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Monetary Policy Regime Shifts Under the Zero Lower Bound: An Application of a Stochastic Rational Expectations Equilibrium to a Markov Switching DSGE Model

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Abstract

I extend a simple new Keynesian model with the Markov-switching-type Taylor rule introduced by Davig and Leeper (2007) by incorporating the constraint of the zero lower bound (ZLB), using the concept and algorithms of the stochastic rational expectations equilibrium proposed by Billi (2013). According to the calibration, when an economy does not face the ZLB constraint, there is no gap in the fluctuation of output and inflation between stochastic expectations and perfect foresight because of the linear policy functions. In contrast, once negative aggregate demand shocks make the nominal interest rate hit the ZLB under stochastic expectations, unlike perfect foresight, intensifying uncertainty plays an important role in further declines of the output and price level even in response to the same shock, regardless of the monetary policy regime adopted. The calibration also indicates the possibility that the steady states of a model, in the absence of the ZLB, are underestimated in periods of deflation, since the means often used as estimates of the steady states are biased downward from these. The analysis sheds light on an exit strategy from the zero interest rate policy, since a passive policy regime reduces the expected interest rate and induces both the expected output and the inflation to increase under the ZLB.

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Keywords: new Keynesian model, Markov switching DSGE model, Taylor rule, zero lower bound, stochastic rational expectations, perfect foresight.
JEL codes: C63, E32, E52.

1 Introduction

The effectiveness of monetary policy in stabilizing inflation and business cycles has been extensively studied both theoretically and empirically; the literature discussing these topics is too extensive to mention at present. However, the important issues of monetary policy have continuously surfaced as new problems in the global real economy, and some of these issues have not yet been resolved. One problem concerns how to manage monetary policy subject to the zero lower bound (ZLB) constraint, and another involves the consideration of the dynamics of business cycles under monetary policy regime changes. These matters have frequently been observed in the real economy. These topics are related to the indeterminacy of equilibrium, which is known to induce instability of inflation and real activities.

After the Great Recession developed between 2007 and 2009, the economies of the US and the EU countries encountered the ZLB on the nominal interest rate in a similar fashion to the economy of Japan. The central banks of these countries also had no choice but to implement an unconventional monetary policy, such as the quantitative easing (QE) policy, to stimulate their economies and to withstand deflation. An exit strategy from the zero interest rate policy became not only a big issue within macroeconomic theory but also an actual problem that monetary authorities faced, as the next stage for reverting to the standard monetary policy emerged, along with their economic recovery. In the recent literature, a number of authors have begun to conduct a systematic investigation into the monetary policy implications arising from the ZLB in a rational expectations (RE) model, including a dynamic stochastic general equilibrium (DSGE) model. Research by initiators in this field has been undertaken by Eggertsson and Woodford (2003), Jung et al. (2005) and Kato and Nishiyama (2005)[1]. In particular, a serial study by Adam and Billi (2006, 2007) and Billi (2011, 2012, 2013) has attracted attention because of its numerical approach, since the authors focused on a form of stochastic rational expectations, while most other studies have adopted more deterministic approaches, such as perfect foresight. The above studies have mainly dealt with optimal policy, including commitment and discretionary policy subject to the ZLB constraint. The latest work, such as the study by Billi (2013), has, however, struggled with the Taylor rule as well as the optimal policy constrained by the lower bound on the nominal interest rate. According to Adam and Billi (2006, 2007) and Billi (2011, 2012, 2013), the form of expectations, such

[1]Aoki and Ueno (2012) have estimated linear DSGE models with the zero lower bound on nominal interest rates. Their method makes use of forward rate curves in order to take into account the effects of the zero lower bound on equilibrium endogenous variables without relying on nonlinear techniques for solving rational expectation equilibrium. However, most of the related works have adopted nonlinear techniques for solving it due to improving accuracy of the solution around the ZLB.

as perfect foresight or stochastic rational expectations, is an important factor in the determination of equilibrium in the case of a non-linear DSGE model caused by the ZLB. This paper follows their study.

On the other hand, monetary policy regime changes could often be observed in an actual economy before the Great Recession. For example, the replacement of a chairman of the Federal Reserve or a governor of the Bank of Japan, etc., indicates the shaping of monetary policy regime shifts. In fact, some empirical studies, including Lubik and Schorfheide (2004), have reported that the US inflation of the 1970s was due to the Federal Reserve's failure to obey the Taylor principle, before Paul Volker, the chairman of the Federal Reserve between 1979 and 1987, steered monetary policy based on the Taylor principle and cleaned up the hyper-inflation caused by his predecessors. The analysis of policy regime change using an RE model can be undertaken by adopting a Markov switching (MS) linear RE (MSLRE) model. The literature on the MSLRE model has recently been an active field in empirical macroeconomics. For example, the research by Davig and Leeper (2007) and Farmer et al. (2009, 2011) is representative of this field. Davig and Doh (2008), Liu et al. (2011) and Schorfheide (2005) estimated the presence of monetary policy change for the US economy using MSLRE models. However, these models could not deal with the constraint of the ZLB.

The purpose of this study is to combine these two theses; accordingly, I extend an MSLRE model to the model subject to the ZLB on the nominal interest rate. In addition, this task indicates the combination of two different types of regime-switching monetary policies. This is because the model subject to the ZLB is regarded as a kind of state-dependent regime-switching one, in which, if an economy belongs to a regime with a positive interest rate, then the interest rate can operate based on the Taylor rule, but if the economy hits the other regime, in which the interest rate is fixed at zero, then the Taylor rule cannot work, regardless of the status of its economic activities. This type of regime-switching model must be a non-linear model, unlike the MSLRE model. The combination of the two kinds of regime-switching policies is, however, likely to occur as explained above. Accordingly, the challenge in my study could contribute to the consideration of the operation of actual monetary policy in the future. In addition, I consider the difference in the effects on the economy with different expectations, since the lack of equilibrium between perfect foresight and stochastic expectations must be non-negligible under the ZLB, as pointed out by Adam and Billi (2007). In contrast, the gap does not exist in the MSLRE models without the ZLB. To this end, I deal with a simple new Keynesian model with the MS-type Taylor rule introduced by Davig and Leeper (2007) and incorporate the ZLB constraint into the model, using the concept and the algorithm of the stochastic rational expectation equilibrium (SREE) proposed by Billi (2013). Furthermore, I calibrate and evaluate the effects of monetary policy regime shifts under the ZLB constraint. [2]

[2]Recently, Arouba et al (2014) and Mertens and Ravn (2014) have considered Markov-Switching DSGE models which incorporate the ZLB on the nominal interest rate. These two studies embed a sunspot shock with two-state discrete Markov process in the models, and the two-state shock generates two equilibria. In Arouba et al (2014), the sunspot shock triggers moves from a targeted-inflation regime to a deflation regime and vice versa, by switching infla-

According to the calibration result, there are two findings. First, there is no gap in the fluctuations of output and inflation between stochastic expectations and perfect foresight due to the linear policy functions, when an economy does not face the ZLB constraint. In contrast, this calibration shows that once-negative aggregate demand shocks make the nominal interest rate hit the ZLB under stochastic expectations, unlike perfect foresight; intensifying uncertainty measured by the volatilities of shocks would further deepen the declines of the endogenous variables even in response to the same shock, regardless of the monetary policy regime adopted. These results suggest that perfect foresight is biased upward, so that the possibility exists that the steady states of a model in the absence of the ZLB are underestimated in periods of deflation, since the means often used as estimates of the steady states are biased downward from those. Second, the calibration shows the validity of a passive monetary policy under the ZLB, unlike one without the ZLB, in which Davig and Leeper (2007), dealing with the MS Taylor rule, showed that the passive policy always makes the output and inflation fluctuate more than the active policy. Since the passive policy regime reduces the expected interest rate and induces both expected output and inflation to increase under the ZLB, it is suggested that the passive policy might be a candidate for an exit strategy from a zero interest rate policy. It is also suggested that a guidance policy to form expectations would play an important role in recovering an economy by contributing to mitigating the uncertainty of an aggregate demand shock, rather than retaining an active monetary policy regime, since the reduction in the volatility of the shock raises the means of the output and inflation.

The rest of the paper is organized as follows. Section 2 presents the model studied in this paper, the concept of equilibrium and an approach to solving the equilibrium. Section 3 explains the calibration method and parameter values used in this study. Section 4 discusses the calibration results of the effects of monetary policy regime changes under the ZLB. Section 5 provides the policy evaluation. Section 6 presents our conclusions.

2 Motivation

In this section, I describe the reasons why stochastic rational expectations should be considered and why the expectations are introduced into a regime-switching DSGE model in order gain more insights for the remainder of the paper. First of all, I start by offering brief definitions of stochastic rational expectations and their counterpart: perfect foresight. The perfect foresight of endogenous variables is derived using a policy function with the expected value of the exogenous variables, structural shocks, as the domain of the function, as shown below:

tion targeted value of the Taylor rule. In Mertens and Ravn (2014), the two-state discrete sunspot shock determines one out of two regimes such as "optimistic expectation" and "pessimistic expectation", and an equilibrium under the pessimistic expectation regime converges to the one under liquidity trap. However, these two studies fix coefficients of the Taylor rule. On the other hand, my study consider the two-state Taylor rule driven by Markov process under the ZLB.

$$E_t y_{t+1} = f\,(E_t \varepsilon_{t+1}),$$

where $y_t + 1$ is an endogenous variable and ε_{t+1} is an exogenous variable. E_t denotes the expectations in period t. On the other hand, the stochastic rational expectations are derived from the integration of the policy function with respect to the exogenous variables as below:

$$E_t y_{t+1} = \int f(\varepsilon_{t+1})\, dp(\varepsilon_{t+1}),$$

where $dp(\varepsilon_{t+1})$ is the density function of the exogenous shocks. As above, the concept of stochastic rational expectations is more realistic than that of perfect foresight, since the size of the uncertainty of the future endogenous variables is taken into account in terms of the given information on the exogenous variables. Figure 1 (a) intuitively illustrates these two concepts. There is no discrepancy in the expected values of the endogenous variables between the two expectations in the case of a linear DSGE model without restrictions such as the ZLB. In the long run, their unconditional expected value and steady states are equivalent in this case, since the distribution of the endogenous variables is symmetrical. Accordingly, we do not need to select the stochastic rational expectations as a means of calculating the expectations with an effort.[3]

