+1} - \frac{1-\theta}{(1+\theta)\sigma_c}(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{1-\theta}{(1+\theta)\sigma_c}(1-\rho^c)u^c_t$$

   where we set $E_t u^c_{t+1} = \rho^c u^c_t$.

2. Investment Euler equation:

$$\widehat{inv}_t = \frac{1}{1+\beta}\widehat{inv}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{inv}_{t+1} + \frac{\varphi}{1+\beta}\hat{q}_t + \frac{\beta}{1+\beta}(1-\rho^{inv})u^{inv}_t$$

   where we set $E_t u^{inv}_{t+1} = \rho^{inv} u^{inv}_t$.

3. Asset pricing Euler equation:

$$\hat{q}_t = -(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t\hat{q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\hat{r}^k_{t+1} + \varepsilon^q_t$$

4. Wage setting equation.:

$$\hat{w}_t = \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1}$$
$$- \frac{1}{1+\beta}\Psi_w\left[\hat{w}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-\theta}\left(\hat{c}_t - \theta\hat{c}_{t-1}\right) - u^L_t - \varepsilon^w_t\right]$$

   where $\Psi_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w}\right)\xi_w}$

Firm's Equilibrium Conditions

1. Production function:

$$\hat{y}_t = \phi u^a_t + \phi\alpha\hat{k}_{t-1} + \phi\alpha\psi\hat{r}^k_t + \phi(1-\alpha)\hat{L}_t$$

2. Labor demand:

$$\hat{L}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{k}_{t-1}$$

3. Price setting equation.:

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\Psi_p \left[\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p\right]$$

where $\Psi_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$

Miscellaneous Equilibrium Conditions

1. Resource constraint:

$$\hat{y}_t = (1 - \tau k_y - g_y)\hat{c}_t + \tau k_y \widehat{inv}_t + \bar{r}^k \psi k_y r_t^k + g_y u_t^g$$

2. Capital accumulation equation:

$$\hat{k}_t = (1-\tau)\hat{k}_{t-1} + \tau \widehat{inv}_{t-1}$$

3. Monetary policy rule:

$$\hat{R}_t = \rho_m \hat{R}_{t-1} + (1-\rho_m)\left[\mu_\pi \hat{\pi}_{t-1} + \mu_y \hat{y}_t\right] + \varepsilon_t^m$$

Persistent Shocks

1. : preference shock: $u_t^c = \rho^c u_{t-1}^c + \varepsilon_t^c$
2. : investment shock: $u_t^{inv} = \rho^{inv} u_{t-1}^{inv} + \varepsilon_t^{inv}$
3. : labor shock: $u_t^L = \rho^L u_{t-1}^L + \varepsilon_t^L$
4. : productivity shock: $u_t^a = \rho^z u_{t-1}^a + \varepsilon_t^a$
5. : government spending shock: $u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g$

Forecast Errors

1. Inflation forecast error: $\hat{\pi}_t = E_{t-1}\hat{\pi}_t + \eta_t^\pi$
2. Wage forecast error: $\hat{w}_t = E_{t-1}\hat{w}_t + \eta_t^w$
3. Q forecast error: $\hat{q}_t = E_{t-1}\hat{q}_t + \eta_t^q$
4. Investment forecast error: $\widehat{inv}_t = E_{t-1}\widehat{inv}_t + \eta_t^{inv}$
5. Consumption forecast error: $\hat{c}_t = E_{t-1}\hat{c}_t + \eta_t^c$
6. Capital cost forecast error: $\hat{r}_t^k = E_{t-1}\hat{r}_t^k + \eta_t^{rk}$

Endogenous Variables

y_t : output
π_t : inflation rate
w_t : nominal wage
k_t : capital stock
q_t : shadow price of capital stock
inv_t : physical investment
c_t : consumption
R_t : nominal interest rate
r_t^k : rental rate on capital (cost of capital)
L_t : labor input
$u_t^c, u_t^{inv}, u_t^L, u_t^a, u_t^g$: persistent shocks to consumption, investment, labor, productivity, and government spending, respectively.

