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Non-Wasteful Government Spending in an Estimated Open Economy DSGE Model: Two Fiscal Policy Puzzles Revisited

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Non-Wasteful Government Spending in an Estimated Open Economy DSGE Model: Two Fiscal Policy Puzzles Revisited

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Abstract

This paper examines two fiscal policy puzzles related to the effects of government spending shocks. Contrary to theoretical predictions, recent empirical evidence suggests a crowding-in of consumption and a depreciation of the real exchange rate after a government spending increase. While several studies have been made to reconcile the conflicting results, this paper provides new time-series evidence and proposes an alternative explanation using the Japanese data. The empirical responses of consumption and the real exchange rate after government spending shocks are shown to be well-replicated by an estimated medium-scale open economy dynamic stochastic general equilibrium (DSGE) model augmented with (i) Edgeworth complementarity between private and public consumption, and (ii) productive public capital. Furthermore, sensitivity analysis suggests that the combination of Edgeworth complementarity, home bias, and incomplete asset market allows the model to account for an immediate increase in consumption and for a hump-shaped depreciation of the real exchange rate after a government consumption shock. This result is potentially important in preventing the model from showing the consumption-real exchange rate anomaly after the shock.


Keywords: DSGE modeling, fiscal policy, Bayesian estimation, Japanese economy.

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1 Introduction

Fiscal policy has been gaining renewed attention as a stabilization tool after the Lehman shock, since the zero bound on nominal interest rate has become a binding constraint for monetary policy in major industrial countries. With regard to the consequences of fiscal policy, however, there are two major disagreements between theoretical predictions and empirical evidence on the responses of private consumption and the real exchange rate to a government spending shock. A structural vector autoregressive (VAR) analysis tends to find a crowding-in of consumption and a depreciation of the real exchange rate after an increase in government spending.\(^1\) However, standard dynamic general equilibrium models predict crowding-out of consumption in response to a government spending increase, while the textbook IS-LM models predict a positive response of consumption. In addition, both the international real business cycle (IRBC) models and the new open economy macroeconomics (NOEM) models, as well as traditional Mundell-Fleming IS-LM models, predict an appreciation of the real exchange rate.\(^2\)

Whereas the first puzzle concerning the response of consumption to a government spending shock has been well recognized and several theoretical attempts have been made to account for the anomalies in a closed-economy setting,\(^3\) the second puzzle, which concerns the response of the real exchange rate, has received less attention, at least until recently. Kim and Roubini (2008), Monacelli and Perotti (2010), Corsetti et al. (2012), and Ravn et al. (2012) have documented that government spending shocks in one country depreciate its real exchange rate, based on empirical evidence from VAR models. Since their work has been published, reconciliation between the empirical evidence and theoretical predictions has become an important challenge for fiscal policy analysis in an open economy. Several theoretical approaches have been developed; Monacelli and Perotti (2010) suggest that a model with non-separable preferences over consumption and leisure may generate a depreciation of the real exchange rate in response to a government spending shock. Corsetti et al. (2012) and Ravn et al. (2012) show that models augmented with "spending reversals" and "deep habits" can replicate the responses of the real exchange rate}

\(^1\)Blanchard and Perotti (2002) and Kim and Roubini (2008) are the early studies that have found crowding-in of consumption and a depreciation of the real exchange rate after government spending shocks, respectively. Note that some VAR analyses find the opposite, depending on identification methods and data frequency. VAR evidence based on narrative approaches typically shows that an increase in government spending has negative effects on consumption (see, for example, Ramey (2011)). VAR analyses using low frequency (i.e., annual) data tend to find a real exchange rate appreciation in response to government spending shocks (see, for example, Beetsma and Giuliodori (2011)).

\(^2\)Regarding the general-equilibrium effects of fiscal policy in standard closed and open economy settings, see, for example, Baxter and King (1993) and Backus et al. (1994), respectively.

\(^3\)It has been shown that positive response of consumption to a government spending increase can be generated in a dynamic general equilibrium model by introducing either of the following: (a) non-Ricardian households (see Gali et al. (2007)), (b) non-separable preferences over consumption and leisure (see Linnemann (2006), Bilbiie (2009) and Bilbiie (2011)), (c) "deep habits" (see Ravn et al. (2006)), (d) "spending reversals" (see Corsetti et al. (2010)), (e) productive public capital (see Linnemann and Schabert (2006)), and (f) Edgeworth complementarity between private and public consumption (see Bouakez and Rebei (2007)).
from VAR models, respectively.

It is important to note that these approaches solve the first puzzle and the second puzzle simultaneously, and rely on the international risk-sharing condition, which relates dynamics of the real exchange rate to that of consumption in standard IRBC and NOEM models under complete asset market assumption. The international risk-sharing condition, on the other hand, has been known to cause the consumption-real exchange rate anomaly between predictions of open economy models and empirical evidence, also referred to as the Backus-Smith puzzle.4 Backus and Smith (1993) and Kollmann (1995) have independently shown that empirical observations do not provide supportive evidence for a positive correlation between relative consumption across countries and the real exchange rate, as opposed to the predictions of standard open economy models. The above-mentioned approaches that study the two fiscal policy puzzles have focused on generating proper directions of the responses of consumption and the exchange rate to a government spending shock in models with complete asset market, in which a crowding-in of consumption is always accompanied by a depreciation of the real exchange rate due to the international risk-sharing condition. Thus, timing of the responses has not yet been well considered in the literature so far.

Although the effects of government spending have always been at the center of the policy debate, their composition has been largely ignored, and they have been typically modeled as wasteful in most macroeconomic models. Government spending can be broadly divided into two categories according to the roles they play in the economy: government consumption and government investment. The necessity of modeling two major categories of government spending has long been recognized (see, for example, Barro (1981), Aschauer (1985), and Aschauer (1989)), but very few papers have considered non-wasteful nature of government spending, especially in the context of two fiscal policy puzzles.5 Linnemann and Schabert (2006) and Bouakez and Rebei (2007) are the early attempts that account for the first puzzle by incorporating productive public

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4 Several studies have examined whether the anomaly can be attributed to international asset market incompleteness. While Chari et al. (2002) have shown that introduction of incomplete asset market is not sufficient in eliminating the positive correlation between consumption and the real exchange rate, recent studies have found that it is possible to account for the puzzle in models with incomplete asset markets by generating a strong wealth effect (see Corsetti et al. (2008)), or by incorporating other features, such as local currency pricing and traded and non-traded sectors (see Benigno and Thoenissen (2008) and Rabanal and Tuesta (2010)). On the other hand, Kollmann (2012) shows that if the share of non-Ricardian households is sufficiently large, the correlation can be eliminated even in a model with a complete asset market.

5 Studies that have considered both categories of government spending in a single framework are more limited. Pappa (2009) examines the effects of government consumption and government investment by estimating a structural VAR. The shocks are identified via sign-restrictions, relying on a closed economy model with non-separability between private and public consumption, and productive public capital. Galstyan and Lane (2009) show that the composition of government spending influences the long-run behavior of the real exchange rate using cross country pooled data. Ganelli and Tervala (2010) study the welfare effects of government spending in a calibrated two-country model with utility-enhancing public consumption (i.e., substitutability between private and public consumption is assumed) and productive public capital.
capital and Edgeworth complementarity between private and public consumption, respectively.\textsuperscript{6} As for the second puzzle, Basu and Kollmann (2010) recently showed that a simple two-country model can generate a real exchange rate depreciation if it features productive public capital. To the best of my knowledge, Edgeworth complementarity has not yet been examined within the context of an open economy model.

With respect to empirical testing of the models that address the two fiscal policy puzzles, most of the studies have been confined to a theoretical examination, despite the recent developments in macroeconometrics.\textsuperscript{7} Ravn et al. (2012) is the only study I am aware of that estimates the key structural parameters that define the deep-habit mechanism that contributes toward generating a real exchange depreciation after a government spending shock in an open economy model.\textsuperscript{8} Regarding non-wasteful nature of government spending, Bouakez and Rebei (2007) estimate the parameters that govern Edgeworth complementarity between private and public consumption within a general equilibrium framework. The importance of estimating the key parameter was recently demonstrated by Fève et al. (2012). They argue that the effects of a government spending shock are downward-biased when the parameter governing Edgeworth complementarity between private and public consumption is not included in the estimation.

Furthermore, existing contributions on the two fiscal policy puzzles have focused on data for Anglo-Saxon countries. It is, however, worth exploring the effects of fiscal expansion on the real exchange rate in the context of Japanese fiscal policy. Japan’s current account surplus position is considered to have helped maintain stability in the Japanese Government Bond (JGB) market, despite the high level of government debt.\textsuperscript{9} If expansionary fiscal policy appreciates the real exchange rate and leads to a current-account deterioration, its use as a stabilization tool needs to be restrained in light of Japan’s current fiscal position. Nonetheless, time-series evidence on the effect of government spending on the real exchange rate has not yet been established for the Japanese data. On the other hand, recent empirical studies based on the Japanese data...
tend to suggest a non-wasteful nature of government spending. Kawaguchi et al. (2009) find a positive externality of public capital in Japan. Okubo (2008) concludes that the relationship between private and public consumption in Japan is not a substitute, which is consistent with the early findings by Karras (1994). With regard to the consumption-real exchange rate anomaly, Corsetti et al. (2011) show that the negative correlation between relative consumption and the real exchange rate can be found in most OECD countries, including Japan. This implies that the international risk-sharing condition under complete asset market need to be modified in modeling the Japanese economy.

Against this background, the present paper seeks to solve the two fiscal policy puzzles in a medium-scale open economy DSGE model with incomplete asset market estimating parameters governing non-wasteful nature of government spending by using the Japanese data. In the following, I first estimate a structural VAR model using the same Japanese data as those used to estimate the DSGE model. Then I extend a standard medium-scale open economy model by incorporating non-separability between private and public consumption, and productive public capital. The model is based on the work of Adolfson et al. (2007), which is a small open economy version of the canonical Smets and Wouters (2003) model. As in Adolfson et al. (2007), the model features an incomplete asset market structure by assuming debt-elastic interest rate premium. I also incorporate "spending reversals" to the model to examine whether they work with the Japanese data. Investment-specific technological progress (IST) is also considered in addition to neutral technological progress for the purpose of facilitating parameter identification. As pointed out by Hirose and Kurozumi (2010), relative price of investment goods in Japan shows a quite similar pattern to that of the United States, which indicates the necessity of modeling IST.

The impulse responses from the structural VAR model show that the empirical anomalies found in data for Anglo-Saxon countries can be observed in the Japanese data: consumption increases and the real exchange rate depreciates after both government consumption and government investment shocks. The directions of empirical responses can be well-replicated by the estimated DSGE model. While the empirical relevance of spending reversals in government investment is confirmed, Edgeworth complementarity and productive public capital are shown to

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10 They measure the correlation between relative consumption and the real exchange rate at lower frequency using spectral analysis. The unconditional measure for Japan reported in Corsetti et al. (2008), in contrast, indicates a positive correlation between the two.

11 Introduction of debt-elastic interest rate premium is a quite popular approach to modeling incomplete asset market structure in the literature. See, for example, Kollmann (2002), Schmitt-Grohé and Uribe (2003), Benigno (2009), and Justiniano and Preston (2010) among others. This approach is consistent with the empirical findings by Lane and Milesi-Ferretti (2002) that the net foreign asset positions play an important role in determining real interest rate differentials. Note also that Rabanal and Tuesta (2010) and Benigno and Thoenissen (2008) employ this approach to address the consumption-real exchange rate anomaly in their models with an incomplete asset market structure.
be the main contributory sources for generating responses of consumption and the real exchange rate in the empirically-plausible directions following government consumption and government investment shocks, respectively. In addition, the timing of empirical responses to a government consumption shock is also well addressed. The simulation results reveal that the combination of Edgeworth complementarity, home bias, and incomplete asset market allows the model to account for an immediate increase in consumption and for a hump-shaped depreciation of the real exchange rate after a government consumption shock. This result matches the response patterns generated by the structural VAR model and is potentially important in preventing the model from showing the consumption-real exchange rate anomaly after the shock.

The remainder of this paper is organized as follows. In the next section, I estimate the effects of government spending shocks using a structural VAR model. In Section 3, I set out a medium-scale open economy DSGE model augmented with non-wasteful government spending. Section 4 presents the estimation results. Section 5 investigates the transmission of government consumption and government investment shocks conducting some sensitivity analyses. Finally, Section 6 concludes the paper with a brief discussion of a further research agenda.

2 Time-Series Evidence

2.1 The Structural VAR and Identification Methodology

I start my analysis by presenting time-series evidence from the Japanese data. I consider a VAR model that consists of nine variables: government spending, gross domestic product, private consumption, private investment, budget balance, trade balance (all on a per-capita basis), GDP deflator, real effective exchange rate, and short-term interest rates. I use the logarithm for all variables except for interest rates. Two categories of government spending—government consumption and government investment—are considered. The series come from the Cabinet Office and the OECD Economic Outlook database, with the exception of the short-term interest and real effective exchange rates, which are taken from the Bank of Japan Statistics. Private consumption is defined as personal consumption expenditures on non-durables and services, while private investment is the sum of personal consumption expenditures on durables and gross private domestic investment. Because the VAR analysis aims to provide dynamic properties of the time-series data to assess the empirical performance of the DSGE model to be developed in the next section, the sample period is chosen so as to cover the estimation period of the DSGE model, which starts in 1980:Q1 and ends in 1998:Q4. Nonetheless, because the estimation period of the DSGE model is relatively short, I extend the estimation period forward and backward to provide sufficient length to identify government spending shocks. Two data sets that contain
15 years of data are used. The first data set, 1973:Q1-1998:Q4, has the same end period as
the estimation period of the DSGE model, while the second data set, 1980:Q1-2005:Q4, has
the same starting period. Note that GDP data in the first data set are based on 1968 System
of National Accounts (SNA), while that in the second data set are based on 1993 SNA. The
latter are only available since 1980. Because government final consumption expenditure defined
in the 1993 SNA includes social security benefits in kind, I use actual final consumption (i.e.,
collective consumption expenditure, such as national defense etc.) as government consumption
in the second data set, considering that social security benefits in kind during this period have
clear upward trends due to population aging.

Regarding shock identification, I employ the sign-restrictions approach proposed by Uhlig
(2005). The idea is to require impulse responses to have certain signs, so that the signs are
consistent with the principles of macroeconomic theory. While identification in structural VAR
models usually requires many restrictions, the method imposes restrictions only on the responses
of selected variables for a certain period following the shock of interest. Given that the main
focus of this paper is to account for empirical responses to government spending shocks with a
DSGE model, this approach is attractive because it is more agnostic than other identification
approaches and is firmly grounded in macroeconomic theory. Although this approach does
not achieve exact identification of the structural shock, it does solve the structural parameter
identification problem with sufficient restrictions, as shown by Fry and Pagan (2011).

The basic framework is described as follows. Consider a VAR model in reduced-form:

$$Y_t = B(L)Y_{t-1} + U_t,$$

where $Y_t$ is an $m \times 1$ vector at date $t$, and $B(L)$ is a lag polynomial. Identification in the VAR
model amounts to finding a matrix $A$ such that $AA' = \Sigma$, and $U = AV$, where $\Sigma$ is the the
variance-covariance matrix of $U$ and $V$ is the vector of orthogonal structural shocks. Because I
am solely interested in a single (i.e., government spending) shock, all I need to know is impulse
vector $a \in \mathbb{R}^m$, a single column of $A$. It can be shown that any impulse vector can be expressed
as $a = \tilde{A}\alpha$, where $\tilde{A}$ satisfies $\tilde{A}\tilde{A}' = \Sigma$, and $\alpha$ is $m$-dimensional vector of unit length. Letting
$r_j(k) \in \mathbb{R}^m$ be the vector impulse response at horizon $k$ to the $j$-th shock, the impulse response

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12 Alternatively, three other approaches have been employed to identify fiscal policy shocks in the literature: the recursive approach (see Fatás and Mihov (2001)), the Blanchard-Perotti approach (see Blanchard and Perotti (2002)), and the narrative (or event-study) approach (see Ramey and Shapiro (1998)). See Caldara and Kamps (2008) for an extensive comparative study on these three approaches and the sign-restrictions approach.

13 Differently from other identification approaches that impose parametric restrictions, the number of "sufficient" restrictions for the sign-restrictions approach depends on the underlying data-generating process. For example, Paustian (2007) presents two conditions to be met to deliver the correct sign of impulse responses, but the result depends on the models used to perform Monte Carlo experiments. Fry and Pagan (2011) suggest utilizing parametric restrictions in addition to sign restrictions, considering that sign information is rather weak.
for \( a \) is given by:

\[
ra(k) = \sum_{j=1}^{m} \alpha_j r_j(k).
\]

Sign restrictions to identify an impulse vector are imposed on \( ra(k) \) for some horizon \( k = 0, 1, \ldots, K \). Assuming Bayesian (i.e., Normal-Wishart) prior for \((B(L), \Sigma)\), I take a joint draw from the posterior for \((B(L), \Sigma)\) constructed using the data, and from the \( m \)-dimensional unit sphere for \( \alpha \). For each draw, I construct impulse vector \( a \) and calculate impulse responses \( r_j(k) \). I keep the draw if the impulse responses satisfy the sign restrictions, and discard otherwise. For further methodological details, see Uhlig (2005).

### 2.2 VAR Evidence on Government Spending Shocks with Sign Restrictions

The sign-restrictions approach has been applied to identifying fiscal shocks (see, for example, Mountford and Uhlig (2009) and Pappa (2009)). Among others, Enders et al. (2011) recently provide evidence that government spending depreciates the real exchange rate when this relatively new methodology is employed for the U.S. data. Thus, I impose restrictions along the lines of Enders et al. (2011). Table 1 reports the set of sign restrictions used. The responses of consumption and the real exchange rate to a government spending shock are our main interest, and are therefore unrestricted. In addition, because responses of investment and trade balance depend on the specifications of DSGE models, I also leave the signs of these variables unrestricted. On the other hand, I impose restrictions that a government spending shock increases output, inflation, and interest rate, which are consistent with predictions of a large class of DSGE models.\(^{14}\)

The restriction that an increase in government spending has a negative impact on budget balance is the key identifying restriction that distinguishes a government spending shock from other shocks, such as productivity, or monetary policy shocks. The sign restrictions are imposed for a year after the shock (i.e., \( K = 3 \)), following Mountford and Uhlig (2009). Since our data sets are relatively short, the lag length of the VAR model is limited to three. I employ the same restrictions on responses to a government consumption shock and a government investment shocks.