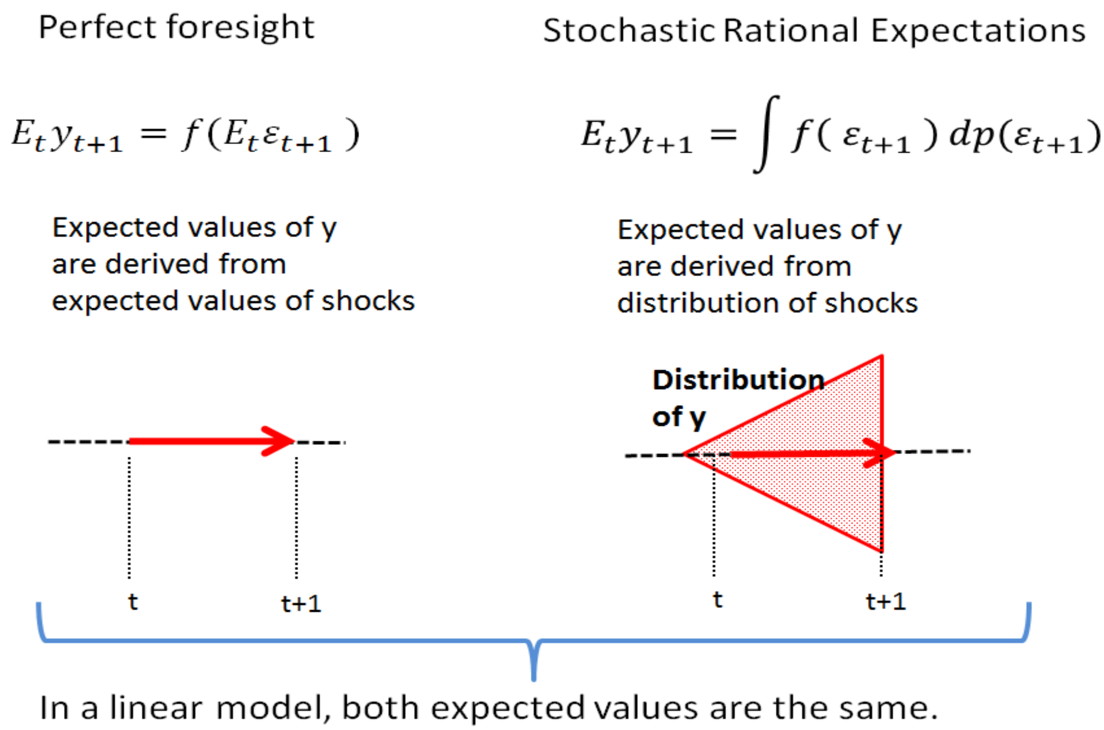
However, the situation must be different in the case of the model with the ZLB constraint, as shown in Figure 1 (b). Under perfect foresight, the expected values of the endogenous variables are not affected by the ZLB, when the policy function derived from the expected value of the structural shocks does not hit the ZLB. In contrast, under stochastic rational expectations, the ZLB truncates the distribution and changes the expected values from the steady state in the long run. In addition, the size of the expected values must depend on the size of the distribution truncated by the ZLB. The bigger the distribution is, the greater the discrepancy between the expected values and the steady states in the long run. Accordingly, we have to take account of the size of the distributions of the endogenous variables and that of the exogenous variables that found the former under the ZLB. Otherwise, the simulation might produce a significant error in a policy analysis.

Now, I turn to the case of a regime-switching DSGE model. In the MS-DSGE developed by Davig and Leeper (2007), there is no difference in the expected values conditional on the specified regime between perfect foresight and stochastic rational expectations, even though the sizes of distribution differ between the different regimes, because the model is a linear one without the ZLB constraint, as shown in Figure 2 (a). Accordingly, the unconditional expected values of the endogenous variables calculated from the equation below are equivalent between the two expectations:

[3]Figure 1, Figure 2, and Figure 8 have been drawn to make the concept of stochastic rational expectations and regime-switching DSGE models easily understood, based on Kosuke Aoki's comments. The author acknowledges Kosuke Aoki.

Figure 1: Perfect foresight vs. Stochastic Rational Expectations

(a) Under the non ZLB



(b) Under the ZLB

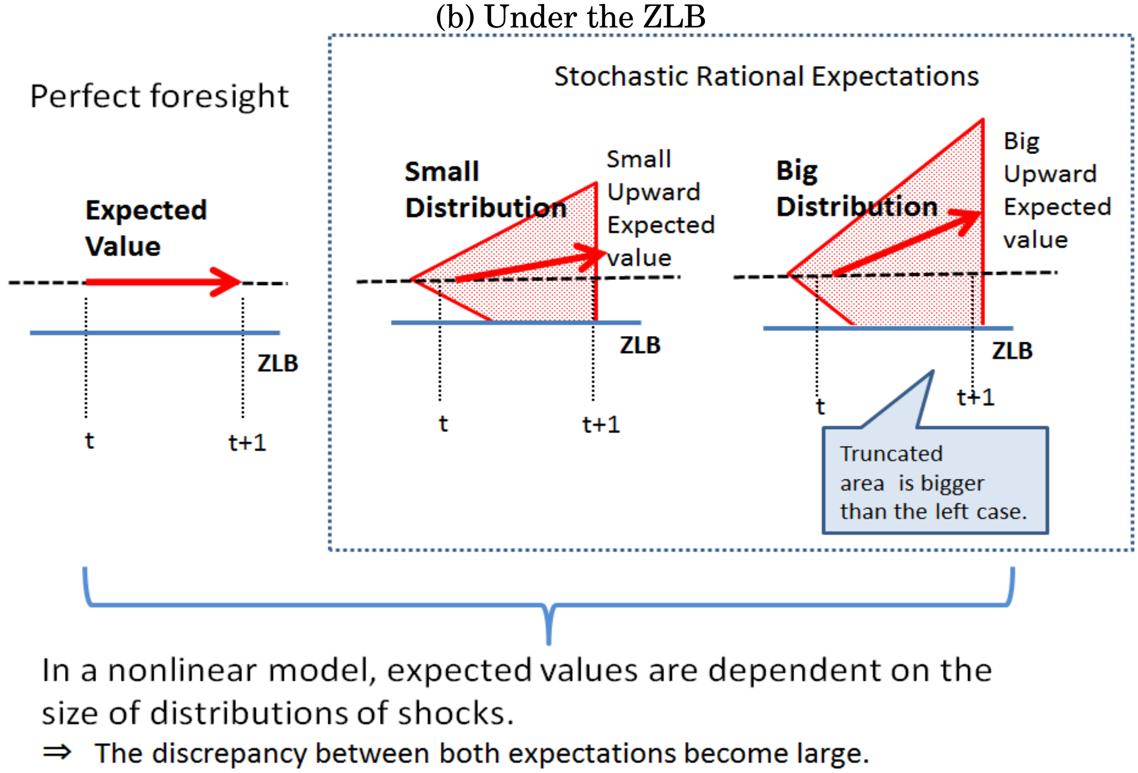
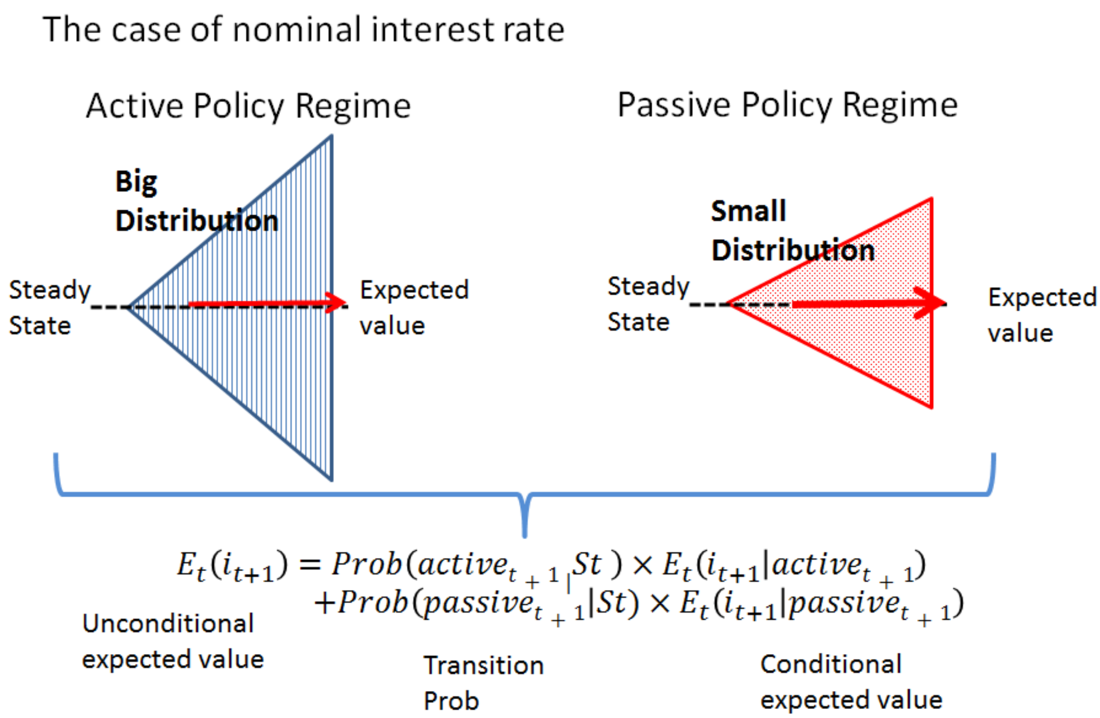
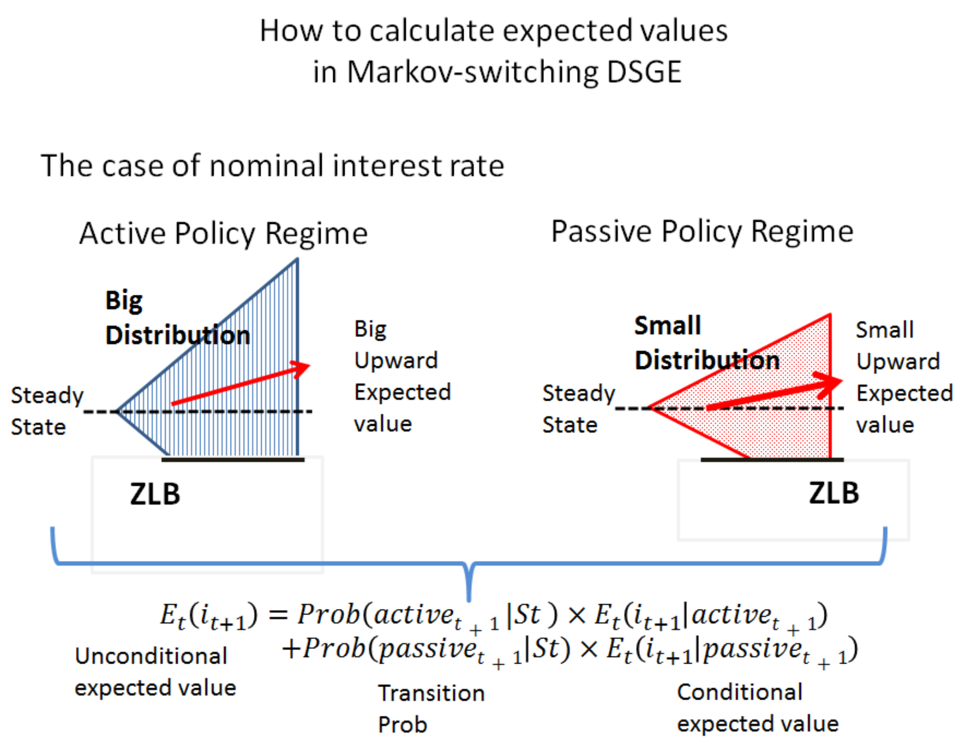


Figure 2: Expected values of Markov-swiching DSGE model under stochastic rational expectations

(a) Under the non-ZLB



(b) Under the ZLB



$$E_t(y_{t+1}) = \sum_{i=1}^{N} Prob(S_{i,t+1}|S_t)\, E_t(y_{t+1}|S_{i,t+1}), \text{ for } i = 1 \cdots N,$$

where $S_{i,t}$ denotes a regime variable when the regime in period t belongs to regime i.

However, the unconditional expected values differ between the two expectations in the regime-switching DSGE model under the ZLB, as shown in Figure 2 (b). This is because the area truncated by the ZLB depends on the size of the distribution of the endogenous variables conditional on the specified regime. This indicates that the policy evaluation seems to be different in each regime as a result of having different expected values. For example, the evaluation of the passive policy might be changed from negative to positive depending on one specified condition, such as deflation or deep recession. This is why we need to consider the stochastic rational expectations in the regime-switching model. It also indicates that whether the stable solution (or a fixed point) of the rational expectations model can be obtained depends on the size of the distributions of the exogenous variables conditional on the regime. Unfortunately, this study does not consider the condition of the stable solution. To overcome this issue, we need to accumulate further general knowledge and calculation techniques of the non-linear DSGE model.

3 Model

3.1 Set-up

This section presents a Markov switching monetary policy process constrained by the ZLB. I use a prototypical new Keynesian dynamic stochastic general equilibrium (DSGE) model, which has frequently been dealt with in the literature studying monetary policy regime shifts and Markov switching DSGE models, for example Davig and Leeper (2007), Farmer et al. (2009, 2011) and Lubik and Schorfheide (2004). This model can be summarized by the following equations, comprised from the private sector and the monetary policy authorities.