Exogenous Shock Variables, (i.i.d. Normal distribution)

ε_t^c : preference shock
ε_t^{inv} : investment shock
ε_t^q : equity premium shock
ε_t^L : labor shock
ε_t^w : wage mark-up shock
ε_t^a : productivity shock
ε_t^p : price mark-up shock
ε_t^g : government spending shock
ε_t^m : monetary policy shock

Forecast Errors

η_t^π : forecast error of inflation
η_t^w : forecast error of real wage
η_t^q : forecast error of equity premium
η_t^{inv} : forecast error of investment
η_t^c : forecast error of consumption
η_t^{rk} : forecast error of rental rate

Preliminary Settings

Estimated Parameters

θ: habit formation, σ_c: inverse long-run IES, σ_L: inverse labor supply elasticity, φ: inverse adj.cost, ϕ: fixed cost share, ψ: capital utilization cost, γ_p: price indexation, γ_w: wage indexation, ξ_p: Calvo price no-revise prob., ξ_w: Calvo wage no-revise prob., ρ_m: lagged interest rate, μ_π: reaction on inflation, μ_y: reaction on output, ρ_c: persitence, preference , ρ_{inv}: persistence, investment, ρ_L: persistence, labor supply, ρ_a: persistence, productivity, ρ_g: persistence, government spending, ε_c: S.D., preference shock, ε_{inv}: S.D., investment shock, ε_q: S.D., equity premium shock, ε_L: S.D, labor supply shock, ε_w: S.D., wage markup shock, ε_z: S.D., productivity shock, ε_p: S.D., price markup shock, ε_g: S.D., gov. spending shock, ε_m: S.D., monetary policy shock.

Values of Calibrated Parameters

discount factor: $\beta = 0.99$,
depriciation rate of capital: $\tau = 0.025$,
share of capital: $\alpha = 0.3$,
capital-output ratio: $k_y = 2.2$,
government spending-output ratio: $g_y = 0.2$,
wage markup: $\lambda_w = 0.05$,
steady-state rental rate: $\bar{r}^k = \frac{1}{\beta} - 1 + \tau$, (Smets and Wouters 2003, p1135)

Canonical LRE Form

$$\Gamma_0 \begin{bmatrix} y_t \\ \pi_t \\ w_t \\ k_t \\ q_t \\ inv_t \\ c_t \\ R_t \\ r_t^k \\ L_t \\ E_t\pi_{t+1} \\ E_t w_{t+1} \\ E_t q_{t+1} \\ E_t inv_{t+1} \\ E_t c_{t+1} \\ E_t r_{t+1}^k \\ u_t^c \\ u_t^{inv} \\ u_t^L \\ u_t^a \\ u_t^g \end{bmatrix} = \Gamma_1 \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ w_{t-1} \\ k_{t-1} \\ q_{t-1} \\ inv_{t-1} \\ c_{t-1} \\ R_{t-1} \\ r_{t-1}^k \\ L_{t-1} \\ E_{t-1}\pi_t \\ E_{t-1} w_t \\ E_{t-1} q_t \\ E_{t-1} inv_t \\ E_{t-1} c_t \\ E_{t-1} r_t^k \\ u_{t-1}^c \\ u_{t-1}^{inv} \\ u_{t-1}^L \\ u_{t-1}^a \\ u_{t-1}^g \end{bmatrix} + \Psi \begin{bmatrix} \varepsilon_t^c \\ \varepsilon_t^{inv} \\ \varepsilon_t^q \\ \varepsilon_t^L \\ \varepsilon_t^w \\ \varepsilon_t^a \\ \varepsilon_t^p \\ \varepsilon_t^g \\ \varepsilon_t^m \end{bmatrix} + \Pi \begin{bmatrix} \eta_t^\pi \\ \eta_t^w \\ \eta_t^q \\ \eta_t^{inv} \\ \eta_t^c \\ \eta_t^{rk} \end{bmatrix}$$

where coefficient matrices Γ_0, Γ_1, Ψ, and Π are set as follows.

