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Measuring Inflation Expectations Using  
Interval-Coded Data

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# Measuring Inflation Expectations Using Interval-Coded Data

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## Abstract

To quantify qualitative survey data, the Carlson–Parkin method assumes normality, a time-invariant symmetric indifference interval, and long-run unbiased expectations. Interval-coded data do not require these assumptions. Since April 2004, the Monthly Consumer Confidence Survey in Japan asks households their price expectations a year ahead in seven categories with partially known boundaries; thus one can identify up to six parameters including an indifference interval each month. This paper compares normal, skew normal, and skew  $t$  distributions, and finds that the skew  $t$  distribution fits the best throughout the period studied. The results help to understand the dynamics of heterogeneous expectations.

KEY WORDS: Survey data; Carlson–Parkin method; Skew normal; Skew  $t$ ; Heterogeneous expectations.

# 1 INTRODUCTION

Under imperfect information, the effectiveness of monetary stabilization policy depends on inflation expectations of the public (the new classical Phillips curve or the Lucas supply function). Under sticky prices, with forward-looking agents, inflation expectations affect the current inflation (the new Keynesian Phillips curve). Hence central banks must monitor not only the current inflation but also inflation expectations. Various survey data are available for that purpose.

A major difficulty in measuring inflation expectations from survey data is that they are often qualitative, or more precisely, ordered data with *unknown* boundaries. To quantify such data, the Carlson–Parkin (C–P) method assumes normality, a time-invariant symmetric indifference interval, and long-run unbiased expectations; see Carlson and Parkin (1975). Subsequent extensions and alternatives also require strong assumptions; see Nardo (2003) and Pesaran and Weale (2006, sec. 3) for recent surveys.

Obviously, such strong assumptions are unnecessary for ordered data with *known* boundaries, or interval-coded data. Since April 2004, the Monthly Consumer Confidence Survey in Japan asks 6,720 households their price expectations a year ahead in seven categories with known boundaries except for an indifference interval; thus one can identify the distribution of household inflation expectations up to six parameters including the indifference interval each month separately.

The Bank of England/GfK NOP Inflation Attitudes Survey in the UK also collects interval-coded data on household inflation expectations quar-

terly since November 1999. Blanchflower and MacCoille (2009) estimate a normal interval regression model for household inflation expectations using individual data not publicly available. For our purpose of extending the C–P method, the released aggregate data suffice.

One may think that quantity data are easier to analyse than interval-coded data; e.g., the Michigan Surveys of Consumers in the US ask households their price expectations a year ahead in integers (after an initial qualitative question). They are not if one takes rounding seriously, since some respondents round to integers while others to multiples of 5 or 10; see Manski and Molinari (2010).

To allow for skewness and excess kurtosis in the distribution of household inflation expectations, this paper uses the skew normal (SN) and skew  $t$  (St) distributions introduced by Azzalini (1985) and Azzalini and Capitanio (2003) respectively. These are natural extensions of the normal and  $t$  distributions, and are easy to use with the `sn` package for R. So far, the only application in econometrics seems Azzalini and Kotz (2002). Juárez and Steel (2010a,b) use alternative SN and St distributions proposed by Fernández and Steel (1998).

Comparing the normal, SN, and St interval regression models with no covariate for household inflation expectations in Japan, we find that the St model fits the best throughout the period studied. Other findings are the following:

1. The skewness parameter is time-varying and consistently positive except for two months (December 2009 and January 2010).

2. The degree of freedom is small and often below 2, in which case the variance is infinite.
3. The indifference interval lies around  $\pm 0.5$ –1%, sometimes asymmetric and unstable over time.
4. The “consensus” inflation forecast, whether it is the mean, the median, or the mode, is consistently higher than the current CPI inflation, and tends to overpredict the actual CPI inflation.
5. The median, the interquartile range (IQR), the skewness parameter, and excess kurtosis of the distribution of household inflation expectations all increase with the current CPI inflation when the latter is positive; the relations are unclear when the latter is negative.

The first four findings show that the assumptions for the C–P method fail. If individual data are available, then demographic factors may explain the first two findings. Capistrán and Timmermann (2009) attribute the fourth finding to asymmetric loss. It also arises from response (cognitive) bias and the CPI inflation bias, though the latter is usually an upward bias relative to the true cost-of-living index inflation; see Hausman (2003) and Lebow and Ruud (2003). The last finding is relevant to the recent literature on rationally heterogeneous expectations due to sticky information (Mankiw and Reis (2002), Mankiw et al. (2003), Carroll (2003)), model uncertainty (Branch (2004, 2007)), or asymmetric loss (Capistrán and Timmermann (2009)).

A possible use of the estimated inflation expectations is inflation forecasting. Ang et al. (2007) find that survey expectations predict US inflation

better than other variables. We compare the mean, median, and mode inflation expectations of the normal, SN, and St models, and find that none of them helps to predict the actual CPI inflation beyond the current CPI inflation, though the estimation period is too short to give a definitive conclusion, covering at most one business cycle.

The paper proceeds as follows. Section 2 introduces the SN and St distributions. Section 3 reviews the C–P method. Section 4 specifies the normal, SN, and St interval regression models. Section 5 estimates these models using the Japanese data, and studies the distributions of household inflation expectations in Japan from April 2004 to March 2010. Section 6 discusses remaining issues.