Private Sector

The basic elements of the private sector in the model include a representative household, a representative firm that produces a final good and a continuum of monopolistically competitive firms that each produce an intermediate good indexed by j $\in$ [0,1]. The behaviour of the private sector is summarized by two log-linearized equations, namely a Euler equation and a Phillips curve, respectively, indicating the demand and the supply side of the economy.

The Euler equation (1) is obtained from the household' optimal choice of consumption and bond holdings:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t\pi_{t+1}) + g_t, \tag{1}$$

where E_t denotes the expectations operator conditional on the information available at time t. y_t is the output, and π is inflation. These variables are percentage deviations from the zero inflation steady state, in the case of output, from a trend path. In addition, i_t denotes the nominal interest rate expressed as the deviation from the interest rate consistent with the zero inflation steady state. g_t denotes an aggregate demand shock, or the net effects of exogenous shifts on the Euler equations. Parameter σ is the intertemporal substitution elasticity.

The Phillips curve (2) is the inflation dynamics obtained from the optimal price-setting behaviour of monopolistically competitive firms, each of which faces a downward-sloping demand curve for differentiated products. The prices are sticky due to Calvo-style rigidity that allows only a fraction of firms to adjust their prices.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t, \tag{2}$$

where u_t is an aggregate supply shock or exogenous shifts of the marginal cost of production. Parameter β is the household's discount factor. κ is the slope parameter which is determined by

$$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega} \frac{\sigma^{-1}+\tau}{1+\tau\theta}$$

where $1-\omega$ is the randomly selected fraction of firms that adjust their prices. The prices are more flexible as $\omega \to 0$, which makes $\kappa \to \infty$. τ is the elasticity of a firm's real marginal cost with respect to its own output level. θ is the price elasticity of demand substitution among differentiated goods produced by firms.

Both aggregate shocks g_t and u_t are assumed to evolve according to univariate AR(1) stochastic processes with coefficients of ρ_g and ρ_u, respectively.

$$g_t = \rho_g \, g_{t-1} + \varepsilon_{g,t}, \tag{3}$$

$$u_t = \rho_u \, u_{t-1} + \varepsilon_{u,t}. \tag{4}$$

Markov Switching Monetary Policy

Following Davig and Leeper (2007), we assume that the monetary authority sets the short-term nominal rate using a two-state regime-switching Taylor rule, such as

$$i_t(S_t) = \phi_\pi(S_t)\, \pi_t + \phi_y(S_t)\, y_t, \tag{5}$$

$$\phi_\pi(S_t) = \begin{cases} \phi_{\pi 1} > 1 & \text{for } S_t = 1 \\ \phi_{\pi 2} \leqq 1 & \text{for } S_t = 2 \end{cases}$$

$$\phi_y(S_t) = \begin{cases} \phi_{y1>}\phi_{y2} & \text{for } S_t = 1 \\ \phi_{y2} & \text{for } S_t = 2 \end{cases}$$

where the deviation of the level of the nominal interest rate $i_t(S_t)$ is the function of S_t, which is a discrete-valued random variable that evolves according to a two-state Markov chain with a transition matrix given by,

$$P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

where $p_{11} = Pr(S_t = 1|S_{t-1} = 1)$ and $p_{22} = Pr(S_t = 2|S_{t-1} = 2)$. The active, or more aggressive, regime corresponds to $S_t = 1$. The less active, or possibly passive, regime corresponds to $S_t = 2$. The nominal interest rate $i_t(S_t)$ is the function of S_t, since it obeys one of the two Taylor rules decided by one of the two policy regimes S_t in the current period t.

In addition, I assumed that the level of the nominal interest rate, $I_t(S_t)$, has to remain positive for both of the above regimes, i.e., $I_t(S_t) \geqq 0$. This requires the deviation of nominal interest rate $i_t(S_t)$ from the steady state r^* : $i_t(S_t) = I_t(S_t) - r^*$ is to be set up by

$$i_t\left(S_t\right) \geqq -r^*$$

where r^* is the real rate of the deterministic zero inflation steady state, or $r^* = 1/\beta - 1$. By accounting for the ZLB constraint as in the above, the regime-switching Taylor rule (5) can be rewritten as[4]

$$i_t(S_t) = \max\{\, -r^*,\ \phi_\pi(S_t)\,\pi_t + \phi_y(S_t)\,y_t \,\}. \tag{6}$$

Note that the monetary policy rule (6) is a combination of two different types of regime switching monetary policies: state-dependent regime switching and Markov switching. This is because the model subject for the ZLB is regarded as a kind of state-dependent regime-switching one, in which if an economy belongs to a regime in which the interest rate is positive, then the interest rate could operate based on the Taylor rule, but if the economy hits the other regime in which the interest rate is fixed at the zero level, then the Taylor rule cannot work regardless of the situation of its economic activities.

Ignoring the existence of the ZLB constraint, the simple problems (1) through (5) can be solved by standard linear-quadratic methods. In contrast, a global numerical procedure must be used to solve the above problem including (6) instead of (5). Following Billi's (2011, 2012, 2013) algorithm, I will solve it by setting up the model as a static one-period problem, as in Subsection 2.3. Beforehand, an approach using the stochastic rational expectation equilibrium model (SREE) to apply to any kind of DSGE model is explained.

3.2 Stochastic Rational Expectation Equilibrium (SREE)

Most studies solving non-linear DSGE models, for example the model constrained by the ZLB, use a standard procedure in which it is assumed that agents have perfect foresight about the evolution of the economy. However, in the case of

[4]Recently, the simple Taylor rule subject to the ZLB constraint has been considered and evaluated by Taylor and Williams (2010).

non-linear DSGE models including the ZLB constraint, we can observe a non-negligible discrepancy of equilibria between perfect foresight and stochastic rational expectations. Following Billi (2011, 2012, 2013), I adopt the concept of stochastic rational expectations and introduce them into policy functions to solve the underlying MS-DSGE model with the ZLB constraint. By finding a fixed point in the space of policy functions numerically, we can obtain the solution to the problem consisting of the first-order conditions (1) through (4) and (6) with stochastic rational expectations, as explained in the following manner.

(1) Expectations function with stochastic shocks characterized by a continuous density function

In equilibrium, private-sector agents, such as households and firms, and policy makers choose a policy vector x_t (or control variables) based on a policy function $x(z_t)$ and a state vector z_t. Since the policy function $x(z_t)$ does not have an explicit representation in the model, only the numerical results of the policy functions are available. In the model of this study, the policy function consists of the three variables,

$$x(z_t) = (\, y_{t,},\, \pi_t,\, i_t\,),$$

for which the policy decision includes the inflation rate, the output and the nominal interest rate. The state vector is

$$z_t = (u_{t,}, g_t),$$

and includes the exogenous shocks. Using these two notations, the law of motion in equilibrium,

$$z_{t+1} \;=\; h\,(\, z_t,\, x(z_t),\, \varepsilon_{t+1}\,),$$

describes how the future state variables in the economy evolve. The future state z_{t+1} depends on the current state z_t and on the current policy $x(z_t)$, which are known to both the monetary authority and the private sector. It also depends on future shock innovations ε_{t+1}, which are unknown. When the private sector knows the current policy, then it can form expectations about the future policy using policy function $x(z_t)$. Therefore, using the policy function with the current policy x_t and state z_t, an expectations function of the future policy $E_t\, x_{t+1}$ can be given by

$$E_t\, x_{t+1}(z_t) = \int_{\varepsilon_{t+1}} x(z_{t+1})\, f(d\,\varepsilon_{t+1}) \;=\; \int_{\varepsilon_{t+1}} x(\, h\,(\, z_t,\, x(z_t),\, \varepsilon_{t+1}\,)\,)\, f(d\,\varepsilon_{t+1}),$$

where $f(\cdot)$ is the probability density function of the shock innovation, and the inside of the function denotes the dynamics of future state $z_{t+1} = \rho z_t + \varepsilon_{t+1}$. As the shock innovation ε_{t+1} can be integrated out on the right-hand side of the above function, only state z_t remains as the input in the expectations $E_t\, x_t(z_t)$ of the left-hand side. In order to obtain unique and bounded values of expectations, the policy functions are requested to satisfy the assumptions such that $\int\; x^+(z_t) < \infty$

, $\int x^{-}(z_t) < \infty$, and elements of domain of function $x(z_t)$, state vectors z_t and shock innovation ε_t, belong to the Borel set, that is , $z_t,\ \varepsilon_t \in B$, described in chapter 7 of Berttsekas and Shreve (1996).

In the model presented in Section 2.1, the expectations function using a stochastic process such as Eqs. (3) and (4) is rewritten as

$$\begin{aligned} E_t\pi_{t+1}(u_t,\, g_t) &= \int_{\varepsilon_u}\int_{\varepsilon_g} \pi(u_{t+1},\, g_{t+1})\, f(d\,\varepsilon_{g,t+1}|\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{u,t+1}), \\ &= \int_{\varepsilon_u}\int_{\varepsilon_g} \pi(\rho_u\, u_t + \varepsilon_{u,t+1},\, \rho_g\, g_t + \varepsilon_{g,t+1})\, f(d\,\varepsilon_{g,t+1}|\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{u,t+1}), \end{aligned} \tag{7}$$

$$\begin{aligned} E_t\, y_{t+1}(u_t,\, g_t) &= \int_{\varepsilon_u}\int_{\varepsilon_g} y(u_{t+1},\, g_{t+1})\, f(d\,\varepsilon_{g,t+1}|\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{u,t+1}). \\ &= \int_{\varepsilon_u}\int_{\varepsilon_g} y(\rho_u\, u_t + \varepsilon_{u,t+1},\, \rho_g\, g_t + \varepsilon_{g,t+1})\, f(d\,\varepsilon_{g,t+1}|\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{u,t+1}). \end{aligned} \tag{8}$$

On the other hand, in the case of perfect foresight, only the expectations values of states are used as inputs for policy functions so that the expectations function of the control variables is expressed as

$$\begin{aligned} E_t\pi_{t+1}(u_t,\, g_t) &\equiv \pi(E_t\, u_{t+1},\, E_t\, g_{t+1}) \\ &= \pi(\rho_u\, u_t,\, \rho_g\, g_t) \end{aligned} \tag{9}$$

$$\begin{aligned} E_t y_{t+1}(u_t,\, g_t) &\equiv y\,(E_t\, u_{t+1},\, E_t\, g_{t+1}) \\ &= y\,(\rho_u\, u_t,\, \rho_g\, g_t) \end{aligned} \tag{10}$$

where two future states implying the inputs of the function, $E_t\ u_{t+1}$ and $E_t\ g_{t+1}$, are derived from $u_{t+1} = \rho_u u_t$, and $g_{t+1} = \rho_g g_t$, and the realized values of future shock innovations are set as $\varepsilon_{t+1} = 0$. Most studies even using non-linear DSGE models adopt perfect foresight like Eqs. (9) and (10), instead of stochastic rational expectations such as Eqs. (7) and (8). Therefore, to show the significance of the expectations form of models for the effects on output and inflation in the presence of the ZLB, I will also simulate a perfect foresight version and compare it with its counterparts. It is noteworthy that the expectations explained above are just expectations and do not necessarily match the realized value. We allow the presence of forecasting errors between expectations and actual values. Similarly, expectations derived from the perfect foresight without distributions can be allowed to deviate from actual values and to take on forecasting errors, as well as stochastic expectations.

(2) Stochastic Rational Expectations Equilibrium

Generally speaking, a solution to a non-linear DSGE model can be obtained from the concept of the *stochastic rational expectations equilibrium* (SREE) proposed by Billi (2013, p.8). The definition of SREE is given as the following. I apply the equilibrium concept to solve the MS-DSGE model with the ZLB constraint.