The estimated impulse responses to a one-standard-deviation expansionary government consumption shock are shown in Figures 1.a and 1.b for the period 1973:Q1-1998:Q4 and 1980:Q1-2005:Q4, respectively. The median responses and the 16 and 84% quantiles are depicted. Inference is obtained from 4,000 draws from the unit sphere for each of 4,000 draws from the VAR posterior. In both sample periods, government consumption shocks generate a crowding-in of

\(^{14}\)Although the effectiveness of fiscal policy in Japan during the 1990s is a debatable issue, existing estimates of the government spending multiplier are found to be positive (see, for example, Bayoumi (2001) and Kuttner and Posen (2002)).
consumption and a depreciation of the real exchange rate. Note that an increase in the real exchange rate is conventionally expressed as a "depreciation" throughout this paper. While private consumption increases immediately after the shock contributing to output rise, the real exchange rate shows a hump-shaped pattern of depreciation. Private investment, on the other hand, shows large decline in later periods. Trade balance deteriorates on impact, but the real exchange rate depreciation induces improvement with some delay. The hump-shaped pattern of trade balance improvement is consistent with Corsetti et al. (2012).\textsuperscript{15} Figures 2.a and 2.b show responses to a one-standard-deviation expansionary government investment shock for the period 1973:Q1-1998:Q4 and 1980:Q1-2005:Q4, respectively. Very similar results to those of a government consumption shock are obtained, except that the depreciation of the real exchange rate after a government investment shock is not enough to improve the trade balance for the first data set. All in all, regardless of the type of government spending, it is confirmed that the empirical anomalies regarding responses of consumption and the real exchange rate to government spending shocks can be observed within the Japanese data. Although VAR evidence on the effect of government spending on the real exchange rate has not yet been established for the Japanese data, the results here are largely in line with the consistent findings across studies that consider data for Australia, Canada, the U.K., and the U.S.\textsuperscript{16} It is indicated that the downside risk of expansionary fiscal policy to Japan’s external position may not as big as standard open economy models predict.

\section{The Model}

I now turn to construct a medium-scale open economy DSGE model to be estimated. I extend the model of Adolfson et al. (2007) by incorporating the following features. First, I relax a common assumption in standard macroeconomic models that government spending is wasteful. When we distinguish the role of government consumption from government investment, non-separability between private and public consumption is assumed. With regard to government investment, on the other hand, I allow for a positive externality of public capital, which increases the productivity of private firms. Second, I introduce "spending reversals," which can be a contributory source to a crowding-in of private consumption and a depreciation of the real exchange rate after a government spending shock. Third, mainly for the purpose of estimation, I allow for an investment-specific technological (IST) progress suggested by Greenwood et al. (1997) for the

\textsuperscript{15}Empirical findings on trade balance responses to government spending shocks from VAR models are rather mixed: Kim and Roubini (2008) document that an expansionary fiscal shock improves the current account using the U.S. data, while Ravn et al. (2012) and Monacelli and Perotti (2010) find deterioration in trade balance in response to a government spending shock using data from Australia, Canada, the U.K., and the U.S.

\textsuperscript{16}See Kim and Roubini (2008), Monacelli and Perotti (2010), Corsetti et al. (2012), and Ravn et al. (2012).
U.S. data following Christiano et al. (2011), in addition to the neutral technological progress already incorporated in the Adolfson et al. (2007) model.

3.1 Firms

There are four types of monopolistically competitive firms: domestic intermediate-good producing, consumption-good importing, investment-good importing, and exporting firms. There are eight types of competitive firms: domestic final-good firms, wholesalers of imported consumption goods and wholesalers of imported investment goods, domestic retailers of consumption goods and domestic retailers of investment goods, domestic export-good wholesalers, foreign retailers, and employment agencies. It should be noted that I conventionally call the rest of the world a "foreign" economy throughout this paper.

3.1.1 Domestic-good producing firms

The domestic final-good firm combines the differentiated goods, $Y_{i,t}$, produced by monopolistically competitive intermediate-good firms indexed by $i \in [0, 1]$, using the following bundler technology:

$$Y_t = \left[ \int_0^1 Y_{i,t} \frac{1}{\lambda_i^d} di \right]^{\lambda_i^d},$$

where an i.i.d.-normal shock, $\varepsilon \lambda_i^d$, is assumed for the price markup, $\lambda_i^d$, which follows the exogenous stochastic process: $\dot{\lambda}_i^d = \rho \lambda_i^d \lambda_{i-1} + \varepsilon \lambda_i^d$. Note that variables marked with a hat denote percent deviations from their steady states, throughout the following. The competitive final-good firm takes its output price, $P_{d,t}$, and its input price, $P_{i,t}^d$, as given and solves:

$$\max_{Y_{i,t}} \sum_{i} P_{d,i,t}^{d} \left[ \int_0^1 Y_{i,t} \frac{1}{\lambda_i^d} di \right]^{\lambda_i^d} - \int_0^1 P_{i,t}^d Y_{i,t} di.$$

Then, the demand for the intermediate goods is expressed as:

$$Y_{i,t} = \left( \frac{P_{d,t}^{d}}{P_{i,t}^d} \right)^{\frac{1}{\lambda_i^d}} Y_t.$$

Putting this demand to the bundler technology of the final-good firm gives a pricing rule:

$$P_{d,t}^{d} = \left[ \int_0^1 P_{i,t}^d \frac{1}{\lambda_i^d} di \right]^{1-\lambda_i^d}.$$
The production function of each intermediate-good firm is given by:

\[ Y_{i,t} = z_i^{1-\alpha} \epsilon_t \tilde{K}_{i,t-1}^\alpha L_{i,t}^{1-\alpha} (K_{G,t-1})^{\alpha g} - z_i^+ \Theta, \]

where \( \alpha > 0, \alpha_g > 0 \) and \( \alpha + \alpha_g < 1 \). \( \tilde{K}_{i,t-1} \) is the effective private capital stock at time \( t \) given by \( \tilde{K}_{i,t} = u_{i,t} K_{i,t-1} \). \( u_{i,t} \) is the degree of capital utilization. \( L_{i,t} \) is the effective labor input bundled by the employment agency (discussed below), and \( \Theta \) represents a fixed cost. \( \epsilon_t \) is a stationary neutral technology shock assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: \( \epsilon_t = \rho \epsilon_{t-1} + \varepsilon_{\epsilon,t} \). The parameter \( \alpha \) measures the cost share of private capital input, and \( \alpha_g \) measures the productivity of public capital, \( K_{G,t} \). The intermediate-good firm faces constant returns to scale in the two private factors and increasing returns to scale in all three factors of production due to the positive externality of public capital.

The economy has two sources of growth: a neutral (or equivalently, labor-augmenting) technological progress, represented by a scaling variable, \( z_t \); and an investment-specific technological (IST) progress in the private sector, represented by a scaling variable, \( \Psi_t \). Note that a fixed cost is included to ensure that profits of the intermediate-good firms are zero in the steady state. Therefore the fixed cost needs to grow at the same rate as output. Furthermore, we assume that public capital grows at the same rate as output on a balanced growth path. Accordingly, the growth of output can be represented by a scaling variable:

\[ z_t^+ = \Psi_t^{\frac{1-\alpha}{1-\alpha_g}} \frac{z_t^{1-\alpha}}{z_t^{1-\alpha_g}}. \]

With the exception of labor, \( L_t \), private capital, \( K_t \), and investment, \( I_t \), all the other real variables at the aggregate level grow at the same rate as output, \( \mu_{z^+,t} = \frac{z_t^+}{z_t^{1-\alpha}} \). \( \mu_{z^+,t} \) is assumed to follow the exogenous stochastic process: \( \mu_{z^+,t} = \rho \mu_{z^+,t-1} + \varepsilon_{\mu_{z^+,t}} \), where \( \varepsilon_{\mu_{z^+,t}} \) represents an i.i.d.-normal shock. These real variables are stationarized by \( z_t^+ \) in the following way: \( y_t = \frac{Y_t}{z_t^+} \). Note that the scaled variables stationarized by \( z_t^+ \) are expressed using the corresponding lower case letters throughout the paper. On the other hand, \( K_t \) and \( I_t \) grow at the same rate, \( \mu_{z^+,t} \mu_{\Psi,t} \), where \( \mu_{\Psi,t} = \frac{\Psi_t}{\Psi_t^{1-\alpha}} \). \( \mu_{\Psi,t} \) is assumed to follow the exogenous stochastic process: \( \mu_{\Psi,t} = \rho \mu_{\Psi,t-1} + \varepsilon_{\mu_{\Psi,t}} \), where \( \varepsilon_{\mu_{\Psi,t}} \) represents an i.i.d.-normal shock. \( K_t \) and \( I_t \) are scaled by \( z_t^+ \Psi_t \) and their scaled variables are expressed as \( k_t = \frac{K_t}{z_t^+ \Psi_t} \) and \( i_t = \frac{I_t}{z_t^+ \Psi_t} \), respectively.

The assumption of an increasing returns to scale with respect to public capital can be found in Baxter and King (1993), Glomm and Ravikumar (1997), Turnovsky (2004), and Leeper et al. (2010). The condition, \( \alpha + \alpha_g < 1 \), is necessary to ensure a stable balanced growth path (see Turnovsky (2004)).
production function of domestic goods at the aggregate level in scaled form can be expressed as:

\[ y_t = \epsilon_t \left( \frac{1}{\mu_{z+t}} \right)^{\alpha + \alpha_g} \left( \frac{1}{\mu_{\Psi,t}} \right)^{\alpha} \tilde{k}_{t-1}^{-\alpha} L_t^{1-\alpha} (k_{t-1}^G)^{\alpha_g} - \Theta, \]  

(1)

where \( \tilde{k}_{t-1} = u_t k_{t-1} \). Taking the real rental cost of capital, \( r_t^k \), and aggregate real wage \( w_t \) as given, cost minimization subject to the production technology yields real marginal cost, \( mc_t^d \), and labor demand:

\[ mc_t^d = \frac{\bar{w}_t L_t}{\epsilon_t (1 - \alpha)} (k_{t-1}^G)^{\alpha_g}, \]  

(2)

\[ \tilde{r}_t^k = \frac{\alpha}{1 - \alpha} \tilde{k}_{t-1} \mu_{z+t} \mu_{\Psi,t}, \]  

(3)

where \( \tilde{r}_t^k \equiv \Psi_t r_t^k \) and \( \bar{w}_t \equiv \frac{w_t}{z_t} \) denote stationarized real rental cost and real wage, respectively.

With regard to the price setting problem, the staggered price contracts à la Calvo (1983) is assumed. A fraction \( 1 - \xi_d \) of intermediate-good firms can re-optimize their prices, unless otherwise they follow the price indexation scheme:

\[ p_{i,t}^d = \left( \frac{p_{i,t-1}^d}{p_{i,t-2}^d} \right)^{\kappa_d} P_{i,t-1}^d, \]  

where \( \kappa_d \) measures the degree of indexation. An intermediate-good firm, which is allowed to re-optimize, knows the probability \( \xi_d \) that the price it chooses in this period will still be in effect \( s \) periods in the future. Taking \( p_{i,t}^d \) and \( Y_t \) as given, the optimal price \( p_{i,t}^{d,\text{opt}} \) is chosen to maximize the discounted sum of expected nominal profits, \( D_t^d \):

\[ p_{i,t}^{d,\text{opt}} \equiv \arg \max_{p_{i,t}^d} E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \left[ D_{i,t+s}^d \right], \]  

where

\[ D_{i,t+s}^d = \left( p_{i,t+s}^d - p_{i,t+s}^d mc_{i,t+s}^d \right) \left( \frac{p_{i,t+s}^d}{p_{i,t}^d} \right)^{\kappa_d} \frac{Y_{i,t+s}}{z_{i,t+s}^d P_{i,t+s}^d mc_{i,t+s}^d \Theta}, \]  

and \( p_{i,t+s}^d = \left( \frac{p_{i,t+s}^d}{p_{i,t-1}^d} \right)^{\kappa_d} p_{i,t}^{d,\text{opt}} \). The aggregate price index for the domestic goods then evolves according to a law of motion:

\[ P_t^d = \left( 1 - \xi_d \right) \left( p_{i,t}^{d,\text{opt}} \right)^{1-\lambda_t^d} + \xi_p \left( \left( \frac{p_{i,t-1}^d}{p_{i,t-2}^d} \right)^{\kappa_d} \right)^{1-\lambda_t^d} \left( p_{i,t}^{d,\text{opt}} \right)^{\lambda_t^d} \left( \frac{p_{i,t-1}^d}{p_{i,t-2}^d} \right)^{\kappa_d}. \]  

(4)
Real profit in scaled form, \( d_t^d \equiv \frac{D_t^d}{p_t z_t^d} \), can be expressed as:

\[
d_t^d = y_t - mc_t^d (y_t + \Theta).
\] (5)

As noted earlier, zero-profit condition \( d_t^d = 0 \) is assumed in the steady state.

### 3.1.2 Importing firms and import-good wholesalers

The two types of importing firms are assumed to have access to differentiating technologies, such as "brand naming." The consumption-good importing and investment-good importing firms both buy homogenous foreign goods at price \( P_t^s \) and convert them into a differentiated consumption good, \( C_{i,i}^m \), and a differentiated investment good, \( I_{i,i}^m \), respectively. The differentiated goods are sold to the competitive domestic wholesalers of imported consumption goods at price \( P_{i,t}^{m,c} \), and to the competitive domestic wholesalers of imported investment goods at price \( P_{i,t}^{m,i} \).

The wholesaler of imported consumption goods produces \( C_t^m \) that is used in producing final consumption goods (discussed below) taking its output price, \( P_{i,t}^{m,c} \), and its input prices, \( P_{i,t}^{m,c} \), as given. The production function of the wholesaler of imported consumption goods is given by:

\[
C_t^m = \left[ \int_0^1 C_i x_i^{1/\lambda_i} \, dx \right]^{1/\lambda_i^m,c},
\]

where an i.i.d.-normal shock \( \varepsilon_{\lambda_i^m,c,t} \) is assumed for the price markup, \( \lambda_i^m,c \), which follows the exogenous stochastic process: \( \lambda_i^m,c = \rho_{\lambda_i^m,c}\lambda_i^m,c_{t-1} + \varepsilon_{\lambda_i^m,c,t} \). The demand for the differentiated imported consumption goods is then expressed as:

\[
C_{i,t}^m = \left( \frac{P_{i,t}^{m,c}}{P_{i,t}^{m,c}} \right)^{\lambda_i^m,c_{t-1}} C_t^m.
\]

As in the case of domestic intermediate-good firms, the consumption-good importing firms are subject to price setting frictions à la Calvo (1983). A fraction \( 1 - \xi_{m,c} \) of consumption-good importing firms can re-optimize their prices, unless otherwise they follow the price indexation scheme:

\[
P_{i,t}^{m,c} = \left( \frac{P_{i,t-1}^{m,c}}{P_{i,t-2}^{m,c}} \right)^{\kappa_{m,c}} P_{i,t-1}^{m,c},
\]

where \( \kappa_{m,c} \) measures the degree of indexation. The consumption-good importing firm knows the probability \( \xi_{m,c}^s \) that the price it chooses in this period will still be in effect \( s \) periods in the future. Taking \( P_{i,t}^{m,c} \) and \( C_t^m \) as given, the optimal price \( P_{i,t}^{m,c,\text{opt}} \) is chosen to maximize the discounted sum of expected nominal profits, \( D_t^{m,c} \):

13
\[ P_{m,c}^{\text{opt},i,t} = \arg \max_{P_{m,c}^{i,t}} E_t \sum_{s=0}^{\infty} (\beta \xi_{m,c})^s [D_{t+s}^{m,c}] , \]

where

\[ D_{t+s}^{m,c} = \left( P_{m,c}^{i,t} - P_{m,c}^{i,t+s} m_{c,t+s} \right) \left( \frac{P_{m,c}^{i,t+s}}{P_{m,c}^{i,t+s}} \right)^{m_{c,t+s}} C_{t+s}^m , \]

\[ mc_{t} = \frac{S_t P_t^s}{P_{m,c}^t} , \text{ and } P_{m,c}^{i,t+s} = \left( \frac{P_{m,c}^{i,t+s}}{P_{m,c}^{i,t+s-1}} \right)^{m_{c,t}} P_{m,c}^{i,t} . \]  

\( S_t \) denotes the nominal exchange rate. Real profit in scaled form, \( \delta_{m,c}^{i,t} \), can be expressed as:

\[ \delta_{m,c}^{i,t} = \left( \frac{P_{m,c}^{i,t}}{P_{d}^t} - \frac{S_t P_t^s}{P_{d}^t} \right) \xi_{m,c} . \]  

(6)

Similarly to domestic intermediate-good producing firms, zero-profit condition \( \delta_{m,c}^{i,t} = 0 \) is assumed in the steady state. Aggregate price law of motion for the imported consumption goods is then expressed as:

\[ P_{m,c}^{i,t} = \left[ \left( 1 - \xi_{m,c} \right) \left( \frac{P_{m,c}^{i,t,\text{opt}}}{P_{m,c}^{i,t-1}} \right)^{1-\lambda_{m,c}^{i,t-1}} + \xi_{m,c} \left( \frac{P_{m,c}^{i,t-1}}{P_{m,c}^{i,t}} \right)^{\kappa_{m,c}} P_{m,c}^{i,t-1} \right]^{1-\lambda_{m,c}^{i,t}} . \]  

(7)

Equivalently, aggregate price law of motion for the imported investment goods is expressed as:

\[ P_{m,i}^{i,t} = \left[ \left( 1 - \xi_{m,i} \right) \left( \frac{P_{m,i}^{i,t,\text{opt}}}{P_{m,i}^{i,t-1}} \right)^{1-\lambda_{m,i}^{i,t-1}} + \xi_{m,i} \left( \frac{P_{m,i}^{i,t-1}}{P_{m,i}^{i,t}} \right)^{\kappa_{m,i}} P_{m,i}^{i,t-1} \right]^{1-\lambda_{m,i}^{i,t}} . \]  