## 2 SKEW NORMAL AND SKEW $t$ DISTRIBUTIONS

### 2.1 Skew Normal Distribution

The SN distribution is due to Azzalini (1985). The pdf of a standard SN distribution with skewness parameter  $\lambda$ , denoted as  $\text{SN}(\lambda)$ , is  $\forall z \in \mathbb{R}$ ,

$$\phi(z; \lambda) := 2\Phi(\lambda z)\phi(z), \quad (1)$$

where  $\Phi(\cdot)$  and  $\phi(\cdot)$  are the cdf and the pdf of  $N(0, 1)$  respectively. This reduces to  $N(0, 1)$  when  $\lambda = 0$ , and approaches to the standard half-normal

distribution as  $\lambda \rightarrow \infty$ . The cdf of  $\text{SN}(\lambda)$  is  $\forall z \in \mathbb{R}$ ,

$$\Phi(z; \lambda) := \Phi(z) - 2T(z, \lambda),$$

where  $T(\cdot, \cdot)$  is the function tabulated by Owen (1956). The `sn` package for R helps to evaluate the cdf and the pdf of  $\text{SN}(\lambda)$ .

Let  $Z \sim \text{SN}(\lambda)$ . Then  $-Z \sim \text{SN}(-\lambda)$ ,  $Z^2 \sim \chi^2(1)$ , and

$$\text{E}(Z) = \sqrt{\frac{2}{\pi}} \frac{\lambda}{\sqrt{1 + \lambda^2}}, \quad (2)$$

$$\text{var}(Z) = 1 - \frac{2}{\pi} \frac{\lambda^2}{1 + \lambda^2}. \quad (3)$$

## 2.2 Skew t Distribution

The skew t distribution is due to Azzalini and Capitanio (2003). Let  $Z \sim \text{SN}(\lambda)$  and  $U \sim \chi^2(\nu)$  be independent. Then  $Z/\sqrt{U/\nu}$  has a skew t distribution with  $\nu$  degrees of freedom and skewness parameter  $\lambda$ , denoted as  $\text{St}(\nu; \lambda)$ , whose pdf is  $\forall x \in \mathbb{R}$ ,

$$f(x; \nu, \lambda) = 2F\left(\lambda x \sqrt{\frac{\nu + 1}{\nu + x^2}}; \nu + 1\right) f(x; \nu), \quad (4)$$

where  $f(\cdot; \nu)$  is the pdf of  $t(\nu)$  and  $F(\cdot; \nu + 1)$  is the cdf of  $t(\nu + 1)$ . The `sn` package for R helps to evaluate the cdf and the pdf of  $\text{St}(\nu; \lambda)$ .

Let  $X \sim \text{St}(\nu; \lambda)$ . Then  $-X \sim \text{St}(\nu; -\lambda)$  and  $X^2 \sim F(1, \nu)$ . If  $\nu > 1$ , then

$$\text{E}(X) = \frac{\Gamma((\nu - 1)/2)}{\Gamma(\nu/2)} \sqrt{\frac{\nu}{\pi}} \frac{\lambda}{\sqrt{1 + \lambda^2}}, \quad (5)$$

and if  $\nu > 2$ , then

$$\text{var}(X) = \frac{\nu}{\nu - 2} - \left( \frac{\Gamma((\nu - 1)/2)}{\Gamma(\nu/2)} \right)^2 \frac{\nu}{\pi} \frac{\lambda^2}{1 + \lambda^2}. \quad (6)$$

### 3 THE CARLSON–PARKIN METHOD

The C–P method is estimation of an ordered probit model applied to inflation expectations data with particular identification restrictions. Let  $\mathbf{y}$  be a random sample of size  $n$ . Assume that for  $i = 1, \dots, n$ ,

$$y_i := \begin{cases} 1 & \text{if } y_i^* \leq a \\ 2 & \text{if } a < y_i^* < b, \\ 3 & \text{if } y_i^* \geq b \end{cases} \quad (7)$$

$$y_i^* \sim \text{N}(\mu, \sigma^2), \quad (8)$$

where  $y_i^*$  is latent. Consider estimation of  $\mu$  given  $\mathbf{y}$ . An ordered probit model usually assumes that  $\sigma^2 := 1$  and  $a := 0$  by rescaling  $y_i^*$ , virtually estimating not  $(\mu, b)$  but  $((\mu - a)/\sigma, (b - a)/\sigma)$ . The C–P method instead identifies  $(a, b)$  from the models for multiple periods, assuming a time-invariant symmetric indifference interval ( $a = -b$ ) and long-run unbiased expectations. Given  $(a, b)$ , the C–P method estimates  $(\mu, \sigma^2)$  by the method of moments.

## 4 INTERVAL REGRESSION

Assume a normal interval regression model, i.e., an ordered probit model with known boundaries, for  $y_i$  such that

$$y_i := \begin{cases} 1 & \text{if } c_0 < y_i^* \leq c_1 \\ \vdots & \\ J & \text{if } c_{J-1} < y_i^* \leq c_J \end{cases}, \quad (9)$$

$$y_i^* = \mu + \sigma u_i, \quad (10)$$

$$u_i \sim N(0, 1), \quad (11)$$

where  $c_0 := -\infty$  and  $c_J := \infty$ . Then for  $i = 1, \dots, n$ , for  $j = 1, \dots, J$ ,

$$\begin{aligned} \Pr[y_i = j] &= \Pr[c_{j-1} < y_i^* \leq c_j] \\ &= \Phi\left(\frac{c_j - \mu}{\sigma}\right) - \Phi\left(\frac{c_{j-1} - \mu}{\sigma}\right). \end{aligned} \quad (12)$$

Since the choice probabilities sum up to 1, we have  $J - 1$  moment restrictions to identify up to  $J - 1$  parameters. If  $J \geq 3$ , then we can identify  $(\mu, \sigma^2)$ . If  $J \geq 4$ , then we can relax the assumption of normality or known boundaries (or both if  $J \geq 5$ ), though we must know at least two boundaries.