Definition 1 (SREE) *A stochastic , rational-expectations equilibrium is given by a policy function $x(z_t)$ and corresponding expectation function $E_t x_{t+1}(z_t)$, respectively , which satisfy the equilibrium conditions such as Eqs. (1) through (4) and (6) described in Section 2.1 .*

Adam and Billi (2006, 2007) adopted the concept of the Markov perfect equilibrium (MPE)[5] for solving an equilibrium, whereas Billi (2013) introduced the new concept of SREE. The difference between the theorems is that the SREE explicitly uses stochastic rational expectations for calculating policy functions or best responses, while the MPE can include perfect foresight as well as stochastic rational expectations.

(3) Discretization of the Expectations Function

To set up the model as a simple static one-period maximization problem in which the first order conditions (1) through (4) and (6) can be used to determine the updated policy functions, we transform the above expectations functions into discrete versions of those functions.[6] The state vector, z, is discretized into a grid of interpolation nodes $\{ z_n \in \mathbf{z} \mid n = 1, \cdots, N \}$, where N is the number of nodes. In particular, state vector $\mathbf{z}_t$ consists of two exogenous shocks u_t and g_t in this model, which are represented as $\mathbf{z}\,(\,i,\,j\,) = [\;g(i),\;u(j)\;]$ by using pointwise multiplication, i.e., $N \otimes N$, and dropping subscript t from states $\mathbf{z}_t$. Here, $g(i)$ and $u(j)$ are the values of exogenous shocks at the $i-$th and $j-$th interpolation nodes, respectively. Policy functions are evaluated at intermediate values resorting to linear interpolation such that $\mathrm{x}(\,i,\,j\,) = \mathrm{x}(\,\mathbf{z}(\,i,\,j\,)\,)$, where the matrix with pointwise elements, $\mathrm{x}(i,j)$, has the number of grids, $N \otimes N$. The expectations $E_t\,x_{t+1}$ defined by (7) and (8) are evaluated at each interpolation node using the M node Gauss-Hermite quadrature scheme which is the approximation of normal distribution described in Chapter 7 of Judd (1998).[7] Using the notations of policy functions with pointwise elements $x(\,i,\,j\,)$ and letting x^e denote *one-period-ahead expectations* $E_t\mathrm{x}_{t+1}$, then the *stochastic rational expectations* can be rewritten as

$$y^e(\,g(i),\,u(j)\,) = \sum_{k=1}^{M}\sum_{l=1}^{M}\omega(k)\,\omega(l)\,y(\,u^e(k,i),\,g^e(l,j)\,), \tag{11}$$

[5]The MPE is explained as a subset of subgame perfect equilibria in Appendix C of Acemoglu (2009).

[6]To obtain a discrete version of policy functions as function approximation, we need to use the interpolation technique, which is explained in chapter 6 of Miranda and Fackler (2002).

[7]Gauss-Hermite quadrature scheme is also explained in Chapter 4 of Brandimarte (2006) who provide its MATLAB code through the web page, to which my study is indebted for the calculation algorithm of the Gauss-Hermite quadrature.

$$g^e(l,j) = \rho_g\, g(j) + \varepsilon_g(l),$$

$$u^e(k,i) = \rho_u\, u(i) + \varepsilon_u(k),$$

$$\text{for} \;\; i,\, j \;= 1,\, 2, \cdots,\, N, \quad \text{and} \;\; k,\, l \;= 1,\, 2, \cdots,\, M.$$

where $\omega(k)$, $\omega(l)$ and $\varepsilon(k)$, $\varepsilon(l)$ are the k-th and l-th weights and k-th and l-th nodes of future shock innovations $\varepsilon_{u,t+1}$and $\varepsilon_{g,t+1}$ calculated by the Gauss-Hermite quadrature scheme, respectively. Indexes k and l are integrated out from the expectations by summing up the interpolation nodes $\varepsilon(k)$, $\varepsilon(l)$ of shock innovations on the right-hand side. On the other hand, the pointwise function of perfect foresight defined by (9) and (10) using future state vectors as the domain is given by

$$y^e(\, g(i),\, u(j)\,) = \; y(\, u^e(i),\, g^e(j)\,), \tag{12}$$

$$u^e(i) = \rho_u\, u(i),$$

$$g^e(j) = \rho_g\, g(j).$$

$$\text{for} \;\; i,\, j \;= 1,\, 2, \cdots,\, N$$

The left-hand side in (12) is one period ahead of the expectations of the control variables, and the right-hand side is the policy function including one period ahead of the expectations of the exogenous shocks as the domain.

(4) Static One-Period Problem of a DSGE Model Subject to the ZLB

Using discrete expectations functions (11) or (12), based on stochastic rational expectations or perfect foresight, we numerically determine out a fixed point, which indicates an equilibrium of the model, in the discrete space, $N \otimes N$, of policy functions. From the notation of the policy function and expectations function obtained by pointwise approximations, the prototypical new-Keynesian DSGE model with a standard Taylor monetary policy rule is transformed into a simple static one-period problem in which the first-order conditions are represented as

$$y(\, g(i),\, u(j)\,) = y^e(g(i),\, (j)\,) - \sigma\bigg(\, i(\, g(i),\, u(j)\,) - \pi^e(\, g(i),\, u(j) \bigg) + g(i), \tag{13}$$

$$\pi(\, g(i),\, u(j)\,) = \beta\, \pi^e(\, g(i),\, u(j)\,) + \kappa\, y(\, g(i),\, u(j)\,) + u(j), \tag{14}$$

$$i(\, g(i),\, u(j)\,) = \max\bigg\{ \; - r^*,\;\; \phi_\pi\, \pi(\, g(i),\, u(j)\,) + \phi_y\, y(\, g(i),\, u(j)\,) \; \bigg\}. \tag{15}$$

Solving for an equilibrium thus requires a search for a fixed point of the three policy functions, $y(\, g(i),\, u(j)\,)$, $\pi(\, g(i),\, u(j)\,)$ and $i(\, g(i),\, u(j)\,)$ at each node(i, j) of states $u(i)$ and $g(j)$, using the global numerical procedure proposed by Billi (2011).

3.3 Application of an SREE to an MS-DSGE Model

(1) Expectations Function of an MS-DSGE Model

The expectations function, $E_t x_{t+1}$, shown in the previous subsection, is derived from state vector z_t characterized by a continuous-valued random variable. Since we extend a standard DSGE model fixed under the unique regime into an MS-DSGE model explained in Section 2.1, one more discrete-valued state such as S_t, denoting the monetary policy regime in period t, has to be introduced into the domains of the expectations as well as the policy function. Accordingly, the expectations function in an MS-DSGE model can be rewritten as

$$E_t\, x_{t+1}(z_{t+1},\, S_t = s) = \sum_{r=1}^{L} p_{sr} \int_{\varepsilon_{t+1}} x(\, z_{t+1},\, S_{t+1} = r\,)\, f(d\,\varepsilon_{t+1}) \tag{16}$$

$$\text{for } s,\, r = 1, \cdots, L$$

where $x(\, z_t,\, S_t\,)$ is a policy function including the current regime state S_t, and p_{sr} denotes a transition probability such as $p_{sr} = \Pr(\, S_{t+1} = r \,|\, S_t = s\,)$. L is the number of regimes. In the case of my model, the value of L is set to 2. In a similar way, the stochastic rational expectations of inflation and output shown in (7) and (8) are also rewritten as

$$E_t \pi_{t+1}(\, u_t,\, g_t,\, S_t = s\,) = \sum_{r=1}^{L} p_{sr} \int_{\varepsilon_{gt+1}} \int_{\varepsilon_{ut+1}} \pi(\, u_{t+1},\, g_{t+1},\, S_{t+1} = r\,)\, f(d\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{g,t+1}), \tag{17}$$

$$E_t y_{t+1}(u_t,\, g_t,\, S_t = s) = \sum_{r=1}^{L} p_{sr} \int_{\varepsilon_{gt+1}} \int_{\varepsilon_{ut+1}} y(\, u_{t+1},\, g_{t+1},\, S_{t+1} = r\,)\, f(d\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{g,t+1}). \tag{18}$$

$$\text{for } s,\, r = 1, \cdots, L$$

where exogenous shocks $\varepsilon_{u,t}$ and $\varepsilon_{g,t}$ are assumed to be independent of each other in order to make the model simple, so that the density functions are multiplied such as $f(d\,\varepsilon_{u,t+1})\, f(d\,\varepsilon_{g,t+1})$ on the right-hand side.

Meanwhile, perfect foresight expressions of expectations in an MS-DSGE model are obtained by

$$E_t \pi_{t+1}(u_t,\, g_t,\, S_t = s) = \sum_{r=1}^{L} p_{sr}\, \pi(u_{t+1}, g_{t+1},\, S_{t+1} = r)\,, \tag{19}$$

$$E_t y_{t+1}(u_t,\, g_t,\, S_t = s) = \sum_{r=1}^{L} p_{sr}\, y\left(u_{t+1}, g_{t+1}, S_{t+1} = r\right), \tag{20}$$

$$\text{for } s,\, r = 1, \cdots, L$$

where $u_{t+1} = \rho_u\, u_t$, $g_{t+1} = \rho_g\, g_t$, and the realized values of future shocks are set as $\varepsilon_{t+1} = 0$. Davig and Leeper (2007) and Farmer et al (2009, 2011) solve an MS-DSGE model using the concept of perfect foresight. As shown in Section 4, there is no discrepancy in the calibration results between stochastic expectations and perfect foresight in the case using log-linearized models. However, if we add the model with the ZLB constraint, we can observe a non-negligible gap between them.

(2) Discretization of the Expectations Function

Following the previous subsection, we transform the above expectations functions into a discrete version of those functions. Using the notation of policy functions with pointwise elements $x(i, j)$ and letting x^e, denote one period ahead of expectations $E_t x_{t+1}$, the stochastic expectations conditional on the current regime $S = s$ can be written as

$$y^e(\, g(i),\, u(j)\, S = s) = \sum_{r=1}^{L} p_{sr} \sum_{k=1}^{M} \sum_{l=1}^{M} \omega(k)\, \omega(l)\, y(\, u^e(\, k,\, i\,),\, g^e(\, l,\, j\,),\, S^e = r\,) \tag{21}$$

$$\text{for } s,\, r = 1, \cdots, L \quad \text{and} \quad k,\, l\, = 1,\, 2, \cdots,\, M.$$

where g^e , u^e, and S^e are one period ahead of the expectations of the state variables. $u^e(\, k,\, i\,) = \rho_u\, u(\, i\,) + \varepsilon_u(\, k\,)$, and $g^e(\, k\,,\, i\,) = \rho_g\, g(\, i\,) + \varepsilon_g(\, k\,)$ are calculated by the Gauss-Hermite quadrature scheme. Perfect foresight under the current regime $S = s$ is similarly given by

$$x^e(g(i),\, u(j),\, S = s\,) = \sum_{r=1}^{L} p_{sr}\, x(u^e(i\,),\, g^e(\, j\,),\, S^e = r\,) \tag{22}$$

$$\text{for } s,\, r = 1, \cdots, L$$

where $u^e(i) = \rho_u u(i)$, and $g^e(i) = \rho_g g(i)$. The left-hand side in (22) is one period ahead of the expectations of the control variables, and the right-hand side is the policy function including one period ahead of the expectations of the exogenous shocks and the regime state as the domain.