(8)

where the optimal price \( P_{m,i}^{i,t,\text{opt}} \) is obtained in the same manner as \( P_{m,c}^{i,t,\text{opt}} \). \( \xi_{m,i} \) is the Calvo parameter, \( \kappa_{m,i} \) is the indexation parameter, and an i.i.d.-normal shock \( \varepsilon_{\lambda_{m,i},t} \) is assumed for the price markup, \( \lambda_{m,i}^{i,t} \), which follows the exogenous stochastic process: \( \dot{\lambda}_{m,i}^{i,t} = \rho_{\lambda_{m,i}} \lambda_{m,i}^{i,t-1} + \varepsilon_{\lambda_{m,i},t} . \)

Real profit in scaled form, \( \delta_{m,i}^{i,t} \), can be expressed as:

\[ \delta_{m,i}^{i,t} = \left( \frac{P_{m,i}^{i,t}}{P_{d}^t} - \frac{S_t P_t^s}{P_{d}^t} \right) \xi_{m,i} . \]  

(9)

and zero-profit condition \( \delta_{m,i}^{i,t} = 0 \) is assumed in the steady state.
3.1.3 Exporting firms and export-good wholesalers

The exporting firms buy the final domestic good at price $P^d_t$ and differentiate it into a differentiated good $X_{i,t}$ through a brand naming technology. The differentiated export goods are sold to competitive domestic export-good wholesalers at price $P^x_{i,t}$. The export-good wholesaler produces export goods, $X_t$, taking its output price, $P^x_t$, and its input price, $P^x_{i,t}$, as given. The production function of the wholesaler is given by:

$$X_t = \left[ \int_0^1 X_{i,t} \frac{1}{X} \, di \right]^{\lambda^x_t},$$

where an i.i.d.-normal shock $\varepsilon_{\lambda^x_t}$ is assumed for the price markup, $\lambda^x_t$, which follows the exogenous stochastic process: $\hat{\lambda}^x_t = \rho^x \hat{\lambda}^x_{t-1} + \varepsilon_{\lambda^x_t}$. Assuming the Calvo price setting frictions, aggregate price index for the export goods evolves according to a law of motion:

$$P^x_t = \left[ (1 - \xi_x) \left( P^x_{i,t} \right)^{1-\lambda^x_t} + \xi_x \left( \left( \frac{P^x_{t-1}}{P^x_{t-2}} \right)^{\kappa_x} P^x_{i,t-1} \right)^{1-\lambda^x_t} \right]^{1-\lambda^x_t}, \quad (10)$$

where $\xi_x$ and $\kappa_x$ denote the Calvo parameter and the indexation parameter, respectively. The optimal price $P^x_{i,t}^{opt}$ is obtained in the same manner as the optimal prices set by importing firms:

$$P^x_{i,t}^{opt} = \arg \max_{P^x_{i,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \left( \beta^{\xi_x} \right)^s \left[ D^x_{t+s} \right],$$

where

$$D^x_{t+s} = \left( P^x_{i,t+s} - P^x_{i,t+s} mc^x_t \right) \left( \frac{P^x_{t+s}}{P^x_{i,t+s}} \right)^{\lambda^x_{t+s}} X_{t+s},$$

and $mc^x_t = \frac{P^x_{i,t}}{P^x_t}$. Real profit in scaled form, $d^x_t = \frac{D^x_t}{P^x_t}$, can be expressed as:

$$d^x_t = \left( \frac{1}{mc^x_t} - 1 \right) \left( \frac{P^x_t}{P^x_{i,t}} \right)^{-\eta_f} \tilde{z}^*_t \tilde{z}^*_t \tilde{z}^*_t \tilde{z}^*_t,$$

where $\tilde{z}^*_t \equiv \frac{\tilde{z}^*_t}{\tilde{z}^*_t}$, and $\tilde{z}^*_t$ represents a scaling variable for the foreign economy. The asymmetry in the technological progress in the domestic and foreign economies, $\tilde{z}^*_t$, is assumed to follow the exogenous stochastic process: $\hat{\tilde{z}}^*_t = \mu_{\tilde{z}^*} \hat{\tilde{z}}^*_{t-1} + \varepsilon_{\tilde{z}^*t}$. It is also assumed that $\hat{\tilde{z}}^*_t = 1$, $\mu_{\tilde{z}^*} = \mu_{\tilde{z}^*}$, in the steady state, where $\mu_{\tilde{z}^*} \equiv \frac{\tilde{z}^*_t}{\tilde{z}^*_t}$. Similarly to other types monopolistically competitive firms, zero-profit condition $d^x = 0$ is assumed in the steady state.
3.1.4 Domestic and foreign retailers

Domestic final consumption and investment goods are produced by the competitive retailer combining domestic final goods and import goods. The consumption-good retailer produces final goods, $C_t$, taking its output price, $P^c_t$, and its input prices, $P^d_t$ and $P^{m,c}_t$, as given, using the following technology:

$$C_t = \left(1 - \omega_c\right)^{\frac{1}{\eta_c}} \left( C^d_t \right)^{\frac{\eta_c - 1}{\eta_c}} \left(1 + \omega_c \left(C^m_t\right)^{\frac{\eta_c - 1}{\eta_c}}\right)^{\frac{\eta_c}{\eta_c - 1}},$$

where $C^d_t$ is domestically produced consumption good and $\omega_c \in [0, 0.5]$ measures the home bias. Profit maximization subject to the budget constraint, $P^d_tC_t + P^{m,c}_tC^m_t = P^c_tC_t$, leads to the following input demand functions in scaled form:

$$c^d_t = (1 - \omega_c) \left( \frac{P^c_t}{P^d_t} \right)^{\frac{\eta_c}{\eta_c - 1}} c_t,$$

$$c^{m,c}_t = \omega_c \left( \frac{P^c_t}{P^{m,c}_t} \right)^{\frac{\eta_c}{\eta_c - 1}} c_t,$$

where the Consumer Price Index (CPI) is given by:

$$P^c_t = \left(1 - \omega_c\right)^{1 - \frac{1}{\eta_c}} + \omega_c \left(P^c_t\right)^{1 - \frac{1}{\eta_c}}.$$

Following Christiano et al. (2011), final investment goods, $\bar{I}_t$, are defined as the sum of investment goods, $I_t$, used in the accumulation of physical capital and those used in capital maintenance:

$$\bar{I}_t = I_t + a(u_t) K_{t-1},$$

where $a(u_t)$ is the cost associated with variations in the degree of capital utilization. The cost function is assumed to satisfy $a(u) = 0$ and $\sigma_a \equiv \frac{a(a)}{a(u)}$ is defined for the steady-state rate of capital utilization, $u = 1$. The investment-good retailer produces final goods, $\bar{I}_t$, taking its output price, $P^i_t$, and its input prices, $P^d_t$ and $P^{m,i}_t$, as given, using the following technology:

$$\bar{I}_t = \Psi_t \left(1 - \omega_i\right)^{\frac{1}{\eta_i}} \left( I^d_t \right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i \left( P^m_t \right)^{\frac{\eta_i - 1}{\eta_i}} \left( P^{i,m}_t \right)^{\frac{\eta_i}{\eta_i - 1}},$$

where $I^d_t$ is domestically produced investment good and $\omega_i \in [0, 0.5]$ measures the home bias. Profit maximization subject to the budget constraint, $P^d_tI^d_t + P^{m,i}_tI^{m,i}_t = P^i_t\bar{I}_t$, leads to the
following input demand functions in scaled form:

\[ i^d_t = (1 - \omega_i) \left( \frac{\Psi_t P_t^d}{P_t^d} \right)^{\eta_i} \left( i_t + a(u_t) \frac{k_{t-1}}{\mu_{t,t} \mu_{z,t}} \right), \]  

\[ i^m_t = \omega_i \left( \frac{\Psi_t P_t^m}{P_t^m} \right)^{\eta_i} \left( i_t + a(u_t) \frac{k_{t-1}}{\mu_{t,t} \mu_{z,t}} \right), \]  

where and investment good price index is given by:

\[ P_t^i = \left( (1 - \omega_i) \left( \frac{P_t^d}{\Psi_t} \right)^{1-\eta_i} + \omega_i \left( \frac{P_t^m}{\Psi_t} \right)^{1-\eta_i} \right)^{\frac{1}{1-\eta_i}}. \]  

The competitive foreign retailer buys the export goods, \( X_t \), from domestic export-good wholesalers \( x \in [0,1] \), at price \( P_t^x \) and and produces final consumption and investment goods, \( C_t^* \) and \( I_t^* \), taking its output price, \( P_t^* \), as given, using the following technology:

\[ C_t^* = \left[ \int_0^1 (C^x_{x,t})^{\frac{\eta_f-1}{\eta_f}} \, dx \right]^{\frac{\eta_f}{\eta_f-1}}, \]

\[ I_t^* = \left[ \int_0^1 (I^x_{x,t})^{\frac{\eta_f-1}{\eta_f}} \, dx \right]^{\frac{\eta_f}{\eta_f-1}}, \]

where \( C_t^* = \int_0^1 C^x_{x,t} \, dx, I_t^* = \int_0^1 I^x_{x,t} \, dx, C_t^* + I_t^* = X_t \), and \( C_t^* + I_t^* = Y_t^* \). Foreign demand for the domestic consumption and investment goods in scaled form are given by:

\[ c_t^* = \left[ \frac{P_t^c}{P_t^*} \right]^{-\eta_f} c_t^o, \]

\[ i_t^* = \left[ \frac{P_t^i}{P_t^*} \right]^{-\eta_f} i_t^o. \]

3.1.5 Relative prices

Following Adolfson et al. (2007) and Christiano et al. (2011), we define the relative prices in the model as follows so that we can make use of these expressions in the computation:

\[ \gamma_{t,mc,d}^{m,c} \equiv \frac{P_t^{m,c}}{P_t^d}, \]

\[ \gamma_{t,mi,d}^{m,i} \equiv \frac{P_t^{m,i}}{P_t^d}, \]

\[ \gamma_{t,c,d}^c \equiv \frac{P_t^c}{P_t^d}. \]
\[
\gamma_{i,t}^{d} = \frac{P_{i}^{d}}{P_{i}^{a}}, \quad (23)
\]
\[
\bar{p}_{t}^{d} = \frac{\Psi_{i} P_{t}^{d}}{P_{t}^{a}}, \quad (24)
\]
\[
\gamma_{t}^{x} = \frac{P_{t}^{x}}{P_{t}^{a}}, \quad (25)
\]
\[
\gamma_{t}^{f} = \frac{P_{t}^{d}}{S_{t} P_{t}^{a}} = mc_{t}^{*} \gamma_{t}^{x}. \quad (26)
\]

3.2 Households

3.2.1 Preferences and constraints

There is a continuum of households indexed by \( j \in [0, 1] \). Each member of households maximizes its lifetime utility by choosing effective consumption, \( \tilde{C}_{j,t} \) (explained below), investment, \( I_{j,t} \), domestic government bonds denominated in domestic currency, \( B_{j,t}^{d} \), international bonds denominated in foreign currency, \( B_{j,t}^{s} \), capital stock, \( K_{j,t} \), and intensity of the capital stock utilization, \( u_{j,t} \), given the following lifetime utility function that is additively separable between consumption and labor:

\[
E_{t} \sum_{t=0}^{\infty} \beta^{t} \left( \zeta_{t}^{c} \ln \left( \tilde{C}_{j,t} - h \tilde{C}_{j,t-1} \right) - \zeta_{t}^{l} A_{L} \frac{L_{j,t}^{1+\sigma_{l}}}{1 + \sigma_{l}} \right),
\]

where, \( \beta \) is the discount factor, \( \sigma_{l} \) is the inverse of the elasticity of work effort with respect to real wages, \( L_{j,t} \) represents the labor supply. \( h \) measures the degree of habit formation in consumption, and \( A_{L} > 0 \) is a scale parameter. Note that the utility function is assumed to be logarithmic in consumption in accordance with the balanced growth property of the model. Two serially correlated shocks, a preference shock, \( \zeta_{t}^{c} \), and a labor supply shock, \( \zeta_{t}^{l} \), are considered and are assumed to follow a first-order autoregressive process with an i.i.d.-normal error term:

\[
\zeta_{t}^{c} = \rho_{\zeta} \zeta_{t-1}^{c} + \varepsilon_{\zeta,t}^{c}, \quad \zeta_{t}^{l} = \rho_{\zeta} \zeta_{t-1}^{l} + \varepsilon_{\zeta,t}^{l}.
\]

I define the effective consumption with the following form:

\[
\tilde{C}_{t} = C_{t} + v G_{t}^{C},
\]

where \( G_{t}^{C} \) denotes government consumption. Note that a negative [positive] \( v \) indicates that an increase in \( G_{t}^{C} \) increases [decreases] the marginal utility of private consumption \( C_{t} \), implying complementarity [substitutability] between \( C_{t} \) and \( G_{t}^{C} \).

\(^{19}\)A function \( V(G_{t}^{C}) \) which satisfies \( V'(\cdot) > 0 \) can be added to the utility function to ensure positive marginal utility with respect to government consumption even when \( v \) takes a negative value. Nonetheless, as long as \( G_{t}^{C} \) is exogenously given to the households, the presence of such a term does not affect the analysis here and therefore
constraint:
\[
(1 + r^c) P^c_t C_{j,t} + P^d_t [I_{j,t} + a (u_{j,t})] K_{j,t-1} + B_{j,t} + S_t B^*_{j,t} \\
= (1 - r^l) W_{j,t} L_{j,t} + (1 - r^k) R^k_t u_{j,t} K_{j,t-1} + r^k P^k_t a (u_{j,t}) K_{j,t-1} + (1 - r^k) D_{j,t} \\
+ R_t B_{j,t-1} + \Phi \left( A_{t-1}, \tilde{\phi}_{t-1} \right) R^*_{t-1} S_t B^*_{j,t-1},
\]

(27)

where \( D_{j,t} \equiv D^d_{j,t} + D^{m,c}_{j,t} + D^{m,i}_{j,t} + D^k_{j,t} \). \( D^d_{j,t}, D^{m,c}_{j,t}, D^{m,i}_{j,t}, \) and \( D^k_{j,t} \) denote dividends distributed by domestic, consumption-good importing, investment-good importing, and exporting firms to the household, respectively. \( R_{t-1} \) is riskless return on domestic government bonds, \( W_{j,t} \equiv P^d_t w_{j,t} \) is nominal wage income, \( R^k_t \equiv P^d_t r^k_t \) is nominal rental rate of capital, and \( R^*_{t-1} \) is riskless return on international bonds. \( r^c, r^l, \) and \( r^k \) represent tax rates on consumption, labor income, and capital income, respectively. The government bond is assumed to be traded only with domestic household. In other words, the international bond is the only asset that is traded with the foreign economy. Notice that the capital stock, domestic government bonds, and international bonds of the current period are denoted here as \( K_{j,t-1}, B_{j,t-1}, \) and \( B^*_{j,t-1} \), meaning that their decisions are made at time \( t - 1 \). \( \Phi(\cdot) \) represents a risk premium on international bond holdings, which is assumed to have the following functional form:

\[
\Phi \left( a_t, \tilde{\phi}_t \right) = \exp \left( -\tilde{\phi}_a (a_t - a) + \tilde{\phi}_t \right),
\]

where \( \tilde{\phi}_a \) is a positive parameter, and \( a_t = A_t \). \( A_t \) stands for a real aggregate net foreign asset position defined as \( A_t \equiv S_t B^*_t / R^*_t \). A risk premium shock, \( \tilde{\phi}_t \) is considered and is assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: \( \tilde{\phi}_t = \rho_{\tilde{\phi}} \tilde{\phi}_{t-1} + \epsilon_{\tilde{\phi},t} \). It is assumed that \( a = 0, \Phi(0,0) = 1 \) in the steady state, and \( \hat{a}_t \) is defined as \( \hat{a}_t \equiv a_t - a = a_t \). The physical capital accumulation law for the household is expressed as:

\[
K_{j,t} = (1 - \delta) K_{j,t-1} + \zeta^i_t \left[ 1 - S \left( \frac{I_{j,t}}{I_{j,t-1}} \right) \right] I_{j,t},
\]

which can be expressed in scaled form:

\[
k_t = (1 - \delta) \frac{1}{\mu_{z+,t} \mu_{s+,t}} k_{t-1} + \zeta^i_t \left[ 1 - S \left( \frac{\mu_{z+,t} \mu_{s+,t} i_t}{v_{t-1}} \right) \right] i_t,
\]

(28)

where \( \delta \) is the depreciation rate, and \( S(\cdot) \) represents the adjustment cost function in investment. In the steady state, the cost function is assumed to satisfy \( S(1) = S'(1) = 0 \) and \( S''(1) \equiv \frac{1}{\chi} > 0 \). \( \zeta^i_t \) is a shock to investment cost function and is assumed to follow a first-order autoregressive is omitted for simplicity, as in Christiano and Eichenbaum (1992) and Karras (1994).
3.2.2 Choice of allocations

Let $\Lambda_t$ and $\Lambda_tQ_t$ denote the Lagrange multipliers and define $\psi_t \equiv P_{t}^d\Lambda_t$, $\psi_{z,t} \equiv \psi_t z_t$. The first-order conditions with respect to $c_{j,t}$, $b_{j,t}$, $b_{j,t}^*$, $i_{j,t}$, $k_{j,t}$, and $u_{j,t}$ are then expressed as follows:

$$
\psi_{z^+,j}P_{t}^c \frac{P_{t}^c}{P_{t}^d} (1 + \tau') = \frac{\zeta_t}{\tilde{c}_{j,t} - h(\frac{1}{\mu_{z,t}}) \tilde{c}_{j,t-1}} - \beta h E_t \frac{\zeta_{t+1}}{\mu_{z+,t+1} \tilde{c}_{j,t+1} - h \tilde{c}_{j,t}},
$$

(29)

$$
\beta R_t E_t \left[ \psi_{z^+,t+1} \frac{P_{t}^d}{P_{t+1}^d} \right] = \psi_{z^+,t},
$$

(30)

$$
\beta \Phi \left( a_t, \tilde{\phi}_t \right) P_t^c E_t \left[ S_{t+1} \psi_{z^+,t+1} \frac{P_{t+1}^d}{P_t^d} \right] = \psi_{z^+,t},
$$

(31)

$$
\psi_{z^+,j}P_t^c = \psi_{z^+,j}q_t \zeta_t \left[ 1 - S \left( \frac{\mu_{z+,t}^* \mu_{t}^{i,j,t}}{i_{j,t-1}} \right) - S' \left( \frac{\mu_{z+,t}^* \mu_{t}^{i,j,t}}{i_{j,t-1}} \right) \frac{\mu_{z+,t}^* \mu_{t+1}^{i,j,t+1}}{i_{j,t-1}} \right]
$$

$$
+ \beta E_t \left[ \psi_{z^+,t+1}q_{t+1} \zeta_{t+1} S' \left( \frac{\mu_{z+,t}^* \mu_{t+1}^{i,j,t+1}}{i_{j,t-1}} \right) \frac{\mu_{z+,t}^* \mu_{t+1}^{i,j,t+1}}{i_{j,t-1}} \right],
$$

(32)

$$
q_t = \beta E_t \left[ \psi_{z^+,t+1}q_{t+1} \zeta_{t+1} \left[ (1 - \delta) q_{t+1} + (1 - \tau^k) r_{t+1}^k u_{j,t+1} \right] - (1 - \tau^k) p_{t+1}^i a(u_{j,t+1}) \right] + \varepsilon_t^q,
$$

(33)

$$
r_t^k = p_t^i a'(u_{j,t}).
$$

(34)

Here, $q_t \equiv \frac{\Psi_t Q_t}{P_t^d}$ represents the shadow price of additional unit of capital. An i.i.d.-normal error term, $\varepsilon_t^q$, is introduced to capture an equity premium shock. It can be shown that $\frac{1}{\beta} = R = \frac{1}{\mu_{z,t}^*} \left[ 1 - \delta + (1 - \tau^k) \frac{\varepsilon_t^q}{p_t^i} \right]$ and $q = p_t^i$ in the steady state.