The log-likelihood function of  $(\mu, \sigma)$  given  $\mathbf{y}$  is  $\forall (\mu, \sigma) \in \mathbb{R} \times \mathbb{R}_{++}$ ,

$$\begin{aligned} \ell(\mu, \sigma; \mathbf{y}) &= \sum_{i=1}^n \sum_{j=1}^J [y_i = j] \ln \left( \Phi\left(\frac{c_j - \mu}{\sigma}\right) - \Phi\left(\frac{c_{j-1} - \mu}{\sigma}\right) \right) \\ &= \sum_{j=1}^J n_j \ln \left( \Phi\left(\frac{c_j - \mu}{\sigma}\right) - \Phi\left(\frac{c_{j-1} - \mu}{\sigma}\right) \right), \end{aligned} \quad (13)$$

where  $n_j := \sum_{i=1}^n [y_i = j]$ . Let  $\alpha := 1/\sigma$ ,  $\beta := \mu/\sigma$ , and  $\mathbf{c} := (c_1, \dots, c_{J-1})'$ . Then the log-likelihood function is concave in  $(\alpha, \beta, \mathbf{c})$  and hence unimodal in  $(\mu, \sigma, \mathbf{c})$ ; see Pratt (1981). ML estimation of a normal interval regression model (with known boundaries) is easy on some econometric software packages; e.g., gretl and Stata have a command `intreg`.

An SN interval regression model assumes that  $u_i \sim \text{SN}(\lambda)$ . An St interval regression model assumes that  $u_i \sim \text{St}(\nu; \lambda)$ . The `sn` package for R helps to evaluate the cdf in the the log-likelihood function. Note that the log-likelihood function may not be unimodal.

## 5 APPLICATION

### 5.1 Data

The Economic and Social Research Institute (ESRI) of the Cabinet Office, Government of Japan, integrated three old surveys to renew the Monthly Consumer Confidence Survey in April 2004. The survey covers 6,720 households selected by stratified sampling, and asks about various perceptions, price expectation a year ahead, and the state of the household. Individual data are not available. Aggregate data are downloadable from the ESRI's home page. Some disaggregate data are also available; e.g., by household type (one person or not), by sex and age of the head of the household, by income, by region, etc. The response rates were almost 100% by March 2006, and then dropped to around 75% in 2007.

The respondents select their price expectations a year ahead from the

following seven categories (excluding “don’t know”):

1. go down by 5% or more,
2. go down by 2% or more, but less than 5%,
3. go down by less than 2%,
4. stay the same (0%),
5. go up by less than 2%,
6. go up by 2% or more, but less than 5%,
7. go up by 5% or more.

Thus except between 3, 4, and 5, i.e., the indifference interval, the questionnaire gives the boundaries between the categories. We exclude “don’t know” from the sample in the following analysis. Two more categories, “go up/down by 10% or more,” appeared since April 2009, but we maintain the above seven categories as released in the summary report of the Survey. Figure 1 plots the sample size for each month, which is decreasing over time but still quite large.

Figure 2 plots the relative frequencies of the categories for each month. The expectations are heterogeneous, but on average, household inflation expectations in Japan rose since 2007, reaching a peak in mid-2008, and then dropped. Note that the number of households expecting deflation (categories 1–3) increased substantially since late 2008.

In the following, we estimate the distributions of household inflation expectations in Japan from April 2004 to March 2010 using three interval re-

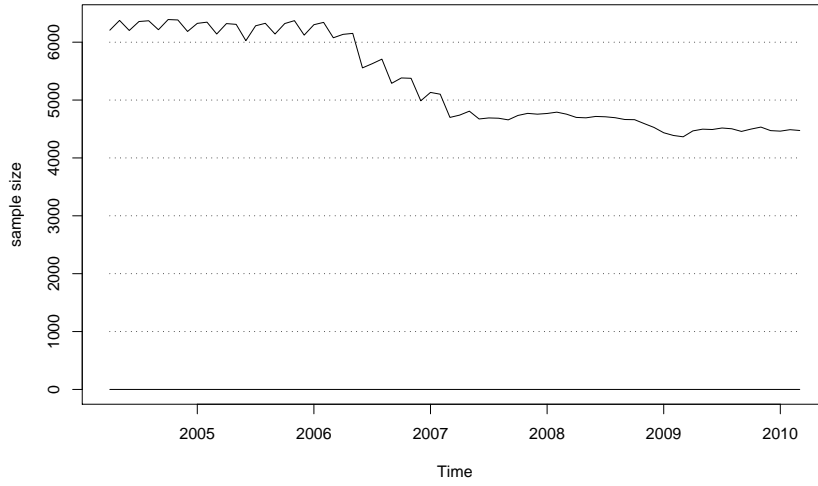


Figure 1: Sample Size (Excluding “Don’t Know”).

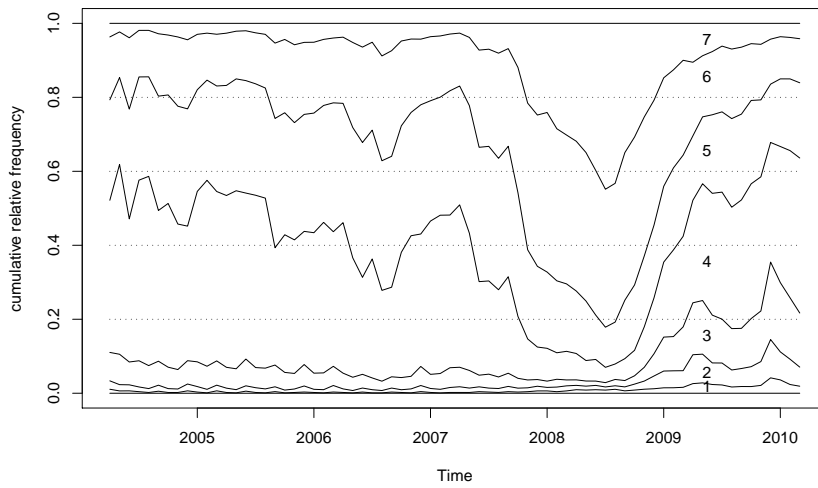


Figure 2: Relative Frequencies of the Categories.

gression models (normal, SN, and St). We use the `bbmle` package for R for numerical maximization of the log-likelihood function.