$$
\Gamma_0 = \begin{bmatrix}
y_t & \pi_t & w_t & k_t & q_t & inv_t & c_t & R_t \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1-\theta}{(1+\theta)\sigma_c} \\
0 & 0 & 0 & 0 & -\frac{\varphi}{1+\beta} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & \frac{1+\beta\gamma_w}{1+\beta} & 1+\frac{\Psi_w}{1+\beta} & 0 & 0 & 0 & -\frac{\sigma_c\Psi_w}{(1+\beta)(1-\theta)} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -\frac{\Psi_p(1-\alpha)}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -\tau k_y & -(1-\tau k_y-\tau g_y) & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-(1-\rho_m)\mu_y & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\left.\begin{array}{ccccccccccccc}
r_t^k & L_t & E_t\pi_{t+1} & E_t w_{t+1} & E_t q_{t+1} & E_t inv_{t+1} & E_t c_{t+1} & E_t r_{t+1}^k & u_t^c & u_t^{inv} & u_t^L & u_t^a & u_t^g \\
0 & 0 & -\frac{1-\theta}{(1+\theta)\sigma_c} & 0 & 0 & 0 & -\frac{1}{1+\theta} & 0 & -\frac{(1-\theta)(1-\rho^c)}{(1+\theta)\sigma_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\beta}{1+\beta} & 0 & 0 & 0 & -\frac{\beta(1-\rho^{inv})}{1+\beta} & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & -\frac{1-\tau}{1-\tau+Rk^*} & 0 & 0 & -\frac{Rk^*}{1-\tau+Rk^*} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\sigma_L\Psi_w}{1+\beta} & -\frac{\beta}{1+\beta} & -\frac{\beta}{1+\beta} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Psi_w}{1+\beta} & 0 & 0 \\
-\phi\alpha\psi & -\phi(1-\alpha) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\phi & 0 \\
-(1+\psi) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\Psi_p\alpha}{1+\beta\gamma_p} & 0 & -\frac{\beta}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Psi_p}{1+\beta\gamma_p} & 0 \\
-Rk^*\psi k_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

$$\Gamma_1 = \begin{bmatrix}
y_{t-1} & \pi_{t-1} & w_{t-1} & k_{t-1} & q_{t-1} & inv_{t-1} & c_{t-1} & R_{t-1} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\theta}{1+\theta} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{1+\beta} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\gamma_w}{1+\beta} & \frac{1}{1+\beta} & 0 & 0 & 0 & -\frac{\sigma_c \Psi_w \theta}{(1+\beta)(1-\theta)} & 0 \\
0 & 0 & 0 & \phi\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{\gamma_p}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1-\tau & 0 & \tau & 0 & 0 \\
0 & (1-\rho_m)\mu_\pi & 0 & 0 & 0 & 0 & 0 & \rho_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

r^k_{t-1}	L_{t-1}	$E_{t-1}\pi_t$	$E_{t-1}w_t$	$E_{t-1}q_t$	$E_{t-1}inv_t$	$E_{t-1}c_t$	$E_{t-1}r^k_t$	u^c_{t-1}	u^{inv}_{t-1}	u^L_{t-1}	u^a_{t-1}	u^g_{t-1}
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	ρ^c	0	0	0	0
0	0	0	0	0	0	0	0	0	ρ^{inv}	0	0	0
0	0	0	0	0	0	0	0	0	0	ρ^L	0	0
0	0	0	0	0	0	0	0	0	0	0	ρ^z	0
0	0	0	0	0	0	0	0	0	0	0	0	ρ^g

$$
\Psi = \begin{bmatrix}
\varepsilon_t^c & \varepsilon_t^{inv} & \varepsilon_t^q & \varepsilon_t^L & \varepsilon_t^w & \varepsilon_t^a & \varepsilon_t^p & \varepsilon_t^g & \varepsilon_t^m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\Psi_w}{1+\beta} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\Psi_p}{1+\beta\gamma_p} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}, \qquad
\Pi = \begin{bmatrix}
\eta_t^\pi & \eta_t^w & \eta_t^q & \eta_t^{inv} & \eta_t^c & \eta_t^{Rk} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Appendix C: Solving DSGE model

Equation (60)

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t}$$

can be solved as follows.