3.2.3 Wage setting

An independent and perfectly competitive employment agency bundles differentiated labor $L_{j,t}$ into a single type of effective labor input $L_t$ using the following technology:

$$
L_t = \left[ \int_0^1 L_{j,t} \frac{1}{\lambda_w} dj \right]^{\lambda_w},
$$

where $\lambda_w$ is the wage markup. The employment agency solves:

$$
\max_{L_{j,t}} W_t \left[ \int_0^1 L_{j,t} \frac{1}{\lambda_w} dj \right]^{\lambda_w} - \int_0^1 W_{j,t} L_{j,t} dj,
$$

(20)
where $W_t$ is aggregate nominal wage index. The labor demand schedule for each differentiated labor service is then expressed as:

$$L_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{1-\lambda_w} L_t.$$ 

Putting the labor demand to the bundler technology of the employment agency gives:

$$W_t = \left[ \int_0^1 W_{j,t} - \frac{1}{\lambda_w - 1} d_j \right]^{1-\lambda_w}.$$

With probability $1 - \xi_w$, each household $j$ is assumed to be allowed to reset its wage optimally, unless otherwise it adjusts its wage partially according to the following indexation scheme:

$$W_{j,t} = \left( \frac{P_{t-1}^c}{P_{t-2}^c} \right)^{\kappa_w} W_{j,t-1},$$

where $\kappa_w$ measures the degree of indexation. The household $j$, which is allowed to optimally reset its wage, is assumed to maximize its lifetime utility taking aggregate nominal wage $W_t$ and effective labor $L_t$ as given. Since the household knows the probability $\xi_w$ that the wage it chooses in this period will still be in effect $s$ periods in the future, the optimal wage $W_{j,t}^{\text{opt}}$ is given by:

$$W_{j,t}^{\text{opt}} = \arg \max_{W_{j,t}} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[ \zeta_{t+s}^c \ln \left( \tilde{C}_{j,t+s} - h \tilde{C}_{j,t+s-1} \right) - A_L \frac{\zeta_{t+s}}{1 + \sigma_L} \left( \frac{W_{j,t+s}}{W_{t+s}} \right)^{1-\lambda_w} L_{t+s} \right]^{1+\sigma_L},$$

subject to the households’ budget constraint. Aggregate nominal wage law of motion is then expressed as:

$$W_t = \left[ (1 - \xi_w) (W_{j,t}^{\text{opt}})^{1-\lambda_w} + \xi_w \left( \frac{P_{t-1}^c}{P_{t-2}^c} \right)^{\kappa_w} W_{j,t-1} \right]^{1-\lambda_w}. \quad (35)$$

### 3.3 Fiscal and Monetary Authorities

#### 3.3.1 Fiscal policy

The flow budget constraint for the fiscal authority is expressed as follows:

$$P_t^d C_t^C + P_t^d G_t^I + R_{t-1} B_{t-1} = \tau^c P_t^c C_t + \tau^l W_t L_t + \tau^k R_t^k u_t K_{t-1} - \tau^k P_t^d a (u_t) K_{t-1} + \tau^k D_t + B_t,$$
where $G_t^I$ denotes government investment. The budget constraint can be rewritten in scaled form as:

$$g_c^t + g_i^t + \frac{R_t b_t}{\pi_t^d \mu_{z^t\psi^t}} = \tau^c \gamma_{c^d} c_t + \tau^i \bar{w}_t L_t + \frac{\tau^k k_t b_t}{\mu_{z^t\psi^t}} \left[ \tau^k u_t - p_t^i (u_t) \right] + \tau^d d_t + b_t,$$

where $b_t = \frac{B_t}{P_t}$. The stock of public capital, which is accumulated by government investment, evolves according to:

$$k^g_t = (1 - \delta_g) \frac{1}{\mu_{z^t\psi^t}} k^g_{t-1} + \zeta^g_{i^t} \left[ 1 - S^g \left( \frac{\mu_{z^t\psi^t} g_t}{g_{t-1}} \right) \right] g_t^i,$$

where $\delta_g$ is the depreciation rate, $S^g(\cdot)$ represents the adjustment cost function in government investment. In the steady state, the cost function is assumed to satisfy $S^g(1) = S^g(1) = 0$ and $S^{g'}(1) > 0$. $\zeta^g_{i^t}$ is a shock to government investment cost function and is assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\zeta^g_{i^t} = \rho_{z^g} \zeta^g_{i^t-1} + \varepsilon^g_{z^g,i^t}$, where $\zeta^g_{i^t} = \zeta^g_{i^t} - \zeta^g_{i}$ and $\zeta^g_{i} = 1$. The time paths of government consumption and government investment are described by the log-linear feedback rules below:

$$g_c^t = \rho_{g_c} g_{c^t-1} + (1 - \rho_{g_c}) \phi_{g_c} \hat{b}_{t-1} + \varepsilon_{g_c}^t,$$

$$g_i^t = \rho_{g_i} g_{i^t-1} + (1 - \rho_{g_i}) \phi_{g_i} \hat{b}_{t-1} + \varepsilon_{g_i}^t,$$

where i.i.d.-normal shocks $\varepsilon_{g_c}^t$ and $\varepsilon_{g_i}^t$ are assumed. If $\phi_{g_c} [\phi_{g_i}] < 0$, government consumption [government investment] follows a debt-stabilizing spending rule called "spending reversals" (see Corsetti et al. (2012)). Because all tax rates are assumed to be time-invariant, at least one of $\phi_{g_c}$ and $\phi_{g_i}$ needs to be negative in order to prevent government debt from exploding.

3.3.2 Monetary policy

The monetary authority sets nominal interest rates according to a simple feedback rule in log-linearized form:

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \phi_{r\pi^c} \hat{\pi}_{t-1}^c + (1 - \rho_r) \phi_{ry} \hat{y}_t + \varepsilon_t^R,$$

where $\pi_{t-1}^c = \frac{P_{t-1}^c}{P_{t-2}^c}$ represents the inflation rate measured by the CPI and an i.i.d.-normal shock $\varepsilon_t^R$ to the interest rate is assumed.
3.4 Market Clearing Conditions

3.4.1 The aggregate resource constraint

The aggregate production function of domestic good is given by:

\[ Y_t = \epsilon_t (u_t K_{t-1})^\alpha L_t^{1-\alpha} (K_{t-1}^G)^{\alpha_g} - z_t^t \Theta, \]

which can be rewritten in scaled form as:

\[ y_t = \epsilon_t (\mu_{z^t,t})^{-(\alpha+\alpha_g)} (\mu_{x^t,t})^{-\alpha} (u_t k_{t-1})^\alpha L_t^{1-\alpha} (k_{t-1}^G)^{\alpha_g} - \Theta. \] (41)

The aggregate demand equation for domestic good is given by:

\[ Y_t = C^d_t + I^d_t + G^C_t + G^I_t + C_x_t + I_x_t, \]

which implies that both government consumption and government investment are assumed to fall entirely on domestic good. Using the relation (12), (15), (18), and (19), the aggregate demand in stationary form becomes:

\[ y_t = (1 - \omega_t) \left( \gamma_{C^d_t}^\eta c_t + (1 - \omega_t) \left( p_t^i \right)^\eta \left[ i_t + a (u_t) \frac{k_{t-1}}{\mu_{x^t,t}} \right] \right) \]

\[ + \gamma_{I^d_t}^\eta i_t + \left( \gamma_{c^*_x}^\eta \right) \gamma_{I^d_t}^\eta y_t^t z_t^t. \] (42)

The equilibrium resource constraint is given by (41) and (42).

3.4.2 Evolution of net foreign assets

The domestic economy’s net foreign assets evolve according to:

\[ S_t P^x_t (C_t^x + I_t^x) - S_t P^x_t (C_t^m + I_t^m) = S_t B_t^* - \Phi \left( A_{t-1}, \phi_{t-1} \right) R_{t-1}^* S_t B_{t-1}^*, \]

which implies that the economy’s trade balance equals its net holding of international bonds at the aggregate level. The evolution of net foreign assets in scaled form is expressed as:

\[ \frac{(\gamma_{c^*_x}^\eta)}{mc_t^x} y_t^t z_t^t - \left( \gamma_{I^d_t}^\eta \right)^{-1} (c_t^m + i_t^m) = a_t - R_{t-1}^* \Phi \left( a_{t-1}, \phi_{t-1} \right) a_{t-1} \frac{S_t}{S_t^* \pi_{x^t,t}^{z^t,t}}. \] (43)
It is assumed that \( R^* = R \) and \( mc^* = \gamma^* = \gamma^f = 1 \) in the steady state. The real exchange rate, \( rex_t \), is defined as follows:

\[
rex_t = \frac{S_t P_t^*}{P_t} = \frac{S_t P_t^* P_t^d}{P_t^c} = \frac{1}{\gamma^f} e^c d_t. \tag{44}
\]

### 3.5 Foreign Economy

Following Adolfson et al. (2007), foreign inflation, output, and interest rate are assumed to be exogenously given. Letting \( X_t^* \equiv [\pi_t^{* \text{data}}, \Delta \ln Y_t^{* \text{data}}, \Delta R_t^{* \text{data}}, \gamma^f] \), the foreign economy is modeled as a structural VAR model:

\[
F_0 X_t^* = F(L) X_{t-1}^* + \varepsilon_t^*. \tag{45}
\]

Note that variables marked with a superscript, \( \text{data} \), represents observed data matched to the model variables. Prior to the Bayesian estimation of the model, the structural VAR model for the foreign economy is estimated assuming that \( F_0 \) in (45) has a following structure:

\[
F_0 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-\gamma_x^* & -\gamma_y^* & 1
\end{bmatrix}.
\]

### 4 Bayesian Estimation of the Model

#### 4.1 Data and Measurement Equations

In estimating the model parameters, I first log-linearize the model around the deterministic steady state and conduct Bayesian inference using the Markov Chain Monte Carlo (MCMC) method. The log-linearized version of the model and the steady-state computation are presented in Appendices A and B, respectively. The parameters are estimated on the Japanese data covering the period from 1980:Q1 to 1998:Q4.\(^{20}\)

As in Hirose (2008), the end of the estimation period is determined so that it does not include the zero interest rate period in Japan. The zero lower bound on interest rates requires us to deal with two difficult problems in a DSGE framework: non-linearity and indeterminacy (see Braun and Waki (2006)). Furthermore, non-linearity complicates Bayesian estimations. In evaluating the likelihood function, we need to resort to the particle filter instead of the Kalman filter when the state space representation of the DSGE model is not linear (see Fernández-Villaverde and Rubio-Ramírez (2005) and Fernández-Villaverde and Rubio-Ramírez (2007)).
interest rates.\footnote{The domestic series are obtained from the Cabinet Office and the Bank of Japan. The foreign output and prices series are calculated as a weighted average index of, respectively, the GDP and GDP deflator series for Japan’s major trading partners: the U.S., Germany, Korea, Taiwan, Hong Kong, and China. Data used in the calculation are derived from Japan External Trade Organization, International Financial Statistics of the IMF, and the Taiwan Statistical Bureau. The Fed fund rate, taken from the FRED database, is used as a proxy for the foreign interest rate.} Private consumption is defined as personal consumption expenditures on non-durables and services, while private investment is the sum of personal consumption expenditures on durables and gross private domestic investment. I take logs and first differences except for interest rates and inflation rates, respectively. Following Adolfson et al. (2007), the foreign variables are assumed to be given by the structural VAR model, the pre-estimated parameters of which are kept fixed throughout the Bayesian estimation of the model. The observed foreign variables are included in the estimation so that they help identify the asymmetry in the technological progress in the domestic and foreign economies. The model is estimated using the following variables:

\[
\begin{align*}
\ln \pi_t^{d,\text{data}}, \ln \pi_t^{c,\text{data}}, \ln \pi_t^{i,\text{data}}, \Delta \ln Y_t^{\text{data}}, \Delta \ln C_t^{\text{data}}, \Delta \ln I_t^{\text{data}}, \\
\Delta \ln C_t^{d,\text{data}}, \Delta \ln G_t^{d,\text{data}}, \Delta \ln M_t^{d,\text{data}}, \Delta \ln X_t^{d,\text{data}}, \Delta \ln L_t^{\text{data}}, \Delta \ln w_t^{\text{data}}, \\
\Delta R_t^{\text{data}}, \Delta \ln \text{rex}_t^{\text{data}}, \Delta \ln Y_t^{*,\text{data}}, \ln \pi_t^{*,\text{data}}, \Delta R_t^{*,\text{data}},
\end{align*}
\]

where $\Delta$ denotes the temporal difference operator. While the model features two stochastic trends in neutral and investment-specific productivity, the variables grow at substantially different rates. Therefore I remove the mean from each of the time series as in Christiano et al. (2011). Similarly to Adolfson et al. (2007) and Christiano et al. (2011), I allow for measurement errors, except for interest rates. The measurement equations that link the model to data are reported in Appendix A together with the log-linearized model, where $\varepsilon_{V_t}^{\text{me}}$ denote the measurement errors for the variables $V$. The standard deviations of the measurement errors are calibrated so that they correspond to $30\%$ of the variance of each data series.

### 4.2 Preliminary Settings

Several parameters are fixed a priori. The discount factor $\beta$ and the depreciation rate of private capital $\delta$ are set to the values used in Sugo and Ueda (2008). The degree of wage indexation $\kappa_w$ and the fixed cost parameter $\theta \equiv 1 + \Theta/y$ are set to 0.2 and 1.9, respectively, based on the estimates of Iwata (2011). The steady-state wage markup is set at $\lambda^w = 1.05$, following Adolfson et al. (2007). The depreciation rate of public capital $\delta_g$ is set to a value that satisfies the ratio $\frac{\delta_g}{\delta} = 1.8$, which is calculated based on data from the National Accounts of Japan. The steady-state debt-to-output ratio $\frac{b}{y}$ is set to 0.6, following Broda and Weinstein (2005). The disutility
of labor scaling parameter $A_L$ is set so that the steady-state hours worked becomes around $1/3$. The share of imports in final consumption and investment goods are calculated utilizing the weights being used in the Indices of Industrial Domestic Shipments and Imports and are set at $\omega_c = 0.35$, $\omega_l = 0.33$. It should be noted that these values imply the presence of home bias in consumption and investment. The steady-state neutral technological growth rate and investment-specific technological growth rate are set at $\mu_{\zeta} = 1.0049$ and $\mu_{\eta} = 1.0029$, utilizing the trend rates of output growth and investment growth during the sample period. The aggregate effective tax rates are set at $\tau^e = 0.076$, $\tau^l = 0.309$, $\tau^k = 0.446$, which are sample period averages of those series calculated following the method of Mendoza et al. (1994). The steady-state ratio between government consumption and private consumption is set at $s^g = 0.283$, so as to match the average of the series during the sample period. Similarly, the steady-state ratio between public capital and private consumption is set at $k^g = 2.027$, utilizing the public capital stock estimate for Japan reported by Kamps (2006).

### 4.3 Estimation Results

Tables 2.a, 2.b, and 2.c report prior distributions, posterior means, and 90%-credible intervals (or Bayesian confidence intervals) of the parameters. The draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for a Metropolis-Hastings algorithm with acceptance rates tuned to 0.279 and 0.280. Prior-posterior plots are presented in Appendix C. The estimated mean of the degree of domestic price stickiness, represented by $\xi_d$, is lower than those of Iiboshi et al. (2006) and Sugo and Ueda (2008) but higher than that of Iwata (2011). I found other price-sticky parameters, $\xi_{m,c}$, $\xi_{m,i}$, and $\xi_x$, are higher than $\xi_d$. All the indexation parameters, $\kappa_d$, $\kappa_{m,c}$, $\kappa_{m,i}$, and $\kappa_x$, are roughly in the interval between 0.2 and 0.3. The estimated mean of sticky wage parameter, $\xi_d$, is in between those of Iiboshi et al. (2006) and Sugo and Ueda (2008). Posterior mean values for capital utilization, $\sigma_a$, and habit persistency, $h$, is very close to those reported by Iwata (2011). With the help of IST modeling, the parameter for the elasticity of investment to the price of adjustment cost, which was difficult to identify in Iwata (2011), is well identified. The inverse of mean estimate, $1/\chi = 8.13$, is close to that reported by Adolphson et al. (2007). The posterior mean of the inverse elasticity of the labor supply, $\sigma_l = 1.06$, is close to the calibrated value set in Adolphson et al. (2007).

Turning to the parameters of our interest, the estimated mean value of $\nu$ is $-0.42$ in favor of Edgeworth complementarity, although $\nu$ is not reliably different from zero. The results indicate that the relationship between private and public consumption may be complementary, which is consistent with the findings of Okubo (2008) and Karras (1994). The estimated mean value of $\alpha_g$ is 0.05, in favor of a positive externality of public capital. Note that the value is very close to the
recent estimate by Kawaguchi et al. (2009) and is also not so different from the official estimate by the Cabinet Office.\textsuperscript{22} It is also worth noting that the value is equal to the benchmark value employed in Baxter and King (1993). "Spending reversals" are observed for government investment, whereas hardly observed for government consumption. The estimated debt-sensitivity of government investment is larger than those assumed in the literature.\textsuperscript{23} Because tax rates are kept fixed, debt stabilization is attained mainly through reduction in government investment in this model.