## 5.2 Parameter Estimates

### 5.2.1 Normal interval regression

Figure 3 plots the ML estimates of the parameters in the normal interval regression model and their asymptotic 95% confidence intervals for each month. Apart from possible misspecification bias, the estimation errors are extremely small, though increasing slightly since 2006 because of the decreasing sample size. As noted in the recent literature on rationally heterogeneous expectations, the mean and the standard deviation of household inflation expectations have positive correlation. The indifference intervals are asymmetric and unstable over time.

### 5.2.2 SN interval regression

Figure 4 plots the ML estimates of the parameters in the SN interval regression model and their asymptotic 95% confidence intervals for each month. Note that  $\mu$  and  $\sigma$  are location and scale parameters, and not the mean and the standard deviation unless  $\lambda = 0$ . Until mid-2007,  $\mu$  is negative and  $\lambda$  is positive; then  $\mu$  rises and  $\lambda$  drops to 0. Moreover,  $\lambda$  turns negative occasionally since late 2008. Since the information matrix of an SN distribution is singular when  $\lambda = 0$ , the estimation errors increase as  $\lambda \rightarrow 0$ . The indifference intervals are asymmetric and unstable over time.

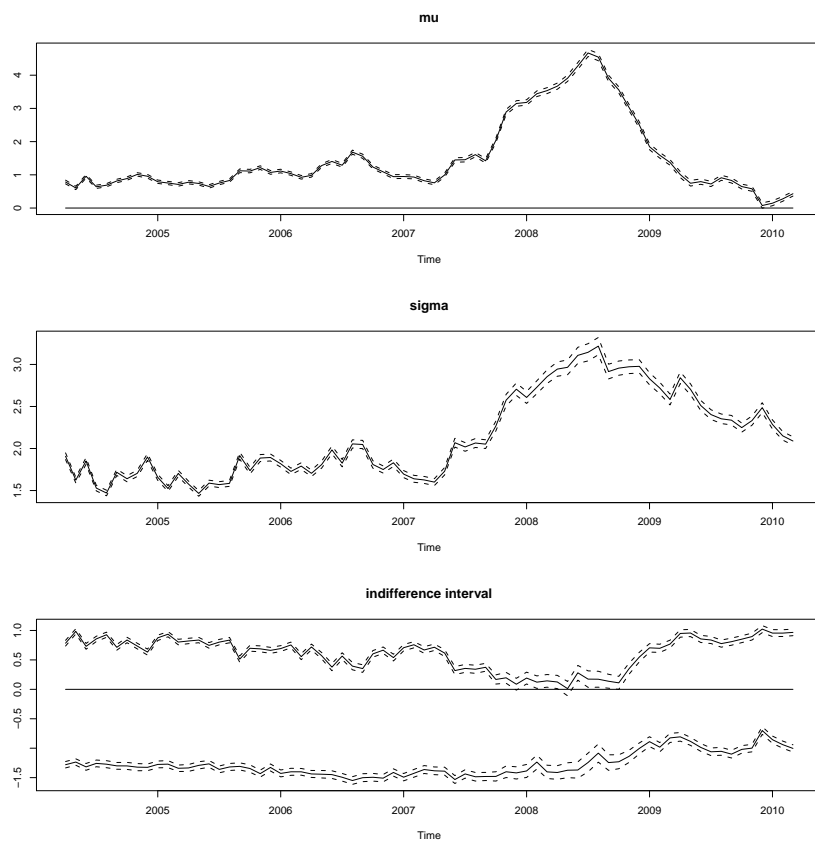


Figure 3: ML Estimates of the Parameters in the Normal Model. The dashed lines are asymptotic 95% confidence intervals.