The matrices Γ_0 and Γ_1 are decomposed by QZ decomposition as below.

$$Q'\Lambda Z' = \Gamma_0, \quad Q'\Omega Z' = \Gamma_1$$

where $Q'Q = Z'Z = I$, and Q and Z are both possibly complex. Also Ω and Λ are possibly complex and upper triangular. Note that the above QZ decomposition always exists. Letting $\omega_t = Z's_t$,, and premultiplying the both sides of equation (60) by Q, then we get

$$\Lambda \boldsymbol{\omega_t} = \Omega \boldsymbol{\omega_{t-1}} + Q\Psi \boldsymbol{\varepsilon_t} + Q\Pi \boldsymbol{\eta_t} \tag{C1}$$

Although QZ decomposition is not unique, the ratio of diagonal elements of Ω and Λ, $\{\omega_{ii}/\lambda_{ii}\}$, which is referred to generalized eigenvalue, is generally unique. The matrix Ω and Λ are ordered with respect to the absolute value of the ratio $\{\omega_{ii}/\lambda_{ii}\}$ (or generalized eigenvalue) by ascending order. By partitioning equation (C1) in two blocks so that the stable generalized eigenvalues corresponding to $|\omega_{ii}/\lambda_{ii}| < \bar{\xi}$ and the unstable generalized eigenvalue corresponding to $|\omega_{ii}/\lambda_{ii}| \geq \bar{\xi}$, it is rewritten as equation (C2). The upper and the lower in equation (C2) are the stable block and the unstable block, respectively. Here, $\bar{\xi}$ is the bound of maximal growth rate of endogenous variables $\boldsymbol{s_t}$, that holds the transversality condition.

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} \\ Q_{2\cdot} \end{bmatrix} \Big(\Psi\varepsilon(t) + \Pi\eta(t) \Big) \tag{C2}$$

where $Q_{1\cdot}$ and $Q_{2\cdot}$ denote the first and the second rows of the matrix Q. For canceling out the term of expectation errors $\boldsymbol{\eta(t)}$ from equation (C2), we premultiply equation (C2) by $[I \quad -\Phi]$ and translate its stable block into the upper of equation (C3). Note that Φ is set to satisfy a linear combination, $Q_{1\cdot}\Pi = \Phi Q_{2\cdot}\Pi$, and this linear combination of expectation errors $\boldsymbol{\eta(t)}$ is the stability condition of the DSGE model.

Meanwhile, on the unstable block (i.e. the lower) in equation (C2), the last term, $Q_{2\cdot}\Pi\eta_{t+1}$, is solved forward[3], and then it becomes $Q_{2\cdot}\Pi\eta_{t+1} = \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi\epsilon_{t+s}$. Here, we set $M = \Omega_{22}^{-1}\Lambda_{22}$. Substituting it into equation (C2), we get

[3]This derivation is described in Sims (2002).

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12}-\Phi\Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12}-\Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix} + \begin{bmatrix} Q_{1\cdot}-\Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi\varepsilon(t) + E_t \begin{bmatrix} 0 \\ \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1} Q_{2\cdot}\Psi\varepsilon_{t+s} \end{bmatrix} \tag{C3}$$

Here, we set $E_t(\varepsilon_{t+s}) = 0$ for $s = 1 \cdots T$ in the last term of equation (C3) and remind that $\omega_t = Z' s_t$, then we get the recursive equilibrium law of motion such as equation (C4).

$$\boldsymbol{s_t} = \boldsymbol{\Theta_1 s_{t-1}} + \boldsymbol{\Theta_0 \varepsilon_t} \tag{C4}$$

where

$$\boldsymbol{\Theta}_1 = Z_{\cdot 1}\Lambda_{11}^{-1}[\Omega_{11} \ \ (\Omega_{12}-\Phi\Omega_{22})]Z$$

$$\boldsymbol{\Theta}_0 = H \begin{bmatrix} Q_{1\cdot}-\Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi$$

$$H = Z \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}(\Lambda_{12}-\Phi\Lambda_{22}) \\ 0 & I \end{bmatrix} Z$$

where $Z_{\cdot 1}$ denotes the first column of matrix Z. Equation (C4) traces the stable path converging to the equilibrium and corresponds to our target, say, the solution of the DSGE models.