To explore the importance of non-wasteful nature of government spending, I also estimate the model imposing restrictions on the parameters, $\nu$ and $\alpha_g$. In addition to the benchmark model without restrictions ($\alpha_g \neq 0$, $\nu \neq 0$) (labeled M1), I estimate a model in which $\alpha_g$ is constrained to zero ($\alpha_g = 0$, $\nu \neq 0$) (labeled M2); a model in which $\nu$ is constrained to zero ($\alpha_g \neq 0$, $\nu = 0$) (labeled M3); a "plain vanilla" model in which $\alpha_g$ and $\nu$ are both constrained to zero ($\alpha_g = 0$, $\nu = 0$) (labeled M4). Note that the restrictions imposed in the model labeled M4 are implicitly imposed in most standard DSGE models in which government spending is typically assumed to be wasteful. Tables 3.a and 3.b report the estimated posterior means of these models. The estimated mean value of $\nu$ in M2 is equal to that in M1, $-0.42$, although it is also not reliably different from zero. Furthermore, the estimated mean value of $\alpha_g$ in M3 is equal to that in M1, 0.05. Overall, the estimated mean values of parameters are stable across different specifications.

With regard to the evaluation and comparison of the models, the posterior mean and likelihood for alternative specifications are reported in Table 4. In calculating log-marginal density, I reestimate the models M1-M4 by calibrating the parameters that govern Edgeworth complementarity between private and public consumption (EC) and productive public capital (PPC), namely, $\alpha_g$ and $\nu$, to the mean values originally estimated for M1, so that the prior distributions across the models M1-M4 do not differ. This aims to avoid the Lindley's paradox, which is known to occur because posterior odds are sensitive to prior distributions on parameters. The likelihood seems to speak in favor of the models with non-wasteful government spending. Although a credible interval for the parameter governing non-separability between private and public consumption includes slightly positive values, the posterior odds show "strong to very strong" evidence in favor of models with EC. On the other hand, the credible interval for the parameter governing productivity of public capital indicates that it takes positive value with high probability, whereas the posterior odds show only "very slight" evidence in favor of models

\textsuperscript{22}The Annual Report on the Japanese Economy and Public Finance 2010 of the Cabinet Office estimates the parameter defining productivity of public capital in Japan as 0.102.

\textsuperscript{23}The implied parameter value for debt-sensitivity of government investment is $(1 - \rho_{pi}) \phi_{pi} = -0.03$, while the values assumed in Corsetti et al. (2010) and Corsetti et al. (2012) are $-0.011$, and $-0.02$, respectively.
5 Non-Wasteful Government Spending in Open Economies

I now turn to investigate the role of Edgeworth complementarity and productive public capital in response to government spending shocks. For comparison purposes and to quantify the importance of these parameters, all parameter values are calibrated to the estimated means of the posterior distributions in the following exercise.

5.1 Impulse Responses to Government Spending Shocks

5.1.1 Government consumption shocks

Figures 3.a and 3.b illustrate the selected dynamic responses to a government consumption shock equal to one percent of the steady-state output across the models. Each dynamic response is depicted as a percentage deviation from the steady state and hence can be interpreted as the impact multiplier. The values of impact multipliers for M1 and M4 are reported in Table 5. Because the transmission mechanism of a government consumption shock is substantially affected by the parameter \( \nu \), the models with \( \nu = -0.42 \) (i.e., M1 and M2) show quite similar patterns in dynamic responses. For the same reason, the models with \( \nu = 0 \) (i.e., M3 and M4) show quite similar patterns.

The models with EC (i.e., M1 and M2) deliver immediate increases in consumption after a government consumption shock. The output multipliers exceed 1.2 on impact because of the sharp rise in consumption. The real exchange rate appreciates initially, but depreciates in later periods. Regarding trade balance, the crowding-in of consumption induces increases in imports and therefore the models with EC show the "twin deficits" phenomenon in initial periods after the shock. In later periods, however, the real exchange rate depreciation improves the trade balance. The hump-shaped responses of the real exchange rate and trade balance are consistent with the VAR evidence shown in Section 2. In contrast, the models without EC (i.e., M3 and M4) deliver crowding-out of consumption. The real exchange rate shows a large appreciation initially, and goes back to its trend level (M2) or depreciates only to a very slight extent (M4) in later periods. Private investment declines more in models with EC than in models without EC, because the initial consumption rise calls for a sharp tightening of monetary policy in models with EC.

For guidance on interpreting posterior odds, see, for example, Dejong and Dave (2007).

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24 For guidance on interpreting posterior odds, see, for example, Dejong and Dave (2007).
5.1.2 Government investment shocks

Figures 4.a and 4.b illustrate the selected dynamic responses to a government investment shock equal to one percent of the steady-state output across the models. Again, I report the values of impact multipliers for M1 and M4 in Table 5. Because the transmission mechanism of a government investment shock is substantially affected by the parameter $\alpha_g$, the models with $\alpha_g = 0.05$ (i.e., M1 and M3) show quite similar patterns in dynamic responses. For the same reason, the models with $\alpha_g = 0$ (i.e., M2 and M4) show quite similar patterns.

The models with PPC (i.e., M1 and M3) deliver declines in consumption and an appreciation of the exchange rate in initial periods after a government investment shock. However, a crowding-in of consumption and a depreciation of the real exchange rate occur in later periods as the new public capital is put in place. The positive externality of public capital increases private investment and accordingly, private consumption. The output multipliers are close to one on impact. They are smaller than those the models with EC exhibit in response to a government consumption shock, but they decline more slowly and reflect the increases in investment and consumption in later periods. Although the models with PPC deliver the crowding-in of consumption, they do not show the "twin deficits" phenomenon after a government investment shock because the crowding-in occurs only in later periods and to a small extent. The difficulty with this type of evidence is the movements in private consumption and investment. The VAR evidence shows immediate rise in consumption after a government consumption shock; however, the models with PPC predict a crowding-in of consumption in later periods. In addition, the models deliver a hump-shaped increase in investment, which can not be observed in the VAR evidence. Although the degree of a real exchange rate depreciation generated by PPC after a government investment shock is not so different from the one generated by EC after a government consumption shock, the trade balance does not show a clear improvement in later periods because of the hump-shaped increase in consumption and investment. The ambiguous response pattern of the trade balance can be seen in the responses of the structural VAR model estimated for the period 1973-1998, whereas the VAR evidence for the period 1980-2005 indicates a clear improvement in the trade balance. The models without PPC (i.e., M2 and M4), on the other hand, deliver crowding-out of consumption. In these models, the real exchange rate shows a large appreciation initially, and depreciates only to a very slight extent in later periods.
5.2 The Transmission Mechanism

5.2.1 International risk-sharing condition

The tight link between consumption and the real exchange rate in standard open economy models has been known for some time, since the works of Backus and Smith (1993) and Kollmann (1995). Before investigating the transmission mechanism, it is worth looking at the condition that forms the basis of the real exchange rate dynamics in the model. Assuming symmetric economic structure between the domestic and foreign economies, the first-order conditions for the domestic and foreign households with respect to consumption and international bond holdings (or international financial transactions) yield the following international risk-sharing condition:

\[
\frac{U_{c,t}}{U_{c,t+1}} = \frac{1}{\Phi(a_t, \phi_t)} \frac{r_{xt} - \tilde{z}_{t}^*}{r_{xt+1} - \tilde{z}_{t+1}^*}
\]  (46)

where \(U_{c,t}\) and \(U_{c,t}^*\) denote the marginal utility of consumption in the domestic and the foreign economies, respectively. Note that only the domestic households are assumed to be charged a risk premium on international bond holdings. When an asset market is complete (i.e., \(\Phi(a_t, \phi_t) = 1\)), the condition predicts a positive correlation between relative consumption across countries and the real exchange rate, which is at odds with the data. The consumption-real exchange rate anomaly is closely related to the two fiscal policy puzzles. Taking the consumption of the foreign economy as a given, the condition causes consumption and the real exchange rate to move in opposite directions. Therefore, in standard open economy models, the real exchange rate tends to appreciate after a country-specific government spending shock because consumption is typically crowded-out in response to a rise in government spending.\(^25\) International financial markets allow households to import when its marginal utility of consumption increases following the crowding-out of consumption caused by a government spending shock. The real exchange rate adjusts to accommodate the international transaction. Therefore, crowding-out of consumption is always accompanied by an appreciation of the real exchange rate under the international risk-sharing condition, and vice versa. This is the reason why the solutions to the first puzzle basically solve the second puzzle.\(^26\)

Consider now the transmission mechanism of government spending shocks in the benchmark model (M1). The model is augmented with three features that help cause a crowding-in of

\(^{25}\) In a general equilibrium framework, an increase in government spending needs to be financed through taxation eventually, which creates a negative wealth effect on consumption. See, for example, Aiyagari et al. (1992) and Baxter and King (1993).

\(^{26}\) The only exception is inclusion of non-Ricardian households. Because non-Ricardian households do not have access to asset markets, their consumption behaviors are irrelevant to the international risk sharing condition. The real exchange rate, therefore, appreciates due to the consumption behavior of Ricardian households after a government spending shock.
consumption in response to a government spending shock: EC, PPC, and "spending reversals" in government investment. These features also contribute to a depreciation of the real exchange rate, in accordance with the relationship between consumption and the real exchange shown by the international risk-sharing condition. In the presence of EC, an increase in government consumption raises the marginal utility of private consumption. When the increase in marginal utility is strong enough to offset the negative wealth effects caused by an increase in government consumption, private consumption increases and accordingly the real exchange rate depreciates. In the presence of PPC, on the other hand, an increase in public capital shifts up the marginal product schedule for private investment. This leads to an increase in output, which, in turn, increases consumption in later periods. In addition, spending reversals observed in a government investment rule also help cause an increase in output, because future reduction in government spending below trend level entails positive wealth effects.

Spending reversals, however, do not play a central role in generating a crowding-in of consumption and a depreciation of the real exchange rate in the benchmark model, the relatively large estimated mean of the parameter governing spending reversals notwithstanding. Recall that the "plain vanilla" model augmented with spending reversals (M4) delivers crowding-out of consumption and only a very slight depreciation of the real exchange rate in later periods in response to both types of government spending shocks. It follows that EC and PPC are the main sources for replicating the empirical responses of consumption and the real exchange rate to government spending shocks in the benchmark model.

5.2.2 Home bias, incomplete asset markets, and the consumption-real exchange rate anomaly

The international risk-sharing condition predicts a high correlation between consumption and the real exchange rate, but their empirical correlation has been found to be low or negative (see Backus and Smith (1993) and Kollmann (1995)). This anomaly has been a long-lasting problem in international business cycle models. Nonetheless, as seen in Figure 3.a, the benchmark model shows an immediate increase in consumption and a hump-shaped depreciation of the real exchange rate after a government consumption shock indicating the low or negative correlation between the two variables.

To understand the exchange rate movements after a government consumption shock in the benchmark model, I first consider the effects of home bias. International financial transactions cause a depreciation of the real exchange rate when the marginal utility of consumption decreases.

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27Recall that I define the real exchange rate as $\text{rex}_t \equiv \frac{S_tP_t^*}{P_t}$ and an increase in the real exchange rate is expressed as a "depreciation" in this paper. Thus, a depreciation of the real exchange rate in accordance with crowding-in of consumption indicates a positive correlation between the two.
creases, following a crowding-in of consumption. However, home bias in private spending moves the real exchange rate in the opposite direction. In the presence of home bias, a crowding-in of consumption contributes to an increase in domestic price of domestically produced goods relative to that of foreign produced goods, leading to an appreciation of the real exchange rate. In order to obtain some intuition for the effects of home bias, I examine the sensitivity of the responses to a government consumption shock, by considering a case without home bias ($\omega_c = \omega_i = 0.5$). Figure 5 shows responses of the real exchange rate and trade balance to a government consumption shock without home bias. I also put in the responses generated by the benchmark model with home bias for comparative purposes. Without home bias, the real exchange rate depreciates in initial periods after a government consumption shock and the trade balance shows improvement with a lag. This implies that the presence of home bias prevents the initial depreciation caused by EC.

Next, I consider the effects of incomplete asset market to investigate the mechanism underlying the hump-shaped depreciation of the real exchange rate. Due to the assumption of an incomplete asset market, the international risk-sharing condition (46) contains the risk premium term, $\Phi(a_t, \tilde{\sigma}_t)$, which hampers the co-movements between consumption and the real exchange rate. Figure 6 shows responses of the real exchange rate and trade balance to a government consumption shock for a case in which an international asset market is complete ($\tilde{\sigma}_a = 0$). Responses under incomplete asset market assumption are also shown for comparative purposes. The hump-shaped real exchange rate depreciation after a government consumption shock disappears if a complete asset market is assumed. Accordingly, the trade balance does not turn into surplus in this case.

With an incomplete asset market, households are assumed to be charged a premium over the international interest rate if the net foreign asset position turns negative. An initial sharp rise in private consumption after a government consumption shock caused by EC is not large enough to generate the real exchange rate depreciation in the presence of home bias. However, the consumption increase leads to deterioration in the net foreign asset position, as shown in Figure 7. As the net foreign asset position deteriorates, the risk premium increases, thereby depreciating the real exchange rate. On the other hand, a government investment shock does not stimulate consumption on impact, and hence the net foreign asset position does not show sharp deterioration, indicating that the effects of a government investment shock on the real exchange rate through the risk premium channel are limited. In a nutshell, EC causes an initial rise in consumption and a deterioration in the net foreign asset position in response to

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28Note that government spending is assumed to fall entirely on domestic goods in the model. Therefore, a government spending shock itself also contributes to an appreciation of the real exchange rate.
a government consumption shock. The presence of home bias prevents the initial depreciation of the real exchange rate, but the incomplete asset market assumption helps cause depreciation in the real exchange rate in later periods. The depreciation reflects a deterioration in the net foreign asset position.

To illustrate the novel result of the benchmark model, I calculate the correlation between consumption and the real exchange rate using model-generated data for 400 periods after government spending shocks. The benchmark model (M1) predicts a negative correlation equal to -0.56 for a government consumption shock, and a positive correlation equal to 0.77 for a government investment shock. The "plain vanilla" model (M4), on the other hand, predicts a positive correlation equal to 0.92 for both government consumption and investment shocks. The results lead us to conclude that the combination of Edgeworth complementarity, home bias, and incomplete asset market enable the benchmark model to generate a crowding-in of consumption and a depreciation of the real exchange rate showing a negative correlation in response to a government consumption shock. Although the model only accounts for the negative correlation between consumption and the real exchange rate after a government consumption shock, the result is potentially important in preventing the model from showing the consumption-real exchange rate anomaly after the shock.

6 Conclusion

This paper has investigated the two fiscal policy puzzles, the anomaly between the standard model predictions and the VAR evidence, and proposes a new but simple approach. First, I present new VAR evidence from Japan on the responses of consumption and the real exchange rate to government consumption and government investment shocks by employing the sign-restrictions approach. In accordance with the results of previous studies on Anglo-Saxon countries, the VAR analysis shows evidence against standard model predictions; consumption increases and the real exchange rate depreciates after both government spending shocks. Although the “twin deficits” phenomenon appears on impact, the trade balance is likely to improve as the real exchange rate depreciates. This implies that the downside risk of expansionary fiscal policy to Japan’s external position may not as big as standard open economy models predict. Second, I have estimated a medium-scale open economy DSGE model introducing (i) non-separability between private and public consumption and (ii) productive public capital, to explain the two puzzles. Using the recently flourishing Bayesian method, I estimate four specifications of the model with and without zero restrictions on the key structural parameters that govern Edgeworth complementarity between private and public consumption, and productive public capital.
The posterior odds favor inclusion of non-wasteful nature of government spending, especially the Edgeworth complementarity. Third, I have shown that the estimated model delivers a crowding-in of consumption and a real exchange rate depreciation after government spending shocks, in line with the empirical evidence obtained from the VAR analysis. The model also replicates the trade balance improvement in later periods due to the real exchange rate depreciation. While the empirical relevance of spending reversals in government investment is confirmed, their presence does not allow the model to account for the two fiscal policy puzzles. Edgeworth complementarity and productive public capital are shown to be the main contributory sources for generating responses of consumption and the real exchange rate in the empirically-plausible directions following government consumption and government investment shocks, respectively.

Furthermore, it should be worth noting that the Edgeworth complementarity also does a good job in explaining the timing of the responses of consumption and the real exchange rate to a government consumption shock with the estimated model. The existing studies have implicitly relied on the international risk-sharing condition to solve the two fiscal policy puzzles. Therefore, these studies predict a positive correlation between consumption and the real exchange rate, which contradicts empirical observation. This paper also shows that the combination of Edgeworth complementarity, home bias, and incomplete asset market allows the model to exhibit a negative correlation between the responses of consumption and the real exchange rate to a government consumption shock. Although the analysis is limited to the responses to government spending shocks, the result is potentially important in preventing the model from showing the consumption-real exchange rate anomaly after the shock.