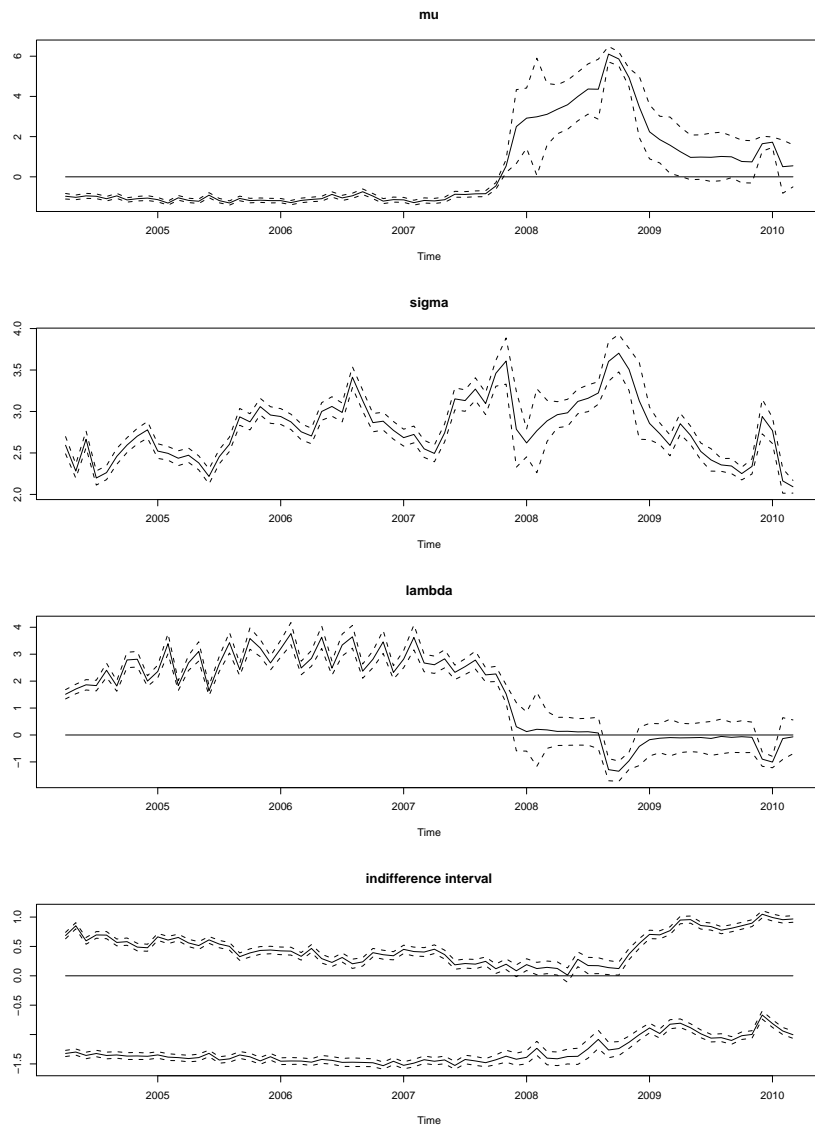


Figure 4: ML Estimates of the Parameters in the SN Model. The dashed lines are asymptotic 95% confidence intervals.

### 5.2.3 St interval regression

Figure 5 plots the ML estimates of the parameters in the St interval regression model and their asymptotic 95% confidence intervals for each month. With six parameters to estimate, the estimation errors increase substantially. Contrary to the SN model,  $\lambda$  remains positive except December 2009 and January 2010. Interestingly,  $\nu$  is small and often below 2, in which case the variance is infinite. The indifference intervals are more symmetric and stable over time than those of the normal and SN models, staying around  $\pm 5$ –1%.

## 5.3 Model Selection

Figure 5 already suggests that the St model is preferable to the normal and SN models throughout the period studied, since  $\lambda$  is significantly different from 0 and  $\nu$  is small.

To find the best model, one can also use a model selection criterion such as Akaike's information criterion (AIC) or Schwarz's Bayesian information criterion (SBIC). Davidson and MacKinnon (2004, pp. 676–677) define

$$\begin{aligned} \text{AIC} &:= \ell - k, \\ \text{SBIC} &:= \ell - \frac{k \ln n}{2}, \end{aligned}$$

where  $\ell$  is the log-likelihood,  $k$  is the number of parameters estimated, and  $n$  is the sample size.

Figure 6 plots AIC and SBIC of the SN and St models relative to those of the normal model, i.e., AIC (SBIC) of the SN (St) model minus that of the

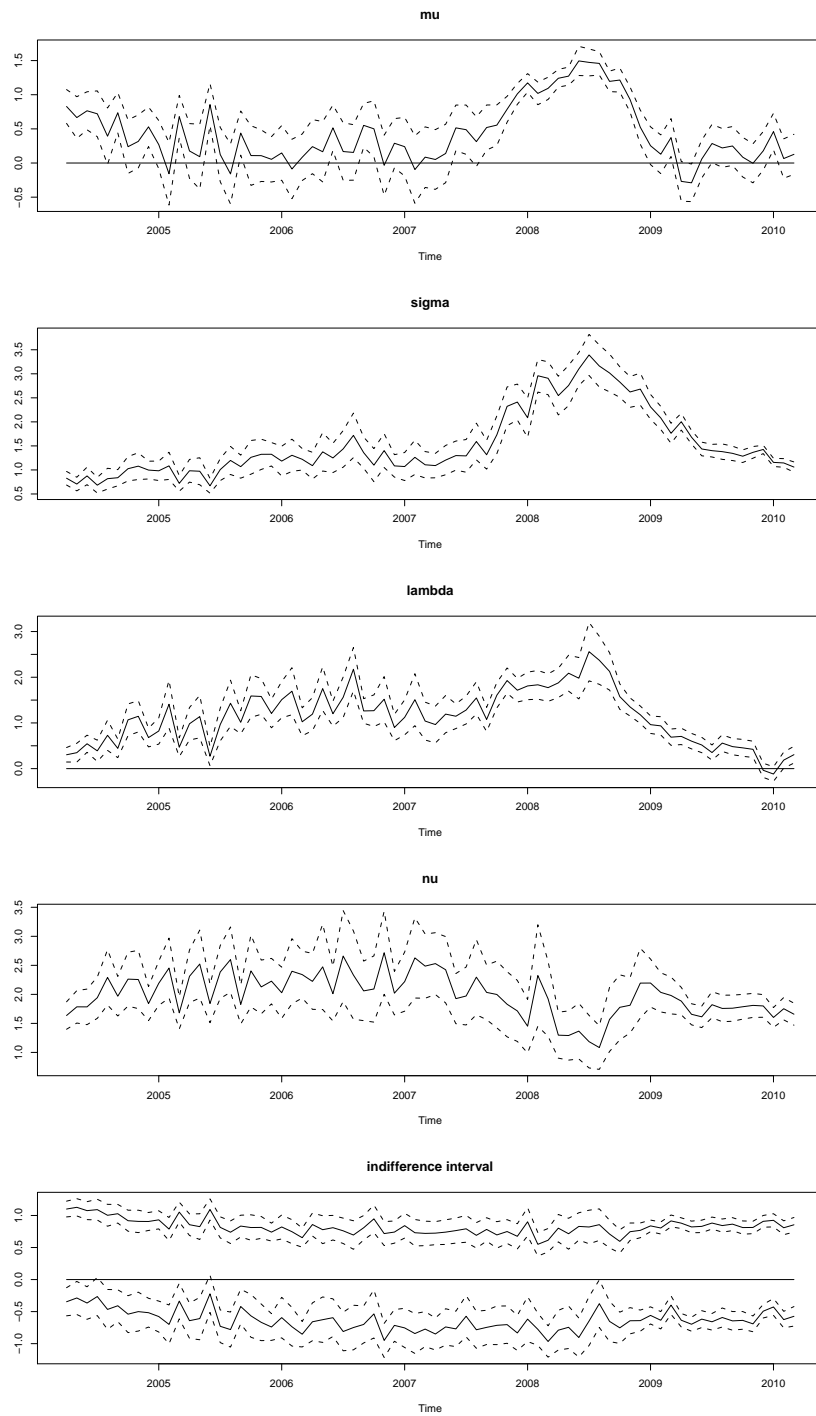


Figure 5: ML Estimates of the Parameters in the St Model. The dashed lines are asymptotic 95% confidence intervals.