(3) Static One-Period Problem of an MS-DSGE Model Subject to the ZLB

Using discrete expectation functions (21) or (22) based on stochastic rational expectations or perfect foresight, we numerically determine a fixed point in the discrete space, $N \otimes N$, of policy functions. Using the notation of policy functions with pointwise elements and expectations functions, the new Keynesian

DSGE model with a two-state regime switching Taylor monetary policy rule constrained by the ZLB, i.e., equations (1) through (4) and (6) conditional on the current regime $S = s$, is transformed into a simple static one-period problem in which the first-order conditions are represented as

$$y(\,g(i),\,u(j),\,S=s) = y^e(\,g(i),u(j),S=s\,) - \sigma\Bigg(\,i(\,g(i),u(j),S=s\,) - \pi^e(g(i),u(j),S=s)\,\Bigg) + g(i), \tag{23}$$

$$\pi(\,g(i),u(j),\,S=s\,) = \,\beta\,\pi^e(\,g(i),\,u(j),\,S=s\,) + \kappa\,y(\,g(i),u(j),S=s\,) + u(j), \tag{24}$$

$$i(\,g(i),u(j),\,S=s\,) = \max\Bigg\{\,-r^*,\;\phi_\pi\,\pi(\,g(i),u(j),S=s\,) + \phi_y\,y(g(i),u(j),S=s\,)\,\Bigg\}, \tag{25}$$

$$\text{for } s = 1,\cdots,L.$$

Solving for an equilibrium thus requires a search for fixed point of the three policy functions, $y(\,g(i),\,u(j),\,S=s)$, $\pi(\,g(i),\,u(j),\,S=s)$ and $i(\,g(i),\,u(j),\,S=s)$ at each node *(i,j,s)* of states $u(i)$ $g(j)$ and $S = s$, using the *global numerical procedure* proposed by Billi (2011).

4 Calibration

4.1 Method

Based on the *global numerical procedure*[8] as described in Appendix I of Billi (2011), I calibrate the model explained in Section 2. However, I modify his algorithm to a more efficient method, which consists of the following three steps, to produce the convergence of policy functions much faster and more accurately.

- **Step 1. Solve an MS-DSGE model without the ZLB and with perfect foresight.**

- **Step 2. Solve an MS-DSGE model with the ZLB and perfect foresight using the results of Step 1 as the initial values.**

[8]Heer and Maussner (2009) discerned solution methods along two dimensions: (1) approximate solutions to Euler equations (FOC) vs. those to the model's policy functions and (2) local methods vs. global methods, as shown in Table 1.3 of Heer and Maussner (2009, pp. 59–60). The procedure that I use obtains the solutions to Euler equations as well as those to policy functions. The approximations solved by local methods, such as perturbation methods, are valid only around particular steady-state points. In contrast, global methods, such as the procedure that I use, are globally valid in any area and do not need to use information about the true model at the steady-state point.

- **Step 3**. **Solve an MS-DSGE model with the ZLB and stochastic rational expectations using the results of Step 2 as the initial values.**

When calculating the policy functions, I fix the values of the policy functions for all three control variables at no shock as steady states of these variables, i.e., $x(u(i) = 0,\, g(j) = 0) = 0$. Futhermore, I set the number of grids of the nodes in the state variables, such as g_t and u_t , as $N = 21$, and the number of grids of the nodes in the Gauss-Hermite quadrature scheme as $M = 3$. The convergence tolerance of the prediction error of the three controlled variables is set to $2.5{\times}10^{-3}$ % at each grid in their policy functions. [9]

4.2 Calibration Parameters

I calibrate the model to the US economy employing the main structural parameters ($\beta,\, \sigma,\, \kappa$) and shock parameters ($\rho_u,\, \rho_g,\, \sigma_u$) of Adam and Billi (2006, 2007) except for the standard deviation of the aggregate demand shock (or real rate shock), σ_g , which is twice their values so as to hit the ZLB frequently.[10] The steady states of these three variables are fixed at the same values as zero for all the regimes and models. The parameter values are summarized in Table 1 and serve as the baseline calibration of the model. The implied steady-state real interest rate for parameterization is 3.5 % annually.

Throughout this study, all the variables are expressed in terms of percentage point deviations from deterministic steady-state values. The interest rates and inflation rates are expressed in annualized percentage deviations, while the aggregate demand shock (or real rate shock) and the aggregate supply shock (or markup shock) are stated as quarterly percentages.

The time-varying coefficients ($\phi_{\pi 0},\, \phi_{y0}$,$\phi_{\pi 1},\, \phi_{y1}$) of the Taylor rule changed by regime shifts, except for transition probabilities ($p_{11},\, p_{22}$), are selected from the empirical results by Lubik and Schorfheide (2004). These values are used for calibrations by Davig and Leeper (2007) and Farmer et al. (2009, 2011). Under the fixed regime used for comparisons as shown in the next section, the policy parameters of the non-switching Taylor rule are adopted as the same values as those of the active regime of the MS-Taylor rule. The transition probabilities,

[9]The prediction error (or Euler error) is represented as

$$err \;=\; \gamma^{k+1} - \gamma^{k}$$

whereγ^k is the controlled variable generated from the k-th iteration of Eq. (23) through Eq. (25). Since the the mapping of Eq. (23) through Eq. (25) are essentially non-linear equations, this procedure need not converge even if the fixed point exists. Den Haan and Marcet (1990) propose to iterate on

$$\gamma^{k+1} \;=\; (1-\lambda)\,\gamma^{k} \;+\; \lambda\,\Gamma(\gamma^{k})$$

for some $\lambda \,\in\, (0,\, 1]$ to foster convergence. Here, $\Gamma(\gamma^k)$ is the mapping of Eq. (23) through Eq. (25). Following Appendix I of Billi (2011), the step size λ that is set to achieve stability in iterations is based on chapter 4 of Bertsekas (1999).

[10]The sizes of these parameters are chosen based on Tables 5.1 and 6.1 of Woodford (2003)

Table 1: Parameter Values

parameter	Economic interpretation	assigned value
β	quarterly discount factor	0.9913 = $(1+\frac{3.5\%}{4})^{-1}$
σ	real rate elasticity of output	6.25
κ	slope of the Phillips curve	0.024
$\phi_{\pi 0}$	reaction cofficient of inflation under active regime	2.2
ϕ_{y0}	reaction cofficient of output under active regime	0.5
$\phi_{\pi 1}$	reaction cofficient of inflation under passive regime	0.8
ϕ_{y1}	reaction cofficient of output under passive regime	0.15
ρ_u	AR-coefficient aggregate supply shocks	0.0
ρ_g	AR-coefficient aggregate demand shocks	0.8
σ_u	S.d. aggregate supply shock innovations (quarterly %)	0.154
σ_g	S.d. aggregate demand shock innovations (quarterly %)	3.048
p_{11}	transition probability from active regime to active regime	0.75
p_{22}	transition probability from passive regime to passive regime	0.75

Notes: S.d. denotes standard deviation. The main structural parameters (β, σ, κ) and shock parameters (ρ_u, ρ_g, σ_u) are set based on Adam and Billi (2006, and 2007) except the standard deviation of aggregate demand shock (or real rate shock), σ_g , which is twice their values so as to hit the ZLB frequently. The time-varying coefficients ($\phi_{\pi 0}$, ϕ_{y0},$\phi_{\pi 1}$, ϕ_{y1}) of the Taylor rule changed by regime shifts are selected from empirical result by Lubik and Schorfheide (2004). Transition probabilities, p_{11}, p_{22} are set to 0.75 so as to provoke frequent regime changes in which average shifts are one out of four periods.

p_{11}, p_{22}, are set to 0.75 so as to provoke frequent regime changes in which average shifts are one out of four periods.

5 Policy Evaluation

5.1 Analyses of Policy Functions

Figures 3 (a) and (b) show the policy functions of three control variables, output, inflation and nominal interest rate, reacting to the aggregate demand shock differentiated by the absence or the presence of the ZLB. Panel (a) of this figure focuses on the active regime based on stochastic rational expectations, and panel (b) focuses on the passive regime under the same expectations. The solid line represents the policy functions under the ZLB, while the dashed line represents those not subject to the ZLB. As can be seen from Figures 3 (a) and (b), all the policy functions of both regimes under the non-ZLB dashed line are expressed as linear functions. In contrast, when the nominal interest rate hits the ZLB, i.e., $i = -r^*$, the reactions of output and inflation to the demand shock are kinked and induce a deep decline in both regimes; this is far from the case of linear RE models, as the solid lines show in both the figures.

Figures 4 (a) and (b) show how the gap in the policy functions reacted to the aggregate demand shock between stochastic rational expectations and perfect foresight under each regime. The solid line represents the policy functions based on stochastic expectations, while the dashed line embodies those based on perfect foresight. As seen in panel (a), the gaps in the output and inflation between them increase when the interest rate reaches the ZLB under the active regime. On the other hand, these big gaps are revealed around the lower nominal interest rate, even though it reaches the bound under the passive regime as shown in panel (b).

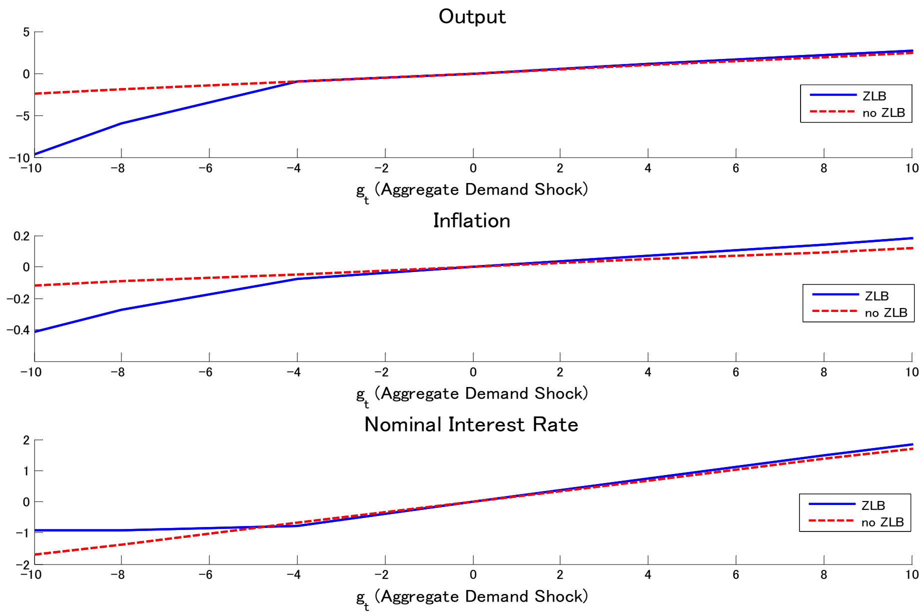
Figure 5 shows a comparison of the functions against the aggregate demand shock between the active policy regime (solid line) and the passive policy regime (dashed line). Panel (a) deals with the case of stochastic expectations. Panel (b) deals with that of perfect foresight. Despite the forms of expectations, the fluctuations of the two variables are much larger for both positive and negative shocks under the passive policy regime than under the active policy regime. However, a decrease in the two variables (the first and second graphs in both panels) under the active policy regime is likely to present the same degree as under the passive regime, when the aggregate shock falls below the -8% level at which the nominal interest rate reaches the ZLB, as shown in the third graph of both panels.