On the other hand, the model fails to explain the timing of an increase in consumption after a government investment shock. While the hump-shaped depreciation of the real exchange rate is consistent with the VAR evidence, a corresponding hump-shaped increase in consumption cannot be observed. The co-movements also indicate a positive correlation between the responses of consumption and the real exchange rate. Thus, a priority for future research should be to examine the timing of responses of consumption to a government investment shock. In this regard, it would be also worth reviewing the distinction between government consumption and government investment. This paper considers different roles of government consumption and government investment in a DSGE model based on the traditional view. In estimating VAR and DSGE models, I use statistical categories in distinguishing government consumption and government investment. However, given that the responses obtained from the VAR model show very similar patterns to both government spending shocks, the one-to-one correspondence I assumed between the two statistical categories of government spending and their roles in the DSGE model might need to be better modified.
References


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Uhlig, Harald (2005) ‘What are the effects of monetary policy on output? Results from an agnostic identification procedure.’ *Journal of Monetary Economics* 52(2), 381–419

Table 1. Set of imposed sign restrictions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sign restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government spending (either gov. consumption or investment)</td>
<td>+</td>
</tr>
<tr>
<td>Output</td>
<td>+</td>
</tr>
<tr>
<td>Private consumption</td>
<td>?</td>
</tr>
<tr>
<td>Private investment</td>
<td>?</td>
</tr>
<tr>
<td>Budget balance</td>
<td>−</td>
</tr>
<tr>
<td>Trade balance</td>
<td>?</td>
</tr>
<tr>
<td>Inflation</td>
<td>+</td>
</tr>
<tr>
<td>Interest rate</td>
<td>+</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>?</td>
</tr>
</tbody>
</table>

Notes: This table reports signs imposed on the impulse responses of the variables to an expansionary government spending (either government consumption or government investment) shock. The question mark (?) indicates that the responses of the variables are unrestricted. A positive sign (+) [negative sign (-)] indicates the response of the variables are restricted to be positive [negative] for four quarters (including the initial quarter).
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>Prior S. D.</th>
<th>Posterior Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_w$</td>
<td>Calvo wages</td>
<td>beta 0.6</td>
<td>0.1</td>
<td>0.387</td>
<td>[0.275 0.499]</td>
</tr>
<tr>
<td>$\xi_d$</td>
<td>Calvo domestic prices</td>
<td>beta 0.6</td>
<td>0.1</td>
<td>0.656</td>
<td>[0.578 0.736]</td>
</tr>
<tr>
<td>$\xi_{m,c}$</td>
<td>Calvo import cons. prices</td>
<td>beta 0.6</td>
<td>0.1</td>
<td>0.827</td>
<td>[0.778 0.876]</td>
</tr>
<tr>
<td>$\xi_{m,i}$</td>
<td>Calvo import inv. prices</td>
<td>beta 0.6</td>
<td>0.1</td>
<td>0.931</td>
<td>[0.895 0.966]</td>
</tr>
<tr>
<td>$\xi_x$</td>
<td>Calvo export prices</td>
<td>beta 0.6</td>
<td>0.1</td>
<td>0.780</td>
<td>[0.709 0.855]</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>Indexation domestic prices</td>
<td>beta 0.4</td>
<td>0.15</td>
<td>0.243</td>
<td>[0.072 0.407]</td>
</tr>
<tr>
<td>$\kappa_{m,c}$</td>
<td>Index. import cons. prices</td>
<td>beta 0.4</td>
<td>0.15</td>
<td>0.201</td>
<td>[0.049 0.342]</td>
</tr>
<tr>
<td>$\kappa_{m,i}$</td>
<td>Index. import inv. prices</td>
<td>beta 0.4</td>
<td>0.15</td>
<td>0.320</td>
<td>[0.082 0.544]</td>
</tr>
<tr>
<td>$\kappa_x$</td>
<td>Indexation export prices</td>
<td>beta 0.4</td>
<td>0.15</td>
<td>0.209</td>
<td>[0.060 0.356]</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit</td>
<td>beta 0.7</td>
<td>0.1</td>
<td>0.436</td>
<td>[0.306 0.566]</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Labor supply elasticity</td>
<td>gamma 2</td>
<td>0.75</td>
<td>1.060</td>
<td>[0.483 1.600]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Edgeworth complementarity</td>
<td>normal 0.1</td>
<td>1.5</td>
<td>-0.415</td>
<td>[-1.281 0.453]</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Subst. elasticity cons.</td>
<td>inv. gamma 1.5</td>
<td>0.25</td>
<td>1.315</td>
<td>[1.071 1.558]</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>Subst. elasticity inv.</td>
<td>inv. gamma 1.5</td>
<td>0.25</td>
<td>1.561</td>
<td>[1.089 1.996]</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>Subst. elasticity foreign</td>
<td>inv. gamma 1.5</td>
<td>0.25</td>
<td>2.039</td>
<td>[1.624 2.450]</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Capital util. adj. cost</td>
<td>gamma 1</td>
<td>0.75</td>
<td>2.306</td>
<td>[1.071 3.478]</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Investment adj. cost</td>
<td>normal 0.2</td>
<td>0.1</td>
<td>0.123</td>
<td>[0.040 0.202]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Productivity capital</td>
<td>beta 0.2</td>
<td>0.05</td>
<td>0.265</td>
<td>[0.190 0.336]</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>Productivity public capital</td>
<td>beta 0.2</td>
<td>0.1</td>
<td>0.046</td>
<td>[0.009 0.079]</td>
</tr>
</tbody>
</table>

Notes: This table reports prior distributions, posterior means, and 90% credible intervals (or Bayesian confidence intervals) of the parameters. The prior-posterior plots can be found in Appendix C. Sample period is 1980:Q1-1998:Q4. In conducting Bayesian MCMC estimation, the draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for Metropolis-Hastings algorithm with acceptance rates tuned to 0.279-0.280.
Table 2.b. Prior and posterior distributions — Benchmark model (M1)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
<th>Prior</th>
<th>Mean</th>
<th>S. D.</th>
<th>Posterior</th>
<th>Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\lambda_d}$</td>
<td>Domestic markup shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.877</td>
<td>[0.790, 0.968]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\lambda_{m,c}}$</td>
<td>Imp. cons. markup shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.423</td>
<td>[0.283, 0.567]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\lambda_{m,i}}$</td>
<td>Imp. inv. markup shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.549</td>
<td>[0.357, 0.740]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\lambda_{x}}$</td>
<td>Export markup shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.755</td>
<td>[0.608, 0.912]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\lambda_{t}}$</td>
<td>Stationary tech shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.877</td>
<td>[0.746, 0.984]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\xi_{c}}$</td>
<td>Cons pref. shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.810</td>
<td>[0.688, 0.945]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\xi_{i}}$</td>
<td>Inv. spec. shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.954</td>
<td>[0.922, 0.987]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\xi_{gi}}$</td>
<td>Gov. inv. spec. shock AR coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.800</td>
<td>[0.650, 0.957]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\mu_{z+}}$</td>
<td>Neutral tech. progress AR coeff.</td>
<td>beta</td>
<td>0.6</td>
<td>0.1</td>
<td>0.559</td>
<td>[0.403, 0.723]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\mu_{\phi}}$</td>
<td>Inv. spec. tech. progress AR coeff.</td>
<td>beta</td>
<td>0.6</td>
<td>0.1</td>
<td>0.436</td>
<td>[0.289, 0.581]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{i^*}$</td>
<td>Asymmetric tech. progress AR coeff.</td>
<td>beta</td>
<td>0.6</td>
<td>0.1</td>
<td>0.598</td>
<td>[0.438, 0.764]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{gc}$</td>
<td>Gov. cons. smoothing coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.776</td>
<td>[0.606, 0.948]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{gi}$</td>
<td>Gov. inv. smoothing coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.761</td>
<td>[0.579, 0.947]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{r}$</td>
<td>Interest rate smoothing coeff.</td>
<td>beta</td>
<td>0.8</td>
<td>0.1</td>
<td>0.856</td>
<td>[0.814, 0.898]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{a}$</td>
<td>International bond risk premium coeff.</td>
<td>normal</td>
<td>0.2</td>
<td>0.1</td>
<td>0.495</td>
<td>[0.374, 0.612]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{gc}$</td>
<td>Gov. cons. reversal coeff.</td>
<td>normal</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.023</td>
<td>[-0.045, 0.088]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{gi}$</td>
<td>Gov. inv. reversal coeff.</td>
<td>normal</td>
<td>-0.2</td>
<td>0.1</td>
<td>-0.127</td>
<td>[-0.238, -0.038]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{r\pi}$</td>
<td>Interest rate inflation coeff.</td>
<td>gamma</td>
<td>2</td>
<td>0.5</td>
<td>1.951</td>
<td>[1.515, 2.345]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{ry}$</td>
<td>Interest rate output gap coeff.</td>
<td>gamma</td>
<td>0.125</td>
<td>0.05</td>
<td>0.046</td>
<td>[0.015, 0.076]</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports prior distributions, posterior means, and 90% credible intervals (or Bayesian confidence intervals) of the parameters. The prior-posterior plots can be found in Appendix C. Sample period is 1980:Q1-1998:Q4. In conducting Bayesian MCMC estimation, the draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for Metropolis-Hastings algorithm with acceptance rates tuned to 0.279-0.280.
### Table 2.c. Prior and posterior distributions — Benchmark model (M1)

<table>
<thead>
<tr>
<th>S. D. of Shocks</th>
<th>Distribution</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{\lambda_d} )</td>
<td>Domestic markup shock</td>
<td>inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>( \varepsilon_{\lambda_{m,c}} )</td>
<td>Imp. cons. markup shock</td>
<td>inv. gamma</td>
<td>0.4</td>
</tr>
<tr>
<td>( \varepsilon_{\lambda_{m,i}} )</td>
<td>Imp. inv. markup shock</td>
<td>inv. gamma</td>
<td>1.2</td>
</tr>
<tr>
<td>( \varepsilon_{\lambda_x} )</td>
<td>Export markup shock</td>
<td>inv. gamma</td>
<td>0.01</td>
</tr>
<tr>
<td>( \varepsilon_{\xi} )</td>
<td>Stationary tech shock</td>
<td>inv. gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>( \varepsilon_{\zeta_c} )</td>
<td>Cons pref. shock</td>
<td>inv. gamma</td>
<td>0.02</td>
</tr>
<tr>
<td>( \varepsilon_{\zeta_i} )</td>
<td>Labor supply shock</td>
<td>inv. gamma</td>
<td>0.2</td>
</tr>
<tr>
<td>( \varepsilon_{\xi_{gi}} )</td>
<td>Inv. spec. shock</td>
<td>inv. gamma</td>
<td>0.02</td>
</tr>
<tr>
<td>( \varepsilon_{\xi_{bi}} )</td>
<td>Gov. inv. spec. shock</td>
<td>inv. gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>( \varepsilon_{\mu_{x+}} )</td>
<td>Neutral tech. progress shock</td>
<td>inv. gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>( \varepsilon_{\mu_{fi}} )</td>
<td>Inv. spec. tech. progress shock</td>
<td>inv. gamma</td>
<td>0.005</td>
</tr>
<tr>
<td>( \varepsilon_{\zeta^*} )</td>
<td>Asymmetric tech. progress shock</td>
<td>inv. gamma</td>
<td>0.02</td>
</tr>
<tr>
<td>( \varepsilon_{gc} )</td>
<td>Gov. cons. shock</td>
<td>inv. gamma</td>
<td>0.008</td>
</tr>
<tr>
<td>( \varepsilon_{gi} )</td>
<td>Gov. inv. shock</td>
<td>inv. gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>Interest rate shock</td>
<td>inv. gamma</td>
<td>0.002</td>
</tr>
<tr>
<td>( \varepsilon_{g} )</td>
<td>External finance premium shock</td>
<td>inv. gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tilde{\phi} )</td>
<td>International bond risk premium shock</td>
<td>inv. gamma</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: This table reports prior distributions, posterior means, and 90% credible intervals (or Bayesian confidence intervals) of the parameters. The prior-posterior plots can be found in Appendix C. Sample period is 1980:Q1-1998:Q4. In conducting Bayesian MCMC estimation, the draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for Metropolis-Hastings algorithm with acceptance rates tuned to 0.279-0.280.
Table 3.a. Posterior mean estimates under different types of parameter restrictions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>M2 $\alpha_g = 0, \nu \neq 0$</th>
<th>M3 $\alpha_g \neq 0, \nu = 0$</th>
<th>M4 $\alpha_g = 0, \nu = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_w$</td>
<td>0.410</td>
<td>0.395</td>
<td>0.401</td>
</tr>
<tr>
<td>$\xi_d$</td>
<td>0.661</td>
<td>0.655</td>
<td>0.660</td>
</tr>
<tr>
<td>$\xi_{m.c}$</td>
<td>0.827</td>
<td>0.828</td>
<td>0.826</td>
</tr>
<tr>
<td>$\xi_{m,i}$</td>
<td>0.930</td>
<td>0.931</td>
<td>0.930</td>
</tr>
<tr>
<td>$\xi_x$</td>
<td>0.775</td>
<td>0.781</td>
<td>0.776</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>0.247</td>
<td>0.244</td>
<td>0.243</td>
</tr>
<tr>
<td>$\kappa_{m.c}$</td>
<td>0.205</td>
<td>0.204</td>
<td>0.199</td>
</tr>
<tr>
<td>$\kappa_{m,i}$</td>
<td>0.314</td>
<td>0.324</td>
<td>0.322</td>
</tr>
<tr>
<td>$\kappa_x$</td>
<td>0.212</td>
<td>0.214</td>
<td>0.217</td>
</tr>
<tr>
<td>$h$</td>
<td>0.442</td>
<td>0.470</td>
<td>0.468</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>1.125</td>
<td>1.024</td>
<td>1.049</td>
</tr>
<tr>
<td>$\nu_\epsilon$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>1.308</td>
<td>1.313</td>
<td>1.309</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>1.575</td>
<td>1.550</td>
<td>1.565</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>2.049</td>
<td>2.041</td>
<td>2.055</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>2.259</td>
<td>2.300</td>
<td>2.295</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.118</td>
<td>0.121</td>
<td>0.115</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.267</td>
<td>0.266</td>
<td>0.268</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>-</td>
<td>0.045</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: This table reports posterior mean estimates of parameters in models under different types of restriction on parameters that govern productivity of public capital ($\alpha_g$) and non-separability between private and public consumption ($\nu$) in their estimation. Parameters of these models are estimated using the same prior distributions as those used for the estimation of the benchmark model M1, which are shown in Tables 2.a, 2.b, and 2.c. Sample period is 1980:Q1-1998:Q4. In conducting Bayesian MCMC estimation, the draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for Metropolis-Hastings algorithm. The acceptance rates are tuned to 0.287-0.292, 0.289-0.284, and 0.293-0.295, in the estimation of M2, M3, and M4, respectively.
Table 3.b. Posterior mean estimates under different types of parameter restrictions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>M2 $\alpha_g = 0, \nu \neq 0$</th>
<th>M3 $\alpha_g \neq 0, \nu = 0$</th>
<th>M4 $\alpha_g = 0, \nu = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\lambda_d}$</td>
<td>0.873</td>
<td>0.870</td>
<td>0.873</td>
</tr>
<tr>
<td>$\rho_{\lambda_{m,c}}$</td>
<td>0.416</td>
<td>0.425</td>
<td>0.421</td>
</tr>
<tr>
<td>$\rho_{\lambda_{m,i}}$</td>
<td>0.550</td>
<td>0.555</td>
<td>0.553</td>
</tr>
<tr>
<td>$\rho_{\lambda_c}$</td>
<td>0.760</td>
<td>0.757</td>
<td>0.757</td>
</tr>
<tr>
<td>$\rho_{\zeta^c}$</td>
<td>0.876</td>
<td>0.869</td>
<td>0.869</td>
</tr>
<tr>
<td>$\rho_{\zeta^i}$</td>
<td>0.793</td>
<td>0.811</td>
<td>0.804</td>
</tr>
<tr>
<td>$\rho_{\xi^i}$</td>
<td>0.301</td>
<td>0.316</td>
<td>0.316</td>
</tr>
<tr>
<td>$\rho_{\zeta^{gi}}$</td>
<td>0.957</td>
<td>0.954</td>
<td>0.954</td>
</tr>
<tr>
<td>$\rho_{\mu^+}$</td>
<td>0.806</td>
<td>0.799</td>
<td>0.803</td>
</tr>
<tr>
<td>$\rho_{\mu^q}$</td>
<td>0.554</td>
<td>0.558</td>
<td>0.555</td>
</tr>
<tr>
<td>$\rho_{\mu^z^+}$</td>
<td>0.442</td>
<td>0.434</td>
<td>0.439</td>
</tr>
<tr>
<td>$\rho_{\mu^z^-}$</td>
<td>0.600</td>
<td>0.599</td>
<td>0.601</td>
</tr>
<tr>
<td>$\rho_{\omega}$</td>
<td>0.770</td>
<td>0.776</td>
<td>0.775</td>
</tr>
<tr>
<td>$\rho_{\omega^{gi}}$</td>
<td>0.760</td>
<td>0.763</td>
<td>0.758</td>
</tr>
<tr>
<td>$\rho_{\omega^r}$</td>
<td>0.858</td>
<td>0.857</td>
<td>0.857</td>
</tr>
<tr>
<td>$\tilde{\phi}_a$</td>
<td>0.481</td>
<td>0.490</td>
<td>0.479</td>
</tr>
<tr>
<td>$\phi_{gc}$</td>
<td>0.031</td>
<td>0.011</td>
<td>0.018</td>
</tr>
<tr>
<td>$\phi_{gi}$</td>
<td>-0.125</td>
<td>-0.134</td>
<td>-0.135</td>
</tr>
<tr>
<td>$\phi_{ri}$</td>
<td>1.972</td>
<td>1.953</td>
<td>1.953</td>
</tr>
<tr>
<td>$\phi_{ry}$</td>
<td>0.049</td>
<td>0.038</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Notes: This table reports posterior mean estimates of parameters in models under different types of restriction on parameters that govern productivity of public capital ($\alpha_g$) and non-separability between private and public consumption ($\nu$) in their estimation. Parameters of these models are estimated using the same prior distributions as those used for the estimation of the benchmark model M1, which are shown in Tables 2.a, 2.b, and 2.c. Sample period is 1980:Q1-1998:Q4. In conducting Bayesian MCMC estimation, the draws from the posterior distribution have been obtained by taking two parallel chains of 300,000 replications for Metropolis-Hastings algorithm. The acceptance rates are tuned to 0.287-0.292, 0.289-0.284, and 0.290-0.293, in the estimation of M2, M3, and M4, respectively.
Table 4. Log marginal data densities and posterior odds

<table>
<thead>
<tr>
<th>Specification</th>
<th>Log marginal data density</th>
<th>Posterior odds versus M4</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 ( \alpha_g = 0.046, \nu = -0.415 )</td>
<td>-2100.86</td>
<td>1.70</td>
</tr>
<tr>
<td>M2 ( \alpha_g = 0.000, \nu = -0.424 )</td>
<td>-2098.78</td>
<td>13.60</td>
</tr>
<tr>
<td>M3 ( \alpha_g = 0.045, \nu = 0.000 )</td>
<td>-2100.90</td>
<td>1.64</td>
</tr>
<tr>
<td>M4 ( \alpha_g = 0.000, \nu = 0.000 )</td>
<td>-2101.39</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: This table reports log marginal data densities for the models M1-M4 and their posterior odds versus M4. The log marginal data densities are computed based on modified harmonic mean estimator. For the purpose of calculating log marginal data densities, the models M1-M3 are reestimated calibrating the parameters that govern productivity of public capital \( \alpha_g \) and Edgeworth complementarity \( \nu \) to the mean values originally estimated for M1, so that the prior distributions across the models M1-M4 do not differ.
Table 5. Impact multipliers