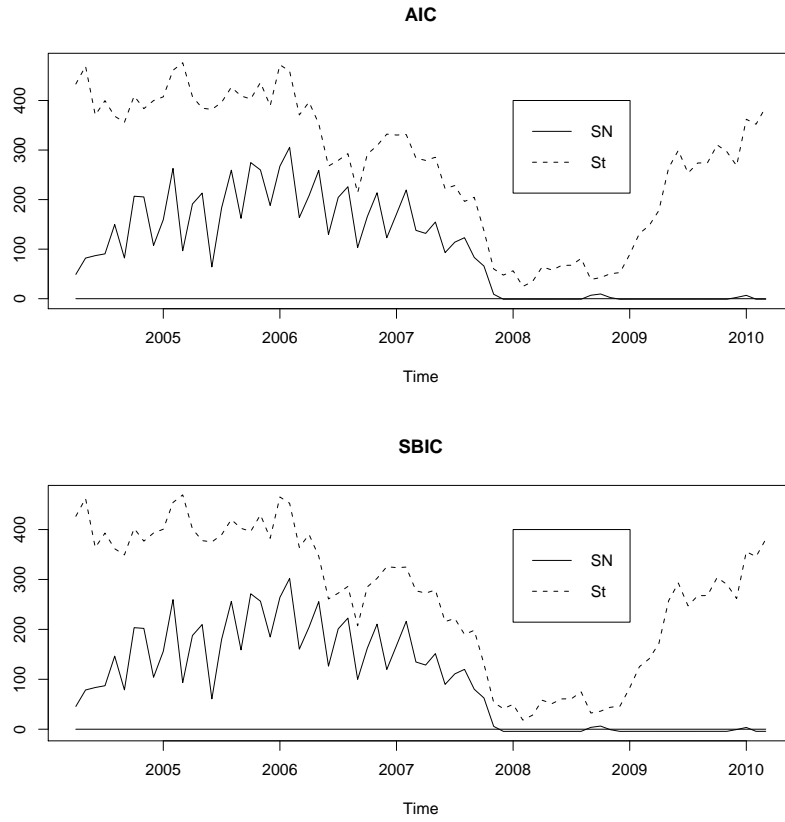


Figure 6: AIC and SBIC of the SN and St Models Relative to the Normal Model.

normal model, for each month. We see that the St model is most preferable throughout the period studied.

## 5.4 Inflation Expectations

For the SN model, the mean inflation expectation is

$$E(y_i^*) = \mu + \sigma \sqrt{\frac{2}{\pi}} \frac{\lambda}{\sqrt{1 + \lambda^2}}. \quad (14)$$

For the St model, if  $\nu > 1$ , then

$$E(y_i^*) = \mu + \sigma \frac{\Gamma((\nu - 1)/2)}{\Gamma(\nu/2)} \sqrt{\frac{\nu}{\pi}} \frac{\lambda}{\sqrt{1 + \lambda^2}}. \quad (15)$$

The top panel of Figure 7 compares the mean inflation expectations implied by the three models with the current CPI inflation (12-month growth rate). For the St model, the mean inflation expectations are too high (25%) in 2008 when  $\nu$  is close to 1. The middle panel compares the median inflation expectations implied by the three models with the current CPI inflation. The three median inflation expectations are close to each other, and mostly higher than the current CPI inflation during the period studied. The SN model gives the lowest median inflation expectations until mid-2007. Since then, the St model gives the lowest median inflation expectations. The bottom panel compares the mode inflation expectations implied by the three models with the current CPI inflation. Until 2007, the SN model gives the lowest mode inflation expectations, which are also closest to (sometimes below) the current CPI inflation. Since then, the St model gives the lowest mode inflation expectations.

Let  $\pi_t$  be the CPI inflation rate from date  $t - 12$  to date  $t$ . Let  $\pi_{t+12|t}^e$  be a survey expectation of  $\pi_{t+12}$  at date  $t$ . Let  $e_t := \pi_{t|t-12}^e - \pi_t$  be a forecast error at date  $t$ . Figure 8 plots the forecast errors of the mean, median, and mode inflation expectations of the St model. The three inflation expectations tend to overpredict the actual CPI inflation during the period studied. On average, the mode inflation expectations have the smallest bias. Capistrán and Timmermann (2009) attribute the bias to asymmetric loss. It also arises