5.2 Comparison of the Impulse Response Functions

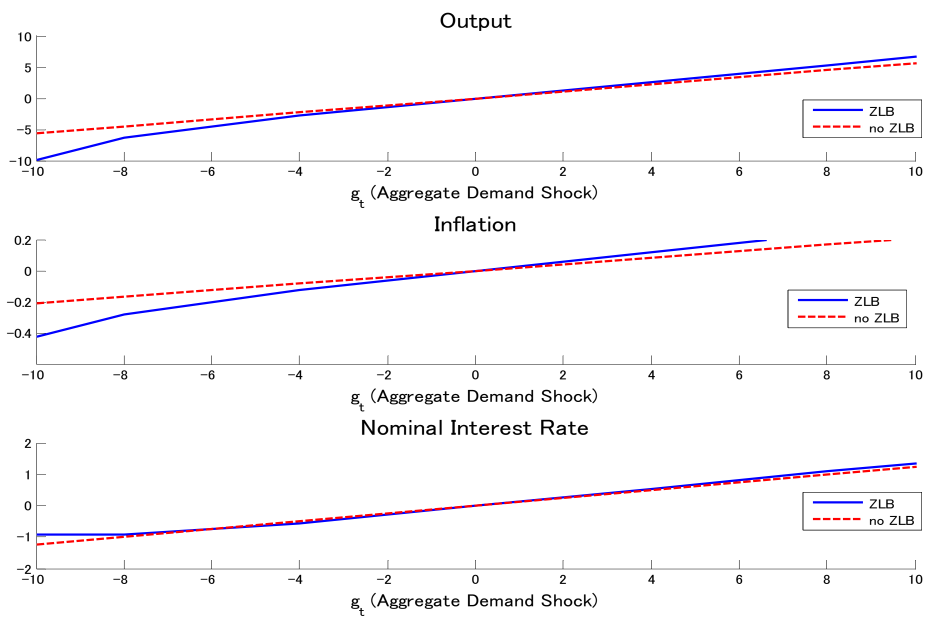
Using the policy functions represented in Figures 3 through 5, the impulse responses in the model are calculated. Figure 6 shows a comparison of three kinds of impulse response functions of the three variables to an aggregate demand shock conditional on the specified regimes. The solid line indicates the response

Figure 3: Policy functions; non-ZLB vs. ZLB

(a) Under Active Regime



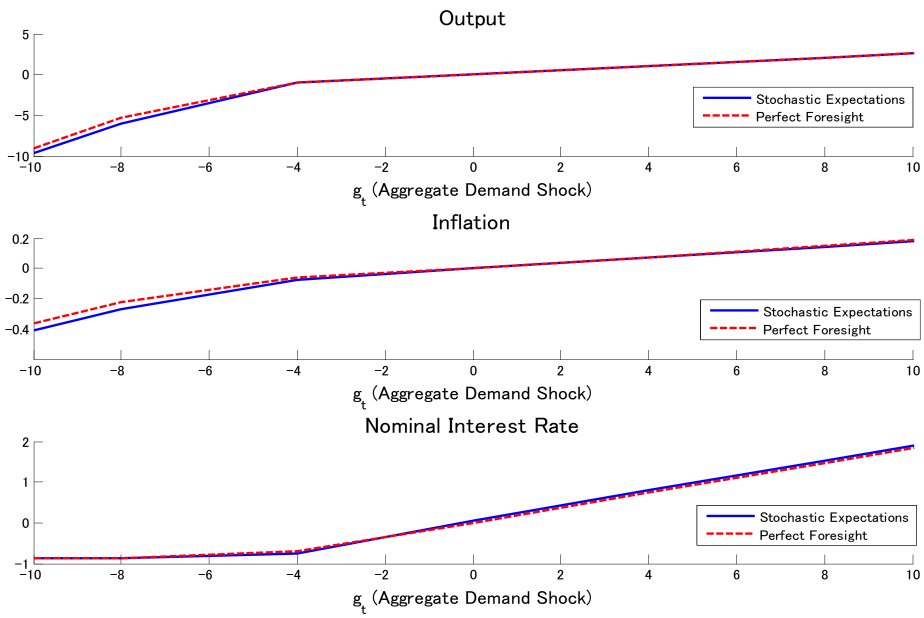
(b) Under Passive Regime



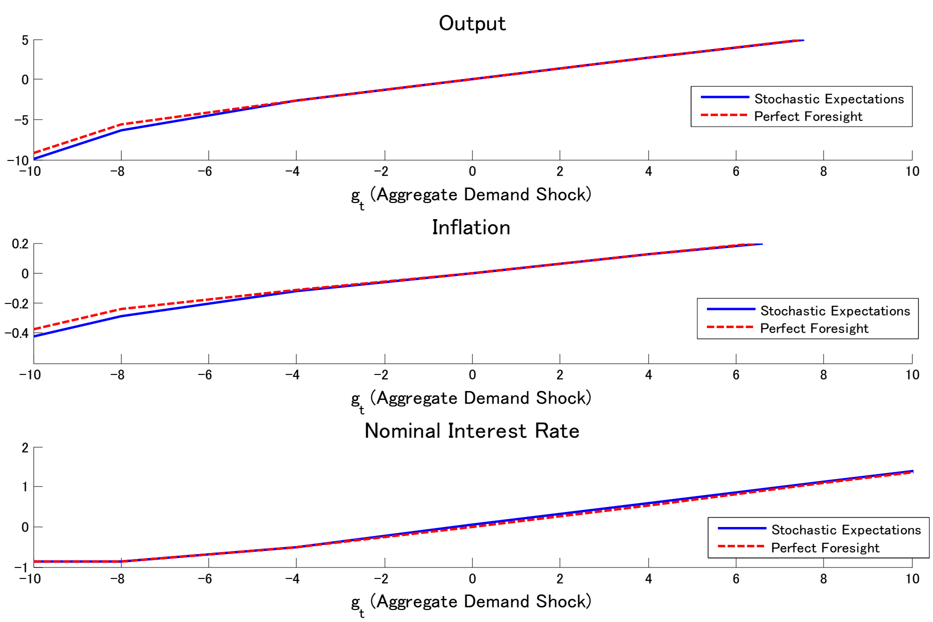
Notes: Solid and dashed lines denote reactions of the three endogenous variables derived from policy functions to aggregate demand shock conditional on that aggregate supply shock is set to zero, with the ZLB and without the ZLB, respectively. Horizontal axis shows the size of aggregate demand shock stated as quarterly percentages. Vertical axis shows the size of reaction of three endogenous variables expressed in terms of percentage point deviations from deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations.

Figure 4: Policy functions; Stochastic Expectations vs. Perfect Foresight

(a) Under Active regime

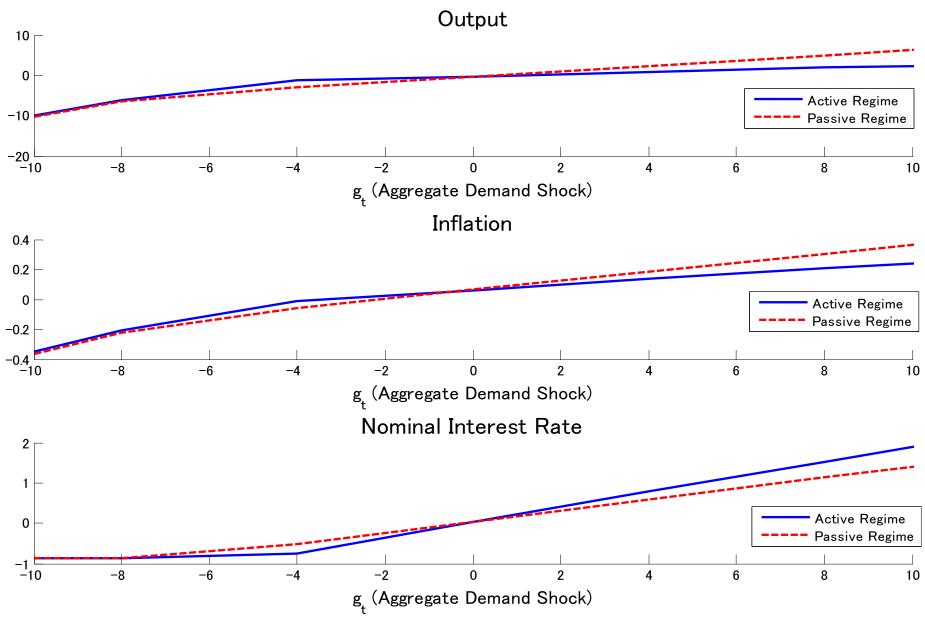


(b) Under Passive Regime

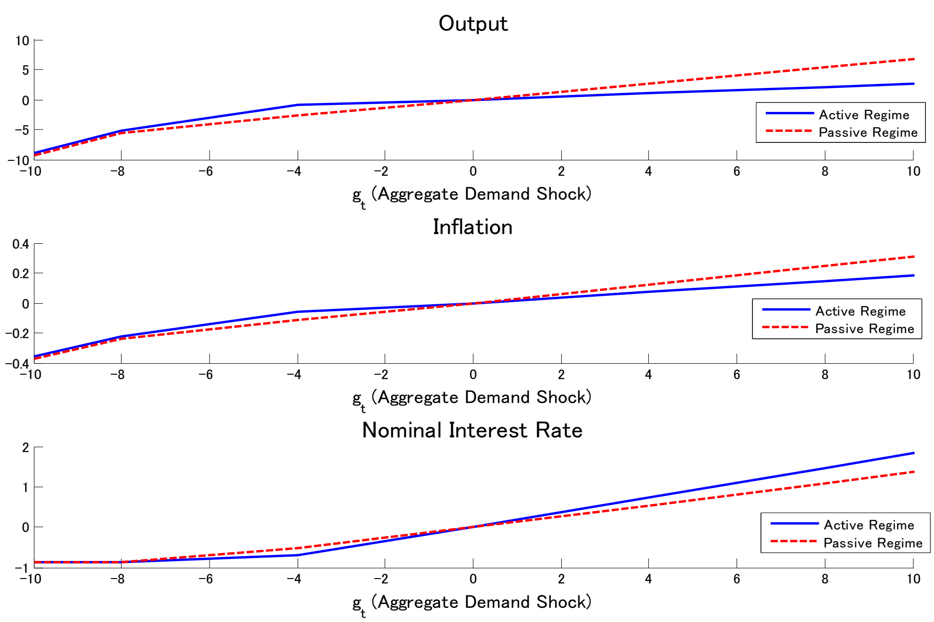


Notes: Solid and dashed lines denote reactions derived from policy functions of the three endogenous variables to aggregate demand shock conditional on that aggregate supply shock is set to zero, under stochastic rational expectations and under perfect foresight, respectively. Horizontal axis shows the size of aggregate demand shock stated as quarterly percentages. Vertical axis shows the size of reaction of three endogenous variables expressed in terms of percentage point deviations from the deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations.

Figure 5: Policy functions; Active Policy Regime vs. Passive Policy Regime
(a) Stochastic Rational Expectation



(b) Perfect Foresight



Notes: Solid and dashed lines denote reactions derived from policy functions of the three endogenous variables to aggregate demand shock conditional on that aggregate supply shock is set to zero, under the active regime and under the passive regime, respectively. Horizontal axis shows the size of aggregate demand shock stated as quarterly percentages. Vertical axis shows the size of reaction of three endogenous variables expressed in terms of percentage point deviations from the deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations.

conditional on the active regime, the dashed line the passive regime and the dotted line the fixed regime, in which the coefficients of the Taylor rule are fixed at the same values without regime switching. The responses to a positive demand shock are drawn in panel (a), in which the reactions of output and inflation are the largest under the passive regime. It is shown that the reactions under the active regime are rather larger than those under the fixed policy regimes. Although the descending order of the three regimes for these reactions to a positive shock is the same as in the case without the ZLB, which was pointed out by Davig and Leeper (2007), it indicates that a regime-switching monetary policy makes the economy fluctuate more than a fixed monetary policy. Meanwhile, in the case of the negative demand shock hitting the ZLB, the responses of those variables would drop by almost the same amount under all three regimes, as shown in panel (b).

Figure 7 shows a comparison of the impulse response functions of the three variables to the aggregate demand shock conditional on active regimes differentiated between stochastic rational expectations and perfect foresight. In panel (a), there is no gap in the reactions of all three variables to a positive shock between stochastic expectations and perfect foresight. This is because the model can be expressed as a linear one under this condition without the ZLB constraint. In contrast, we can observe that the reaction of these functions to the negative aggregate demand shock, which makes the interest rate affect the ZLB, obviously shows a deeper decline in the case of stochastic expectations than in the case of perfect foresight, as can be seen in panel (b).