<table>
<thead>
<tr>
<th>Quarters</th>
<th>M1 (w/ EC, PPC)</th>
<th>M4 (w/o EC, PPC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.28</td>
<td>1.01</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>0.42</td>
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<tr>
<td>8</td>
<td>0.11</td>
<td>0.16</td>
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<tr>
<td>12</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
</tr>
<tr>
<td>1</td>
<td>0.57</td>
<td>-0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
<td>-0.02</td>
</tr>
<tr>
<td>8</td>
<td>-0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>12</td>
<td>-0.05</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>-0.07</td>
<td>-0.02</td>
</tr>
<tr>
<td>4</td>
<td>-0.22</td>
<td>-0.04</td>
</tr>
<tr>
<td>8</td>
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<td>0.01</td>
</tr>
<tr>
<td>12</td>
<td>-0.19</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
</tr>
</tbody>
</table>

Notes: This table reports impact multipliers of government consumption and government investment shocks in M1 (with Edgeworth complementarity and productive public capital) and M4 (without Edgeworth complementarity and productive public capital). The impact multiplier of government spending ($G$) for variable ($V$) in period $k$ is defined as: $\frac{\Delta V_{t+k}}{\Delta V_{t}}$. The values presented in the table correspond to those of dynamic responses of output, consumption and investment, depicted in Figures 3.a and 4.a.
Figure 1.a. Impulse responses to an expansionary government consumption shock one standard deviation in size: Notes: Sample period is 1973:Q1-1998:Q4. The median responses and the 16 and 84% quantiles are depicted. Inference is obtained from 4,000 draws from the unit sphere for each of 4,000 draws from the VAR posterior. The sign restrictions are imposed for four quarters (including the initial quarter).
Figure 1.b. Impulse responses to an expansionary government consumption shock one standard deviation in size: Notes: Sample period is 1980:Q1-2005:Q4. The median responses and the 16 and 84% quantiles are depicted. Inference is obtained from 4,000 draws from the unit sphere for each of 4,000 draws from the VAR posterior. The sign restrictions are imposed four quarters (including the initial quarter).
Figure 2.a. Impulse responses to an expansionary government investment shock one standard deviation in size: Notes: Sample period is 1973:Q1-1998:Q4. The median responses and the 16 and 84% quantiles are depicted. Inference is obtained from 4,000 draws from the unit sphere for each of 4,000 draws from the VAR posterior. The sign restrictions are imposed four quarters (including the initial quarter).
Figure 2.b. Impulse responses to an expansionary government investment shock one standard deviation in size: Notes: Sample period is 1980:Q1-2005:Q4. The median responses and the 16 and 84% quantiles are depicted. Inference is obtained from 4,000 draws from the unit sphere for each of 4,000 draws from the VAR posterior. The sign restrictions are imposed four quarters (including the initial quarter).
Figure 3.a. Model responses to a government consumption shock equal to one percent of the steady-state output: Notes: Solid lines: M1 (with Edgeworth complementarity and productive public capital); dashed lines: M4 (without Edgeworth complementarity and productive public capital).
Figure 3.b. Model responses to a government consumption shock equal to one percent of the steady-state output: Notes: Dash-dotted lines: M2 (with Edgeworth complementarity, without productive public capital); dash-double dotted lines: M3 (without Edgeworth complementarity, with productive public capital).
Figure 4.a. Model responses to a government investment shock equal to one percent of the steady-state output: Notes: Solid lines: M1 (with Edgeworth complementarity and productive public capital); dashed lines: M4 (without Edgeworth complementarity and productive public capital).
Figure 4.b. Model responses to a government investment shock equal to one percent of the steady-state output: Notes: Dash-dotted lines: M2 (with Edgeworth complementarity, without productive public capital); dash-double dotted lines: M3 (without Edgeworth complementarity, with productive public capital).
Figure 5. Sensitivity for home bias — Model responses to a government consumption shock equal to one percent of the steady-state output for a benchmark model (M1) with (dotted lines) and without home bias (solid lines).

Figure 6. Sensitivity for incomplete asset market assumption — Model responses to a government consumption shock equal to one percent of the steady-state output for a benchmark model (M1) with incomplete asset market (dotted lines) and with complete asset market (solid lines).

Figure 7. Model responses to government consumption (left panel) and government investment (right panel) shocks equal to one percent of the steady-state output: Notes: Solid lines: M1 (with Edgeworth complementarity and productive public capital); dashed lines: M4 (without Edgeworth complementarity and productive public capital).
Appendix

A Log-linearized Model

A.1 Firms

A.1.1 Domestic-good producing firms

From (2), (3), and (4):

\[
\hat{d}_t^d = \frac{\beta}{1 + \kappa_d \beta} E_t \hat{d}_{t+1}^d + \frac{\kappa_d}{1 + \kappa_d \beta} \hat{d}_{t-1}^d + \frac{1}{1 + \kappa_d \beta} \left( 1 - \beta \xi_d \right) \left( 1 - \xi_d \right) \left[ \hat{mc}_t^d + \lambda_{d,t} \right], \tag{A.1}
\]

where

\[
\lambda_t^d = \rho_{\lambda}^d \lambda_{t-1}^d + \varepsilon_{\lambda,t}, \tag{A.2}
\]

\[
\hat{mc}_t^d = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^d - \hat{\epsilon}_t - \alpha_g \hat{k}^g_{t-1} + \alpha_g \hat{\mu}_{z+t}, \tag{A.3}
\]

\[
\hat{\epsilon}_t = \rho_{\epsilon} \hat{\epsilon}_{t-1} + \varepsilon_{\epsilon,t}, \tag{A.4}
\]

\[
\hat{r}_t^d = \hat{L}_t - \hat{a}_t - \hat{k}_{t-1} + \hat{\hat{w}}_t + \hat{\hat{\mu}}_{z+t} + \hat{\hat{\mu}}_{\psi,t}. \tag{A.5}
\]

From (5):

\[
\hat{d}_t^d = (1 - \frac{1}{\lambda_d}) \hat{y} \hat{y}_t - y \hat{mc}_t. \tag{A.6}
\]

A.1.2 Importing firms and import-good wholesalers

From (7):

\[
\hat{z}_t^{m,c} = \frac{\beta}{1 + \kappa_{m,c} \beta} E_t \hat{z}_{t+1}^{m,c} + \frac{\kappa_{m,c}}{1 + \kappa_{m,c} \beta} \hat{z}_{t-1}^{m,c} + \frac{1}{1 + \kappa_{m,c} \beta} \left( 1 - \beta \xi_{m,c} \right) \left( 1 - \xi_{m,c} \right) \left[ \hat{mc}_t^{m,c} + \hat{\lambda}_t^{m,c} \right], \tag{A.7}
\]

where

\[
\hat{\lambda}_t^{m,c} = \rho_{\lambda^{m,c}} \hat{\lambda}_{t-1}^{m,c} + \varepsilon_{\lambda^{m,c},t}, \tag{A.8}
\]

\[
\hat{mc}_t^{m,c} = -\hat{\gamma}_t^{f} - \hat{\gamma}_t^{mc,d}, \tag{A.9}
\]

From (6):

\[
\hat{d}_t^{m,c} = \gamma^{mc,d} \epsilon^{m} \left( \hat{\gamma}_t^{mc,d} + \hat{\epsilon}_t^{m} \right) - \epsilon^{m} \left( -\hat{\gamma}_t^{f} + \hat{\epsilon}_t^{m} \right). \tag{A.10}
\]
From (8):

\[
\hat{\pi}^m_i = \frac{\beta}{1 + \kappa_{m,i} \beta} E_t \hat{\pi}^m_{i+1} + \frac{\kappa_{m,i}}{1 + \kappa_{m,i} \beta} \hat{\pi}^m_{i-1} + \frac{1}{1 + \kappa_{m,i} \beta} \frac{(1 - \beta \xi_{m,i}) (1 - \xi_{m,i})}{\xi_{m,i}} \left[ \hat{m}c_t^m + \hat{\lambda}_t^m \right],
\]

(A.11)

where

\[
\hat{\lambda}_t^m = \rho \hat{\lambda}_{t-1}^m + \hat{\varepsilon}_{m,i,t},
\]

(A.12)

\[
\hat{m}c_t^m = -\hat{\gamma}_t^f - \hat{\gamma}_t^m,
\]

(A.13)

From (9):

\[
\hat{\alpha}_t^m = \gamma_{m,d} \hat{\alpha}_t^m \left( \hat{\gamma}_t^m + \hat{\gamma}_t^m \right) - \hat{\gamma}_t^m \left( -\hat{\gamma}_t^f + \hat{\gamma}_t^m \right).
\]

(A.14)

A.1.3 Exporting firms and export-good wholesalers

From (10):

\[
\hat{\pi}^x_t = \frac{\beta}{1 + \kappa_x \beta} E_t \hat{\pi}^x_{t+1} + \frac{\kappa_x}{1 + \kappa_x \beta} \hat{\pi}^x_{t-1} + \frac{1}{1 + \kappa_x \beta} \frac{(1 - \beta \xi_{x}) (1 - \xi_{x})}{\xi_{x}} \left[ \hat{m}c_t^x + \hat{\lambda}_x,t \right],
\]

(A.15)

where

\[
\hat{\lambda}_t^x = \rho \hat{\lambda}_{t-1}^x + \hat{\varepsilon}_{x,t},
\]

(A.16)

\[
\hat{m}c_t^x = \hat{m}c_{t-1}^x + \hat{\pi}_t^d - \hat{\pi}_t^x - (\hat{S}_t - \hat{S}_{t-1}).
\]

(A.17)

From (11):

\[
\hat{\alpha}_t^x = -y^s \hat{m}c_t^x.
\]

(A.18)

A.1.4 Domestic and foreign retailers

From (14):

\[
\hat{\pi}_t^c = \left(1 - \omega_c \right) \left( \gamma_{c,d} \right)^{\eta_{c}-1} \hat{\pi}_t^d + \left( \omega_c \left( \gamma_{mc,c} \right)^{1 - \eta_{c}} \right) \hat{\pi}_t^m,c.
\]

(A.19)

From (13):

\[
\hat{\alpha}_t^m = -\eta_c \hat{\alpha}_t^{mc,d} + \eta_c \gamma_{t} \hat{\alpha}_t^{c,d} + \hat{\alpha}_t.
\]

(A.20)

From (17):

\[
\hat{\pi}_t^i = \left(1 - \omega_i \right) \left( \gamma_{i} \right)^{\eta_{i}-1} \left( \hat{\pi}_t^d - \hat{\mu}_{\psi,t} \right) + \left( \omega_i \left( \gamma_{mi,d} \right)^{1 - \eta_{i}} \right) \left( \hat{\pi}_t^m,i - \hat{\mu}_{\psi,t} \right).
\]

(A.21)
From (16):

\[
\dot{z}_t^m = -\eta_i \dot{z}_t^{mi,d} + \eta_i \dot{p}_t^i + \ddot{\bar{u}}_t + \frac{\bar{p}^k}{(\mu_z + \mu_p - (1 - \delta))} p^d \dot{u}_t. \tag{A.22}
\]

A.1.5 Relative Prices

From (20):

\[
\dot{\gamma}_t^{mc,d} = \dot{\gamma}_{t-1}^{mc,d} + \dot{z}_t^{mc} - \dot{z}_t^d. \tag{A.23}
\]

From (21):

\[
\dot{\gamma}_t^{mi,d} = \dot{\gamma}_{t-1}^{mi,d} + \dot{z}_t^{mi} - \dot{z}_t^d. \tag{A.24}
\]

From (22):

\[
\dot{\gamma}_t^{c,d} = \dot{\gamma}_{t-1}^{c,d} + \dot{z}_t^c - \dot{z}_t^d. \tag{A.25}
\]

From (24):

\[
\dot{p}_t^i = \dot{p}_{t-1}^i + \dot{\bar{u}}_t^i - \dot{z}_t^d + \dot{\bar{q}}_{q,t}. \tag{A.26}
\]

From (25):

\[
\dot{\gamma}_t^{x^*, t} = \dot{\gamma}_{t-1}^{x^*, t} + \dot{\bar{z}}_t^{x^*} - \dot{z}_t^{x^*}. \tag{A.27}
\]

From (26):

\[
\dot{\gamma}_t^{f} = \dot{m}_t^c + \dot{\gamma}_t^{x^*, t}. \tag{A.28}
\]

A.2 Households

A.2.1 Consumption Euler Equation

From (29) and (30):

\[
(\mu_{z^t} - h^q \beta) (\mu_{z^t} - h) \left( \dot{\psi}_{z^t, t} + \dot{\gamma}_t^{c,d} \right) = h^q \beta \mu_{z^t} \tilde{e}_t \tilde{\sigma}_t + (\mu_{z^t} + \tilde{\mu}_{z^t} - h) \left( \mu_{z^t} + \tilde{z}_t^c - h^q \beta E_t \tilde{\zeta}_t^{c} \right), \tag{A.29}
\]

where

\[
\dot{\psi}_{z^t, t} = E_t \left( \dot{\psi}_{z^t, t+1} - \mu_{z^t, t+1} \right) + \dot{R}_t, \tag{A.30}
\]

\[
\dot{\zeta}_t^c = \mu_c^c \zeta_{t-1}^c + \zeta_{z^c, t}, \tag{A.31}
\]

\[
\dot{c}_t^c = c_t^c + v g^c \tilde{g}_t^c. \tag{A.32}
\]
A.2.2 Investment Euler Equation

From (32) and (30):

\[ \hat{t}_t = \frac{1}{1 + \beta} \hat{t}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{t}_{t+1} + \frac{\chi}{(1 + \beta) (\mu_z + \mu_{\psi})} \left( \hat{q}_t + \zeta^i_t - \tilde{p}_t \right) \\
- \frac{1}{1 + \beta} (\mu_{z+} + \mu_{\psi}) + \frac{\beta}{1 + \beta} E_t (\tilde{z}_{z+t+1} + \tilde{z}_{\psi,t+1}), \quad (A.33) \]

where

\[ \zeta^i_t = \rho^i \zeta^i_{t-1} + \varepsilon^i_{t,t}. \quad (A.34) \]

A.2.3 Q Equation

From (33) and (30):

\[ \hat{q}_t = E_t \hat{z}_{z+t+1} - \hat{\psi}_{z+t} - \hat{E}_t \hat{z}_{z+t+1} - \hat{E}_t \hat{\psi}_{t+1} + \frac{\beta (1 - \delta)}{\mu_z + \mu_{\psi}} E_t \hat{q}_{t+1} + \frac{\mu_z + \mu_{\psi} - \beta (1 - \delta)}{\mu_z + \mu_{\psi}} E_t \hat{\tau}_{t+1} + \varepsilon^q. \quad (A.35) \]

A.2.4 Capital Utilization Decision Equation

From (34):

\[ \hat{u}_t = \frac{1}{\sigma_u} \left[ \hat{\tau}^x_t - \hat{p}_t \right]. \quad (A.36) \]

A.2.5 Capital Law of Motion

From (28):

\[ \hat{k}_t = \frac{1 - \delta}{\mu_z + \mu_{\psi}} (\hat{k}_{t-1} - \hat{\mu}_{z+,t} - \hat{\mu}_{\psi,t}) + \left[ 1 - \frac{1 - \delta}{\mu_z + \mu_{\psi}} \right] \left( \hat{t}_t + \zeta^i_t \right). \quad (A.37) \]

A.2.6 Real Wage Law of Motion

From (35):

\[ \hat{w}_t = \frac{\beta}{1 + \beta} E_t \hat{w}_{t+1} + \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{\pi}^d_{t+1} \\
- \frac{1}{1 + \beta} \hat{\pi}^d_t - \frac{\beta \kappa_w}{1 + \beta} \hat{\pi}^c_t + \frac{\kappa_w}{1 + \beta} \hat{\pi}^c_{t-1} \\
- \frac{1}{1 + \beta} \left( (1 - \beta \xi_w) (1 - \xi_w) (1 - \lambda_w) \right) \left( \hat{w}_t - \sigma_t \hat{L}_t - \zeta^l_t + \hat{\psi}_{z+t} \right), \quad (A.38) \]

where

\[ \zeta^l_t = \rho \zeta_{t-1} + \varepsilon_{t,t}. \quad (A.39) \]
A.2.7 Risk-adjusted UIP Condition

From (31) and (30):
\[ \hat{R}_t = \hat{R}_t^i + (\hat{S}_{t+1} - \hat{S}_t) + \left[ -\phi_a \hat{a}_t + \hat{\phi} \right]. \] (A.40)

A.2.8 Evolution of Net Foreign Assets

From (43):
\[ \hat{a}_t - \frac{1}{\beta} \hat{a}_{t-1} = y^* \hat{m} e_t - \eta_f \hat{y} \hat{z}_t^x + y^* \hat{g}_t + y^* \hat{\xi}_t + (c^m + i^m) \hat{\gamma}_t^f \\ - \eta_c \hat{z}_t^{mc,d} + \hat{c}_t \] (A.41)

\[ - \eta_i \hat{z}_t^{mi,d} + \hat{t}_t + \frac{\rho^k}{(\mu_z + \mu_\psi) - (1 - \delta)} \rho^k \hat{u}_t. \]

From (44):
\[ \hat{c}_x \hat{e}_t = -\omega_c (\gamma^{c,mc} - (1 - \eta_c) \hat{z}_t^{mc,d} - \hat{\gamma}_t^x - \hat{m} e_t). \] (A.42)

A.3 Fiscal and Monetary Authorities

A.3.1 Fiscal Policy Rules

From (38), (39), and (36):
\[ \hat{g}_t^c = \rho_{gc} \hat{g}_{t-1} + (1 - \rho_{gc}) \phi_{gc} \hat{b}_{t-1} + \varepsilon_{t}^{gc}, \] (A.43)

\[ \hat{g}_t^i = \rho_{gi} \hat{g}_{t-1} + (1 - \rho_{gi}) \phi_{gi} \hat{b}_{t-1} + \varepsilon_{t}^{gi}, \] (A.44)

\[ g^c \hat{g}_t^c + g^i \hat{g}_t^i + \frac{b}{\beta} \left( \hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t^d - \hat{\mu}_t^+, \right) = \tau^c \gamma^{c,d,c} \left( \hat{\zeta}_t^c + \hat{c}_t \right) + \tau^d \hat{L} \left( \hat{w}_t + \hat{L}_t \right) \] (A.45)

\[ + \tau^k \rho^k \mu \left( \hat{r}_t^k + \hat{k}_{t-1} - \hat{\mu}_t^+, \right) + \tau^k \hat{d}_t + bb_t, \]

where
\[ \hat{d}_t = \hat{d}_t^c + \hat{d}_t^{m,c} + \hat{d}_t^{m,i} + \hat{d}_t^x. \] (A.46)