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Table 1. Prior Distributions of the Parameters in the CEE Model

parameters	meanings	type	mean	S. D.
Structural Parameters				
θ	habit formation	beta	0.7	0.1
σ_c	inverse long-run IES	normal	1	0.375
σ_L	inverse labor supply elasticity	normal	2	0.75
$1/\varphi$	inverse adj. cost	normal	4	1.5
ϕ	fixed cost share	normal	1.45	0.25
ψ	capital utilization cost	normal	0.2	0.075
γ_p	price indexation	beta	0.75	0.15
γ_w	wage indexation	beta	0.75	0.15
ξ_p	Calvo price no-revise prob.	beta	0.75	0.15
ξ_w	Calvo wage no-revise prob.	beta	0.75	0.15
ξ_e	Calvo employment	beta	0.50	0.15
Policy Parameters				
ρ_m	policy, lag interest	beta	0.8	0.1
μ_π	policy, inflation	normal	1.7	0.1
μ_y	policy, output	normal	0.125	0.05
$\mu_{\Delta\pi}$	policy, delta inflation	normal	0.3	0.1
$\mu_{\Delta y}$	policy, delta output	normal	0.0625	0.05
Shock Persistence				
ρ_{target}	persist, target	beta	0.85	0.1
ρ_a	persist, productivity	beta	0.85	0.1
ρ_c	persist, preference	beta	0.85	0.1
ρ_g	persist, gov. expenditure	beta	0.85	0.1
ρ_L	persist, labor supply	beta	0.85	0.1
ρ_{inv}	persist, investment	beta	0.85	0.1
S. D. of Shocks				
ε_c	preference shock	inv. gamma	0.2	2
ε_{inv}	investment shock	inv. gamma	0.1	2
ε_q	equity premium shock	inv. gamma	0.4	2
ε_a	productivity shock	inv. gamma	0.4	2
ε_p	price markup shock	inv. gamma	0.15	2
ε_L	labor supply shock	inv. gamma	1.0	2
ε_w	wage markup shock	inv. gamma	0.25	2
ε_g	gov. expenditure shock	inv. gamma	0.3	2
ε_m	monetary policy shock	inv. gamma	0.1	2
ε_{target}	inflation target shock	inv. gamma	0.02	2

Figure 1: Log-Marginal Likelihood of the CEE Model

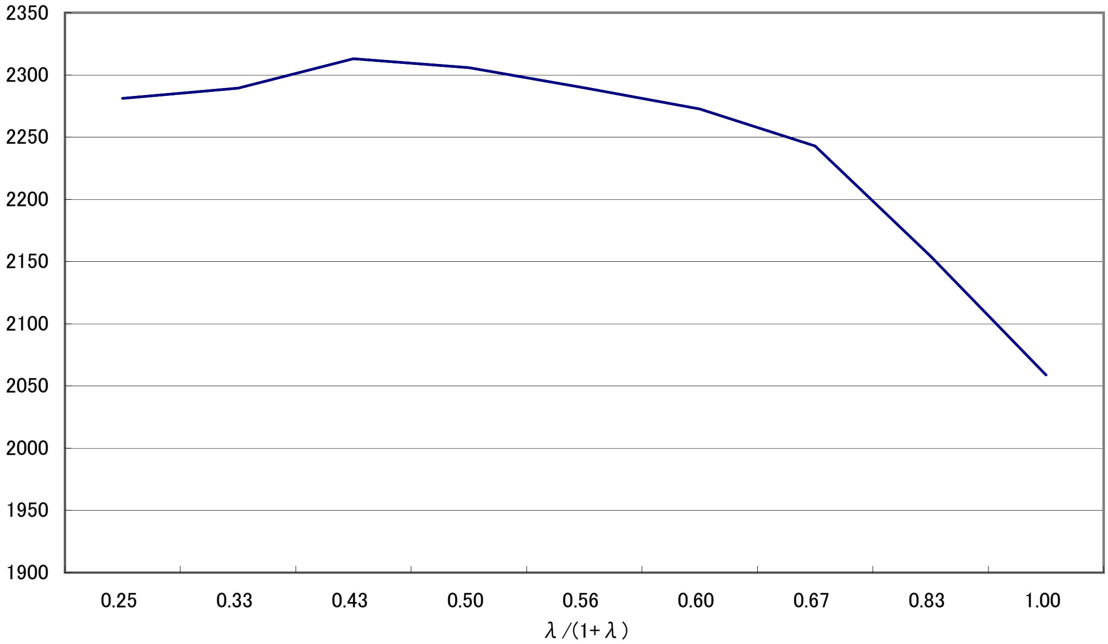

Figure 2: Log-Marginal Likelihoods of the Model with or without Habit Formation