5.3 Monte Carlo Study of the Model

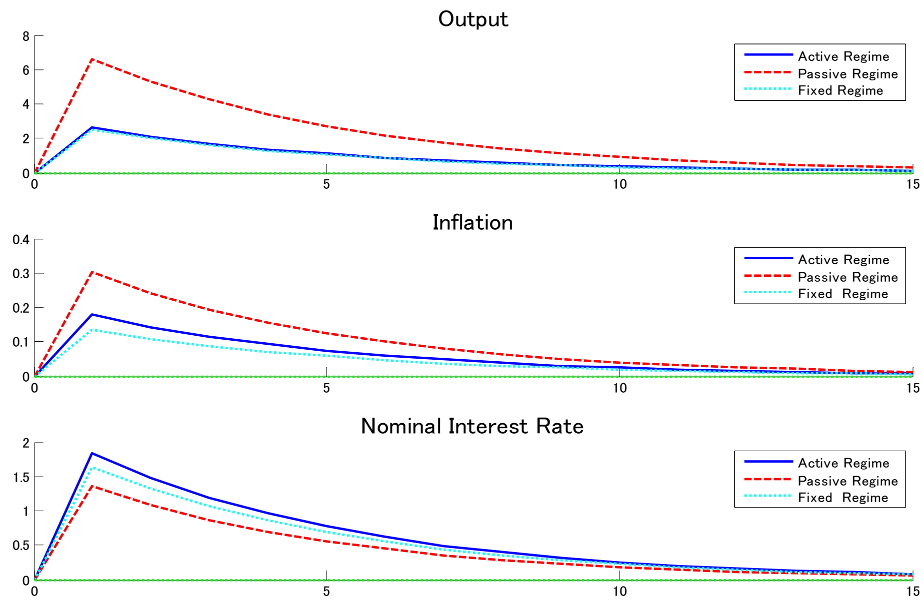
Active Regime vs. Passive Regime

Table 2 summarizes the results of 100,000 replications derived from policy functions under three regimes. Panels (a) and (b) of this table represent the results under the non-ZLB and the ZLB constraints, respectively. The third column shows the means and standard deviations of the three variables of Monte Carlo simulation under the active regime, while the fourth column shows the passive regime case. The fifth column shows the difference between the two regimes. The second row shows those with stochastic rational expectations and the third row those with perfect foresight. In panel (a), we can see that the means of all these variables are close to zero and that the standard deviations of output and inflation under the passive regime are much larger than those under the active regime. This indicates that their means tend to be consistent with their steady states, which are set as zero without the ZLB constraint. In addition, there are no differences in the means and the standard deviations between stochastic rational expectations and perfect foresight.

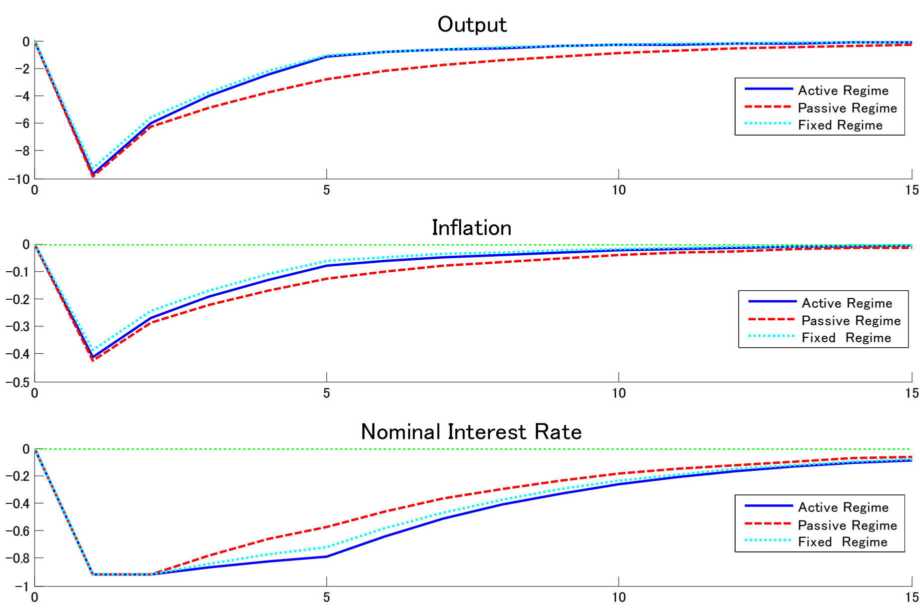
In Panel (b) of Table 2, the simulation results with the ZLB constraint are presented. The standard deviations of output and inflation are almost double the values appearing in panel (a). In addition, the means of output and inflation tend to be below those of their steady state. In particular, the means under the

Figure 6: Impulse response of Aggregate Demand Shock conditional on each Regime

(a) Positive Shock under Stochastic Rational Expectations

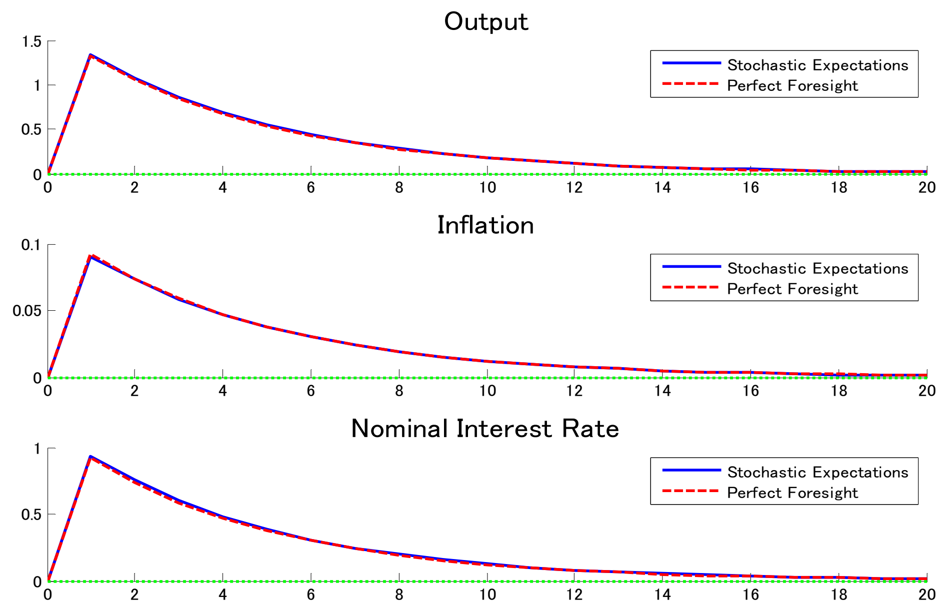


(b) Negative Shock under Stochastic Rational Expectations

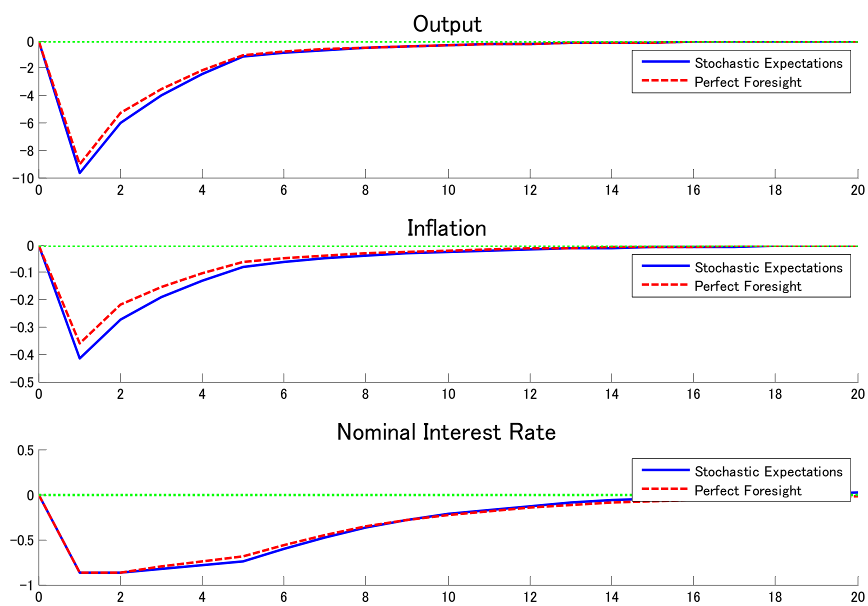


Notes: Horizontal axis shows horizon of periods expressed in terms of quarterly in which the unity size of aggregate demand shock arises at the period 1. Vertical axis shows the size of reaction of three endogenous variables expressed in terms of percentage point deviations from deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations.

Figure 7: Impulse response of Aggregate Demand Shock under each expectations
(a) Positive Shock conditional on Active Policy Regime



(b) Negative Shock conditional on Active Policy Regime



Notes: Horizontal axis shows horizon of periods expressed in terms of quarterly in which the unity size of aggregate demand shock arises at the period 1. Vertical axis shows the size of reaction of three endogenous variables expressed in terms of percentage point deviations from deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations.

active regime decline more than those under the passive regime. Their standard deviations under the active regime are smaller than those under the passive regime. From this table, we can also explicitly observe the definite discrepancy between stochastic expectations and perfect foresight in all three variables.

Figures 8 (a) and (b) illustrate the intuitive interpretation of the simulation results of Table 2. In the case of the absence of the ZLB, as shown in Figure 8 (a), the distribution of the nominal interest rate under the active regime (the striped shaded area) is wider than that under the passive regime (the dotted shaded area). This results in the distribution of the output and inflation under the active regime (the striped shaded area) being narrower than that under the passive regime (the dotted shaded area). The means of these three variables are equivalent to their steady states fixed as the zero value because these three distributions are symmetrical under both regimes. On the other hand, in the case with the presence of the ZLB, as shown in Figure 8 (b), the distributions of the nominal interest rate under both regimes are truncated due to the ZLB constraint. In particular, the area truncated under the active regime (the striped shaded area) is bigger than that under the passive one (the dotted shaded area). This indicates that the means of the nominal interest rates move away from their steady states (zero value) and rise positively. The upward size of the mean under the active regime (dotted line) is bigger than that of the mean under the passive one (solid line). An increase in the means of the nominal rate, in particular under the active regime (dotted line), makes the means of the output and inflation decrease. In addition, the truncated distributions of the nominal interest rate under both regimes result in changing distributions of output and inflation to asymmetry with fat tails in the negative area. In this way, different regimes induce different expected values under the ZLB. It suggests that a passive policy with a bad reputation has a chance to gain a good evaluation as the effective policy lifting up the output and inflation in a situation of deep deflation and that it might be a candidate for an exit strategy from the zero interest rate policy.

In Figures 9 (a) and (b), artificial data conditional on the specified policy regimes generated by Monte Carlo simulations are depicted by distinguishing them according to the presence or the absence of the ZLB, respectively. The same random seed is adopted in panels (a) and (b) to compare the effect of the ZLB constraint, so that the same structural shocks determine the endogenous variables in both graphs. The fluctuations of the variables under the passive regime (the dashed line) have bigger swings than those under the active regime (the solid line) in the case of the absence of the ZLB, as in panel (a). In contrast, positive demand shocks make the reactions of these variables appear as above, whereas negative big shocks cause large declines in these variables, regardless of the policy regimes. The size of decline is bigger in panel (b) than in panel (a); for example, period 80 through 90. As such, the effects of negative shocks on an economy do not depend on the monetary policy regime that is adopted.