From (37):
\[ \hat{k}_{t}^g = \frac{1 - \delta_g}{\mu_{z^+}} \left( \hat{k}_{t-1}^g - \hat{\mu}_{z^+, t} \right) + \left[ 1 - \frac{1 - \delta_g}{\mu_{z^+}} \right] \left( \hat{g}_t^i + \hat{\gamma}_t^i \right), \] (A.47)

where
\[ \hat{g}_t^i = \rho_{gi} \hat{g}_{t-1} + \varepsilon_{t}^{gi}, \] (A.48)
A.3.2 Monetary Policy Rule

From (40):

\[
\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \phi_{r\pi} \hat{z}^\pi_{t-1} + (1 - \rho_r) \phi_{r\eta} \hat{y}_t + \varepsilon^R_t. \tag{A.49}
\]

A.4 Aggregation and Market Clearing

A.4.1 Aggregate Production Equation

From (41):

\[
\hat{y}_t = \theta \left[ \hat{c}_t + \alpha \hat{u}_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{L}_t + \alpha \hat{g} \hat{k}_{t-1} - (\alpha + \alpha_g) \hat{\mu}_{z+,t} + \alpha \hat{\mu}_{\psi,t} \right], \tag{A.50}
\]

where \( \theta \equiv 1 + \Theta / y \), and

\[
\hat{\mu}_{z+,t} = \rho_{\mu_z^+} \hat{\mu}_{z+,t-1} + \varepsilon_{\mu_{z+,t}}, \tag{A.51}
\]

\[
\hat{\mu}_{\psi,t} = \rho_{\mu_{\psi}} \hat{\mu}_{\psi,t-1} + \varepsilon_{\mu_{\psi,t}}. \tag{A.52}
\]

A.4.2 Goods Market Equilibrium Condition

From (42):

\[
y \hat{y}_t = \left(1 - \omega_c\right) \left( \gamma^{c-d} \right)^{\eta_c} \left[ \hat{c}_t + \eta_c \hat{\gamma}^{c-d}_t \right] + (1 - \omega_i) \left( \gamma^i \right)^{\eta_i} \left( \hat{u}_t + \eta_i \hat{\gamma}^i_t \right) \\
+ \left(1 - \omega_i\right) \left( \gamma^i \right)^{\eta_i-1} \frac{k^{f,k}}{\mu_{z+} \mu_{\psi}} \hat{u}_t + g^{\hat{\gamma}^{d}_t} + g^{\hat{\gamma}^i_t} + y^* \left( \hat{y}_t + \hat{\hat{z}}^*_t - \eta_f \hat{\gamma}^{x,*}_t \right), \tag{A.53}
\]

where

\[
\hat{\hat{z}}^*_t = \rho_{\hat{z}^*} \hat{\hat{z}}^*_{t-1} + \varepsilon_{\hat{z}^* t}. \tag{A.54}
\]
A.5 Measurement Equations

\[
\ln \pi_{t, \text{data}}^d = 100 \hat{\mu}_t^d + \varepsilon_{\pi, t}^\text{me} \tag{A.55}
\]

\[
\ln \pi_{t, \text{data}}^c = 100 \hat{\mu}_t^c + \varepsilon_{\pi, t}^\text{me} \tag{A.56}
\]

\[
\ln \pi_{t, \text{data}}^i = 100 \hat{\mu}_t^i + \varepsilon_{\pi, t}^\text{me} \tag{A.57}
\]

\[
\ln \pi_{t, \text{data}}^x = 100 \hat{\mu}_t^x + \varepsilon_{\pi, t}^\text{me} \tag{A.58}
\]

\[
\Delta \ln Y_{t, \text{data}} = 100 \left( \Delta \hat{y}_t + \hat{\mu}_t^x \right) + \varepsilon_{y, t}^\text{me} \tag{A.59}
\]

\[
\Delta \ln C_{t, \text{data}} = 100 \left[ \hat{\mu}_t + \ln \mu + \eta_c \left( \hat{\pi}_t^{\text{c}} - \hat{\pi}_t^d \right) - \frac{c_{t, \text{c}}}{c_t} \eta_c \left( \hat{\pi}_t^{\text{mc}} - \hat{\pi}_t^d \right) + \Delta \hat{\epsilon}_t \right] + \varepsilon_{c, t}^\text{me} \tag{A.60}
\]

\[
\Delta \ln I_{t, \text{data}} = 100 \left[ \hat{\mu}_t + \ln \mu + \eta_c \left( \hat{\pi}_t^{\text{c}} - \hat{\pi}_t^d \right) - \frac{c_{t, \text{m}}}{c_t} \eta_c \left( \hat{\pi}_t^{\text{mc}} - \hat{\pi}_t^d \right) + \Delta \hat{\epsilon}_t \right] + \varepsilon_{i, t}^\text{me} \tag{A.61}
\]

\[
\Delta \ln G_{t, \text{data}} = 100 \left( \hat{\mu}_t + \Delta \hat{g}_t^c \right) + \varepsilon_{g, t}^\text{me} \tag{A.62}
\]

\[
\Delta \ln G_{t, \text{data}}^L = 100 \left( \hat{\mu}_t + \Delta \hat{g}_t^L \right) + \varepsilon_{g, t}^\text{me} \tag{A.63}
\]

\[
\Delta \ln M_{t, \text{data}} = 100 \left[ \hat{\mu}_t + \ln \mu + \eta_c \left( \hat{\pi}_t^{\text{c}} - \hat{\pi}_t^d \right) - \frac{c_{t, \text{c}}}{c_t} \eta_c \left( \hat{\pi}_t^{\text{mc}} - \hat{\pi}_t^d \right) + \Delta \hat{\epsilon}_t \right] + \varepsilon_{m, t}^\text{me} \tag{A.64}
\]

\[
\Delta \ln X_{t, \text{data}} = 100 \left( \hat{\mu}_t - \eta_f \left( \hat{\pi}_t^{\text{x}} - \hat{\pi}_t \right) + \Delta \hat{g}_t^x + \Delta \hat{\epsilon}_t^x \right) + \varepsilon_{x, t}^\text{me} \tag{A.65}
\]

\[
\Delta \ln L_{t, \text{data}} = 100 \Delta \hat{L}_t + \varepsilon_{\text{L}, t}^\text{me} \tag{A.66}
\]

\[
\Delta \ln \psi_{t, \text{data}} = 100 \left( \hat{\mu}_t + \Delta \hat{\psi}_t \right) + \varepsilon_{\psi, t}^\text{me} \tag{A.67}
\]

\[
\Delta \hat{R}_{t, \text{data}} = 100 \Delta \hat{R}_t \tag{A.68}
\]

\[
\Delta \ln \psi_{\text{ext}, t, \text{data}} = 100 \Delta \hat{\psi}_{\text{ext}, t} + \varepsilon_{\text{ext}, t}^\text{me} \tag{A.69}
\]

\[
\Delta \ln Y_{t, \text{data}}^*, = 100 \left( \Delta \hat{y}_t^* + \Delta \hat{\epsilon}_t^* + \hat{\mu}_t^* \right) + \varepsilon_{y^*, t}^\text{me} \tag{A.70}
\]

\[
\ln \pi_{t, \text{data}}^* = 100 \hat{\pi}_t^* + \varepsilon_{\pi^*, t}^\text{me} \tag{A.71}
\]

\[
\Delta R_{t, \text{data}}^* = 100 \Delta \hat{R}_t^* \tag{A.72}
\]
B Computation of the Steady State

Since \( P^{m,c} = \lambda^{m,c} MC^{m,c} = \lambda^{m,c} SP^s \) and \( SP^s = P^d \) in the steady state, we have:

\[
\gamma^{mc,d} = \lambda^{m,c}.
\]

From \( P^c = \left( (1 - \omega_c) \left( \frac{P^d}{P^d} \right)^{1-\eta_c} + \omega_c \left( \frac{P^{m,c}}{P^d} \right)^{1-\eta_c} \right)^{\frac{1}{1-\eta_c}} \), we also have:

\[
\gamma^{c,d} = \frac{P^c}{P^d} = \left( (1 - \omega_c) + \omega_c \left( \lambda^{m,c} \right)^{1-\eta_c} \right)^{\frac{1}{1-\eta_c}}.
\]

Similarly, since \( P^{m,i} = \lambda^{m,i} MC^{m,i} = \lambda^{m,i} SP^s \) and \( SP^s = P^d \) in the steady state, we have:

\[
\gamma^{mi,d} = \lambda^{m,i}.
\]

From \( P^i = \left( (1 - \omega_i) \left( \frac{P^i}{P^i} \right)^{1-\eta_i} + \omega_i \left( \frac{P^{m,i}}{P^i} \right)^{1-\eta_i} \right)^{\frac{1}{1-\eta_i}} \), we also have:

\[
p^i = \frac{\psi P^i}{P^d} = \left( (1 - \omega_i) + \omega_i \left( \lambda^{m,i} \right)^{1-\eta_i} \right)^{\frac{1}{1-\eta_i}}.
\]

Rearranging the relative prices gives:

\[
\gamma^{mc,c} = \frac{P^{m,c}}{P^c} = \frac{P^{m,c} P^d}{P^d P^c} = \frac{\lambda^{m,c}}{\gamma^{c,d}},
\]

and

\[
\frac{\psi P^i}{P^{m,i}} = \frac{\psi P^i P^d}{P^d P^{m,i}} = \frac{P^i}{\gamma^{mi,d}}.
\]

From the private and public capital law of motion, we know that \( i = \left( 1 - \frac{1 - \delta}{\mu + \mu} \right) k \) and \( g^i = \left( 1 - \frac{1 - \delta}{\mu + \mu} \right) k^g \). Let \( \Gamma_1 = (1 - \omega_c) (\gamma^{c,d})^{\eta_c} \), \( \Gamma_2 = \omega_c \left( \frac{\gamma^{c,d}}{\lambda^{m,c}} \right)^{\eta_c} \), \( \Gamma_3 = (1 - \omega_i) (p^i)^{\eta_i} \), \( \Gamma_4 = \omega_i \left( \frac{p^i}{\lambda^{m,i}} \right)^{\eta_i} \), \( n = \frac{\psi^c}{\psi^i} \) and \( m = \frac{k^g}{\psi^c} \), then aggregate resource constraint in the steady state can be expressed as:

\[
y = c^d + i^d + g^c + g^i + c^z + i^z
\]

\[
= \left[ \Gamma_1 + \Gamma_2 + n + \left( 1 - \frac{1 - \delta}{\mu + \mu} \right) m \right] c + \left( \Gamma_3 + \Gamma_4 \right) \left( 1 - \frac{1 - \delta}{\mu + \mu} \right) k.
\]

(B.1)

Let \( \Pi_t^d = P^d_t y_t - P^d_t mc_t (y_t + \Theta) \) and \( \Pi^d = 0 \), and using the relation \( P^d = \lambda^d P^d mc \), then we obtain:

\[
\Theta = \left( \lambda^d - 1 \right) y,
\]
which immediately implies $\lambda^d = \theta$. Hence we can compute $y$ as:

$$y = \frac{L}{\lambda^d} \left( \frac{1}{\mu \Phi} \right)^\alpha \left( \frac{1}{\mu z^+} \right)^{\alpha + \Psi} \left( \frac{k}{L} \right)^\alpha (k^d)^{\alpha g}.$$  \hfill (B.2)

Using $mc^d = \frac{1}{\lambda^d} = \frac{r^k}{\alpha \mu z^+ (\mu z^+ - \alpha g (k^d)^{\alpha g}} (\frac{\bar{w}}{\bar{p}})^{1-\alpha}$, we derive:

$$\frac{k}{L} = \frac{\alpha}{1 - \alpha} \frac{\bar{w}}{\mu z^+ \mu \Phi} = \frac{\alpha}{1 - \alpha} \frac{\mu z^+ \mu \Phi}{(\mu z^+)^{1-\alpha - \alpha g} (mc)^{\alpha g}} \left[ \frac{\alpha (1 - \alpha)^{1-\alpha} (\mu z^+)^{-\alpha g} (mc)^{\alpha g}}{\lambda^d \bar{p}^k} \right]^{\frac{1}{1-\alpha}}$$

$$= (\mu z^+)^{\frac{1-\alpha - \alpha g}{1-\alpha}} \frac{\mu \Phi}{\lambda^d \bar{p}^k} \left[ \frac{\alpha m^{\alpha g}}{\lambda^d \bar{p}^k} \right]^{\frac{1}{1-\alpha}} c^{\frac{\alpha g}{1-\alpha}}, \hfill \text{(B.3)}$$

and

$$\bar{w} = \frac{1 - \alpha}{\alpha} \frac{k}{L} \frac{r^k}{\mu z^+ \mu \Phi}$$

$$= (1 - \alpha) \left[ \frac{1}{\lambda^d} \left( \frac{\bar{p}^k}{\mu z^+} \right)^\alpha \left( \frac{m}{\mu z^+} \right)^{\alpha g} \right]^{\frac{1}{1-\alpha}} c^{\frac{\alpha g}{1-\alpha}}. \hfill \text{(B.4)}$$

In the steady state, the real wage over the wage markup $\lambda_w$ is set to satisfy the first-order condition for the household’s choice of labor input:

$$A_L L^{\sigma_l} = \psi_{z^+} \left( 1 - \tau^l \right) \frac{\bar{w}}{\lambda_w}. \hfill \text{(B.5)}$$

Using the first-order condition with respect to $c_t$, we obtain:

$$A_L L^{\sigma_l} = \frac{1}{\bar{c}} \frac{1 - \tau^l}{(1 + \tau^c) (\mu z^+ - h) \gamma c d \lambda_w}$$

where $\bar{c} = c + \nu g c = (1 + vn) c$. Using (B.4) and (B.5), the steady-state hours worked can be expressed as:

$$L = \frac{\left( 1 - \tau^l \right) (\mu z^+ - \beta h)}{(1 + \tau^c) (\mu z^+ - h) (1 + vn) \gamma c d \lambda_w} \frac{1}{\lambda^d} \left( \frac{\bar{p}^k}{\mu z^+} \right)^\alpha \left( \frac{m}{\mu z^+} \right)^{\alpha g} \frac{1}{1-\alpha} \frac{1}{c^{\frac{1}{1-\alpha}}} \left( \frac{\alpha g + \alpha - 1}{1-\alpha} \right). \hfill \text{(B.6)}$$

Let

$$DA = \frac{\left( 1 - \tau^l \right) (\mu z^+ - \beta h)}{(1 + \tau^c) (\mu z^+ - h) (1 + vn) \gamma c d \lambda_w} \frac{1}{\lambda^d} \left( \frac{\bar{p}^k}{\mu z^+} \right)^\alpha \left( \frac{m}{\mu z^+} \right)^{\alpha g} \frac{1}{1-\alpha} \frac{1}{c^{\frac{1}{1-\alpha}}} \left( \frac{\alpha g + \alpha - 1}{1-\alpha} \right), \hfill \text{(B.7)}$$

then (B.6) can be rewritten as:

$$L = DA \times c^{\frac{1}{1-\alpha}} \left( \frac{\alpha g + \alpha - 1}{1-\alpha} \right). \hfill \text{(B.8)}$$
Combining (B.1) and (B.2) yields:

\[
L \left( \frac{k}{L} \right) \left[ \frac{1}{\lambda^d} \left( \frac{1}{\mu_\Psi} \right)^\alpha \left( \frac{1}{(1 + \mu_+)} \right)^{\alpha + \alpha_g} \left( \frac{k}{L} \right)^{\alpha - 1} (k \gamma)^\alpha_g - \left( \Gamma_3 + \Gamma_4 \right) \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) \right] = \left[ \Gamma_1 + \Gamma_2 + n + \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) m \right] c.
\]

Using (B.3), we obtain:

\[
L \left( \mu_+ \right)^{\frac{1}{1 - \alpha} - \frac{\alpha - \alpha_g}{\alpha}} \mu_\Psi \left[ \frac{\alpha m^{\alpha_g}}{\lambda^d} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{\mathbf{r}^k}{\mu_+ + \mu_\Psi \alpha} - \left( \Gamma_3 + \Gamma_4 \right) \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) \right]^{\alpha_g} = \left[ \Gamma_1 + \Gamma_2 + n + \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) m \right] c.
\]

Let

\[
DB = \left( \mu_+ \right)^{\frac{1}{1 - \alpha} - \frac{\alpha - \alpha_g}{\alpha}} \mu_\Psi \left[ \frac{\alpha m^{\alpha_g}}{\lambda^d} \right]^{\frac{1}{1 - \alpha}} \left[ \frac{\mathbf{r}^k}{\mu_+ + \mu_\Psi \alpha} - \left( \Gamma_3 + \Gamma_4 \right) \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) \right],
\]

and

\[
DC = \left[ \Gamma_1 + \Gamma_2 + n + \left( 1 - \frac{1 - \delta_g}{\mu_+} \right) m \right],
\]

then (B.9) can be rewritten as:

\[
DA \times DB \times c \left[ \frac{\alpha_g + n - 1}{\lambda^d} \right]^{\frac{\alpha_g}{1 - \alpha} + \frac{\alpha_g}{\alpha}} = DC \times c.
\]

Let \( DD = \frac{1}{\sigma_1} \left( \frac{\alpha + \alpha_g - 1}{1 - \alpha} \right) + \frac{\alpha_g}{\alpha} - 1 \), then \( c \) is a solution to:

\[
c = \left[ \frac{DC}{DA \times DB} \right]^{\frac{1}{\lambda^d}}.
\]

Using the solution to (B.12), we can compute \( L \) from (B.8). Subsequently, (B.3) gives \( k \), and (B.1) gives \( y \).
Figure C.1. Prior-posterior plots for the benchmark model (M1): Black lines: posterior distribution; gray lines: prior distribution; dotted lines: modes of posterior distributions.
Figure C.2. Prior-posterior plots for the benchmark model (M1): Black lines: posterior distribution; gray lines: prior distribution; dotted lines: modes of posterior distributions.
Figure C.3. Prior-posterior plots for the benchmark model (M1): Black lines: posterior distribution; gray lines: prior distribution; dotted lines: modes of posterior distributions.