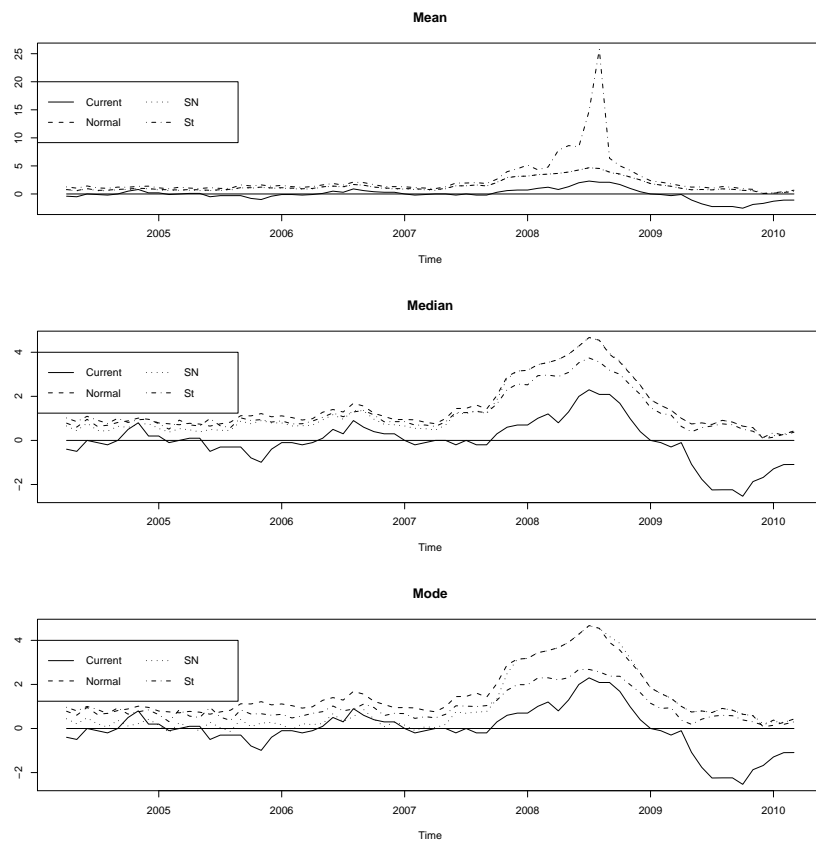


Figure 7: Current Inflation and the Mean, Median, and Mode Inflation Expectations.

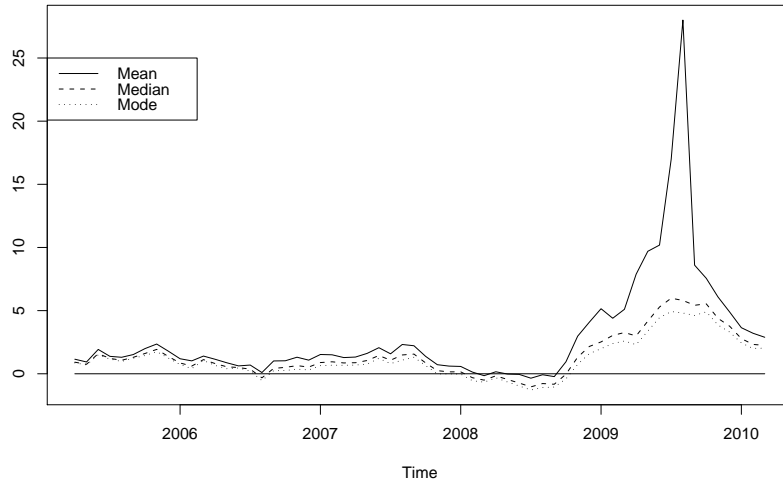


Figure 8: Forecast Errors of the Mean, Median, and Mode Inflation Expectations of the St Model.

from response bias and the CPI inflation bias, though the latter is usually an upward bias relative to the true cost-of-living index inflation; see Hausman (2003) and Lebow and Ruud (2003).

Figure 9 shows the relation between the distribution of household inflation expectations and the current CPI inflation. The median, the IQR, the skewness parameter, and excess kurtosis all increase with the current CPI inflation when the latter is positive (smaller  $\nu$  means higher excess kurtosis); the relations are unclear when the latter is negative. These may arise from sticky information (Mankiw and Reis (2002), Mankiw et al. (2003), Carroll (2003)), model uncertainty (Branch (2004, 2007)), or asymmetric loss (Capistrán and Timmermann (2009)). To study the dynamics of heterogeneous expectations in detail, we need individual data.

A possible use of the estimated inflation expectations is inflation fore-

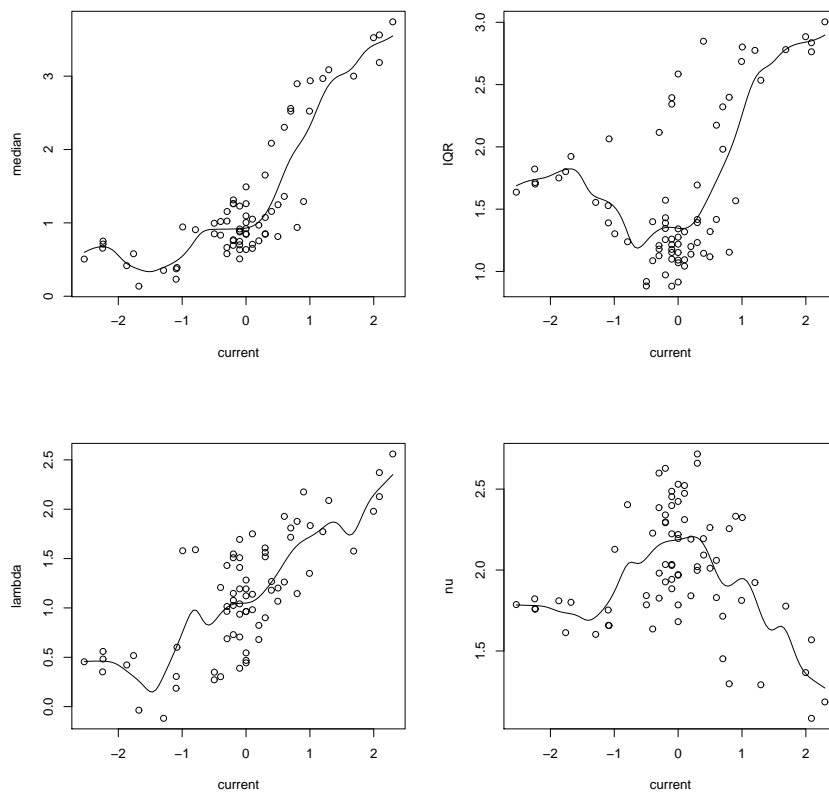


Figure 9: Current Inflation and the Distribution of Inflation Expectations. The nonparametric regression uses the Nadaraya–Watson estimator with a Gaussian kernel.

casting. Ang et al. (2007) find that survey expectations predict US inflation better than other variables. We try the mean, median, and mode inflation expectations of the normal, SN, and St models for  $\pi_{t+12|t}^e$ , and estimate the following predictive regression:

$$E(\pi_t|I_{t-12}) = \alpha\pi_{t-12} + \beta\pi_{t|t-12}^e, \quad (16)$$

where  $I_t$  is the information available at date  $t$ . We use gretl 1.9.0 for OLS. The estimation period is from April 2005 to March 2010 (60 observations).

Table 1 compares the results of predictive regressions using the mean, median, and mode inflation expectations of the SN model and the mean (=median=mode) inflation expectation of the normal model respectively. We see that none of them helps to predict the actual CPI inflation individually beyond the current CPI inflation. Using all three expectation measures of the SN model improves prediction, but this may not work out-of-sample because the estimated coefficients seem unreasonable.

Table 2 compares the results of predictive regressions using the mean, median, and mode inflation expectations of the St model and the mean inflation expectation of the normal model respectively. We see that none of them, even jointly, helps to predict the actual CPI inflation beyond the current CPI inflation.

Note that the estimation period is too short to give a definitive conclusion, covering at most one business cycle. Dropping the last 12 observations, we find that each expectation measure helps to predict the actual CPI inflation beyond the current CPI inflation, with reasonable sign and size of

Table 1: Predictive Regressions for the CPI Inflation (SN vs Normal)

Regressor	SN mean	SN median	SN mode	SN all	Normal
constant	0.10 (0.25)	0.21 (0.21)	0.30 (0.17)	-1.91 (0.44)	0.11 (0.26)
$\pi_{t-12}$	-1.11 (0.44)	-0.97 (0.43)	-0.76 (0.39)	-1.09 (0.44)	-1.09 (0.44)
$\pi_{t t-12}^e$ (mean)	0.10 (0.25)			41.57 (9.80)	0.09 (0.26)
$\pi_{t t-12}^e$ (median)		0.01 (0.24)		-55.37 (13.52)	
$\pi_{t t-12}^e$ (mode)			-0.12 (0.20)	14.44 (3.82)	
adjusted $R^2$	0.38	0.38	0.38	0.67	0.38
AIC	153.61	153.94	153.20	117.31	153.71
SBIC	159.89	160.23	159.48	127.78	159.99

NOTE: Numbers in parentheses are HAC standard errors. In gretl 1.9.0, AIC is  $-2\ell + 2k$  and SBIC is  $-2\ell + k \ln T$ , where  $\ell$  is the log-likelihood,  $k$  is the number of parameters estimated, and  $T$  is the sample length.

Table 2: Predictive Regressions for the CPI Inflation (St vs Normal)

Regressor	St mean	St median	St mode	St all	Normal
constant	0.26 (0.19)	0.18 (0.29)	0.22 (0.29)	0.16 (0.29)	0.11 (0.26)
$\pi_{t-12}$	-0.87 (0.27)	-1.00 (0.41)	-0.96 (0.37)	-1.02 (0.44)	-1.09 (0.44)
$\pi_{t t-12}^e$ (mean)	-0.02 (0.03)			-0.05 (0.04)	0.09 (0.26)
$\pi_{t t-12}^e$ (median)		0.03 (0.32)		1.46 (1.18)	
$\pi_{t t-12}^e$ (mode)			-0.00 (0.39)	-1.67 (1.28)	
adjusted $R^2$	0.38	0.38	0.38	0.37	0.38
AIC	153.65	153.93	153.95	156.34	153.71
SBIC	159.94	160.21	160.23	166.81	159.99

NOTE: See note to Table 1.

the coefficient. Hence the large positive price shock in 2008 may distort the result. The analysis here only illustrates a possible use of estimated inflation expectations.

## 6 DISCUSSION

Quantification of qualitative survey data requires strong assumptions. Interval-coded data do not require many of them. Less assumptions lead to more reliable measurement of inflation expectations.

A time series of estimated distributions of household inflation expectations have several uses. This paper gives two examples. One is to study the dynamic relation between the current inflation and the distribution of household inflation expectations, which may help to develop the theory of rationally heterogeneous expectations. The other is to use the estimated distributions for inflation forecasting. Thorough studies on these issues, however, require a long time series covering multiple business cycles.

If households report their expectations correctly (though Inoue et al. (2009) give evidences that they do not), then different expectations come from different information sets. An interesting question is why the distribution of household inflation expectations has high skewness and excess kurtosis (during the period studied). This paper does not answer that question. The followings are some ideas for future research:

1. Finite mixture models may help to explain the shape of the distribution; e.g., a mixture of two normal distributions assuming a symmetric indifference interval.

2. Demographic variables may explain the shape of the distribution, though such studies require individual data.
3. Each person has a belief, or subjective pdf, over future inflation, but the Monthly Consumer Confidence Survey asks only about the mean of the pdf. One can ask probabilistic questions to learn more about the pdf; see Manski (2004).

In addition, the Monthly Consumer Confidence Survey in Japan has changed slightly since April 2009, and now there are nine intervals to choose from. Thus one can fit even more general distributions.

Finally, the SN and St distributions, both univariate and multivariate, will have other interesting applications; e.g., one can use the multivariate SN and St distributions instead of copulas.

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