Table 2: Monte Carlo Simulations conditional on Specified Regimes

(a) the Case in absence of the ZLB constraint

Expectations	variables	Active		Passive		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
Stoc. Expect.	output	0.00	1.27	0.00	2.87	0.00	-1.60
	inflation	0.00	0.13	0.00	0.16	0.00	-0.03
	interest rate	0.00	0.87	0.00	0.62	0.00	+0.25
Perfect Fore.	output	0.00	1.29	0.01	2.92	-0.01	-1.63
	inflation	0.00	0.13	0.00	0.16	0.00	-0.03
	interest rate	0.00	0.89	0.00	0.64	0.00	+0.25

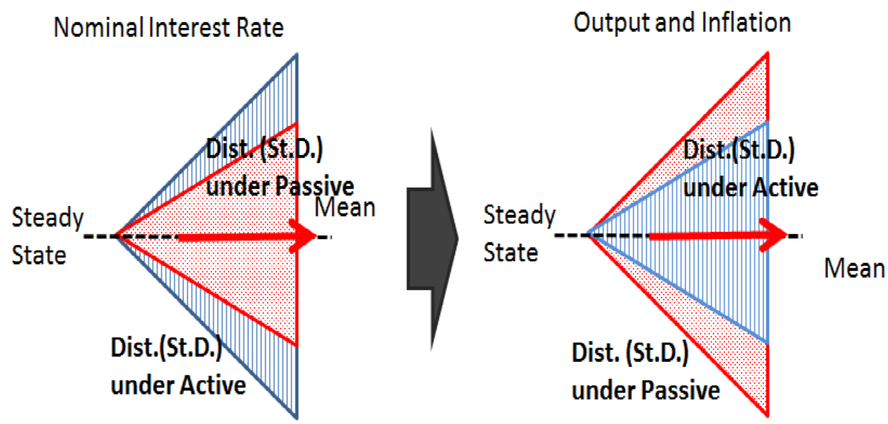
(b) the Case in presence of the ZLB constraint

Expectations	variables	Active		Passive		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
Stoc. Expect.	output	-0.93	3.01	-0.54	4.00	-0.39	-0.99
	inflation	-0.02	0.20	-0.01	0.22	-0.01	-0.02
	interest rate	0.13	0.80	0.07	0.63	+0.06	+0.17
Perfect Fore.	output	-0.57	2.85	-0.12	3.90	-0.45	-1.05
	inflation	-0.01	0.19	0.00	0.21	-0.01	-0.02
	interest rate	0.12	0.78	0.06	0.62	+0.06	+0.16

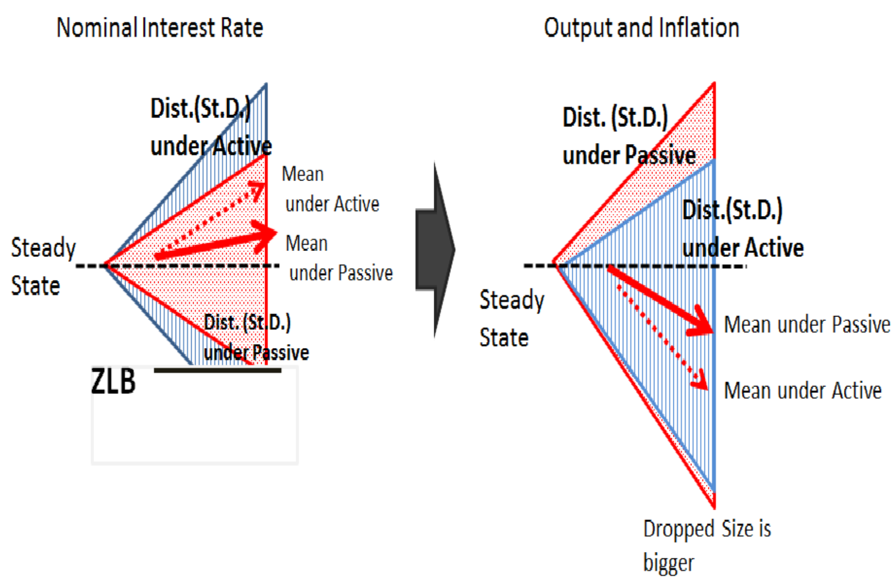
Notes: Means and standard deviations are calculated from 100,000 replications derived from policy functions of the three variables under each regime and under each expectations form. Stoc. Expect. and Perfect Fore. denote stochastic rational expectations and perfect foresight, respectively. Differences means subtraction of the value under the active regime from under the passive regime in each expectations form.

Figure 8: Intuitive Interpretation of Calibration Results; Active Policy vs. Passive Policy

(a) Case in Absence of the ZLB constraint



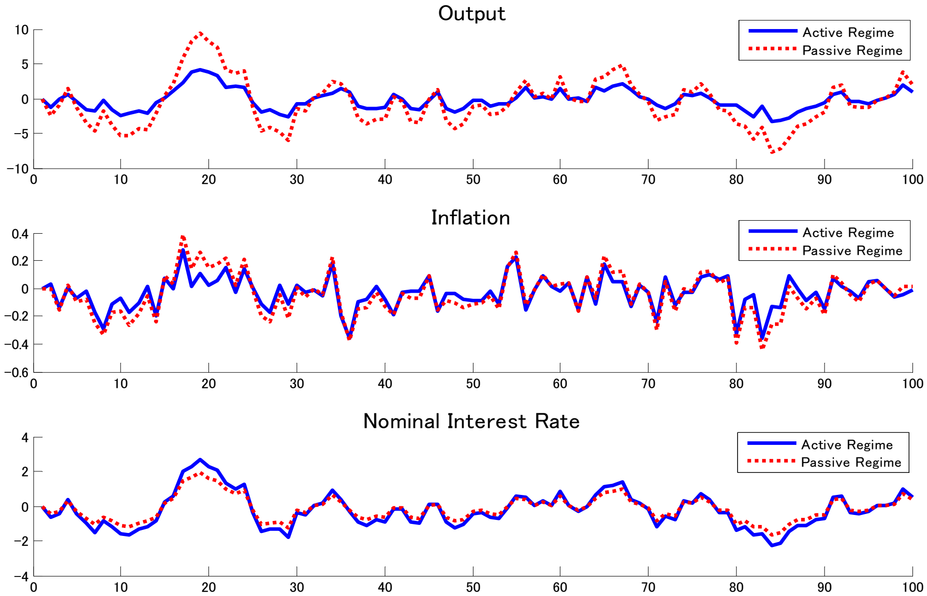
(b) Case in Presence of the ZLB constraint



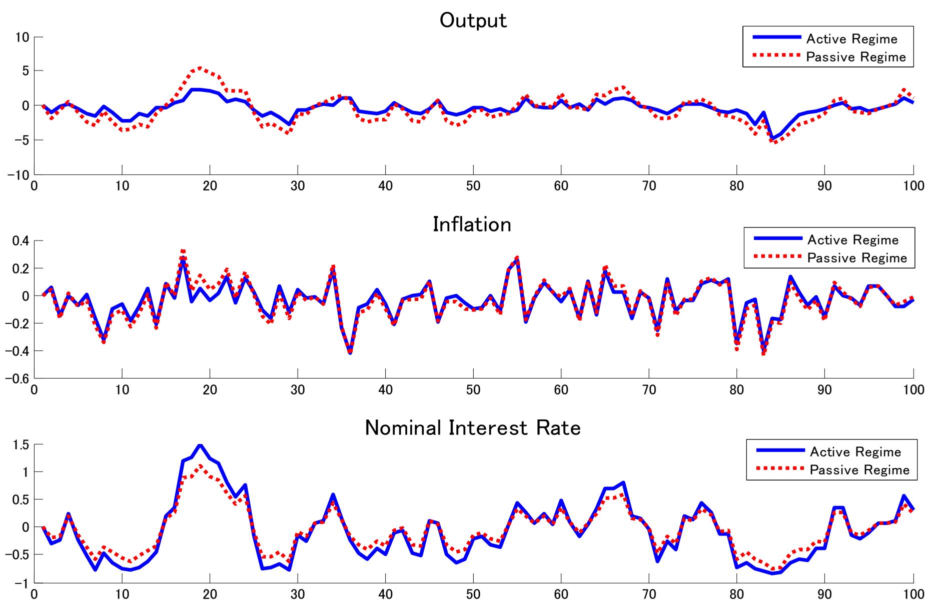
Notes: The above illustrations are summarized from the results of Table 2. The stripe shaded area and the dotted shaded area denote the distributions of endogenous variables under active regime and passive regime, respectively. The solid line and the dotted line denote the means of endogenous variables under active regime and passive regime, respectively.

Figure 9: Calibrations; Active Policy vs. Passive Policy

(a) Case in Absence of the ZLB constraint



(b) Case in Presence of the ZLB constraint



Notes: Artificial data conditional to the specified policy regimes generated from the 100 iterations of Monte Carlo simulations are depicted by distinguishing them according to the presence or the absence of the ZLB, respectively. The same random seed is adopted in panel (a) and (b) for comparing the effect of the ZLB constraint, so that the same structural shocks would determine the endogenous variables in both graphs. Solid line and dashed line represent the active policy regime and the passive policy regime, respectively.

Regime-Switching Policy vs. Fixed Policy

Table 3 summarizes the results of 100,000 replications derived from policy functions for a regime-switching policy and a fixed-regime policy. Panels (a) and (b) represent the results of a mixed-regime policy and a fixed-regime policy differentiated by the absence and presence of the ZLB constraint, respectively. In the case of the absence of the ZLB (panel (a)), the MS Taylor rule indicates an MSLRE model, so that the mean of each variable is consistent with its steady-state value, i.e., zero, regardless of the forms of expectations. Furthermore, the standard deviation of each variable is the same for stochastic rational expectations and perfect foresight. These calibration results are consistent with the theoretical conjecture for the effect of the forms of expectations in an MSLRE model. In addition, the standard deviation is larger under a regime-switching policy than under a fixed monetary policy. This result suggests that a regime-switching policy induces the instability of business cycles, as pointed out by Davig and Leeper (2007). On the other hand, in the case of an economy subject to the ZLB, the effects of these two policies are completely different from the case of its absence. As seen in Table 3, as in Table2, we can also observe the negative values of the means far from steady states for all the variables, regardless of the form of the expectations. The means of the regime-switching policy are smaller than those of the fixed policy. The discrepancy in the standard deviations between the two policies is much smaller than in the case of no ZLB. There are non-negligible sizes in means and standard deviations between stochastic rational expectations and perfect foresight.

In Figures 10 (a) and (b), the artificial data of regime switching and fixed policies generated from Monte Carlo simulation are depicted by distinguishing them according to the presence or the absence of the ZLB, respectively. The same random seed is adopted in panels (a) and (b) to compare the effect of the ZLB constraint, so that the same structural shocks determine the endogenous variables in both graphs. A light shade indicates the passive regime. The fluctuations in the variables under the regime-switching policy (the solid line) have a bigger swing in the passive regime period than under the fixed policy (the dashed line) in the case of the absence of the ZLB, as in panel (a). In contrast, positive demand shocks make the reactions of these variables appear similar to those above, whereas negative big shocks cause big declines of these variables, regardless of the presence of policy regime changes. The size of the decline is larger in panel (b) than in panel (a); for example, period 80 through 90. Accordingly, the effects of negative shocks on an economy do not depend on whether the policy regime changes or remains fixed.

The Impact of Mitigating the Uncertainty of Demand Shocks to an Economy

Here, let us consider some policy implications for recovering recessions under the ZLB. As seen above, the effect of the active policy regime is similar to that of the passive regime. As a simple case, a 20% reduction in the volatility of an aggregate demand shock is introduced into the model. Figure 11 shows that the

Table 3: Monte Carlo Simulations under Regime Switching and Fixed Policies

(a) the Case in absence of the ZLB constraint

Expectations	variables	R.S. Policy		Fixed Policy		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
Stoc. Expect.	output	-0.01	2.22	0.00	1.21	-0.01	+1.01
	inflation	0.00	0.15	0.00	0.12	0.00	+0.03
	interest rate	0.00	0.76	0.00	0.79	0.00	-0.03
Perfect Fore.	output	-0.01	2.21	0.00	1.21	-0.01	+1.01
	inflation	0.00	0.14	0.00	0.12	0.00	+0.02
	interest rate	0.00	0.75	0.00	0.79	0.00	-0.04

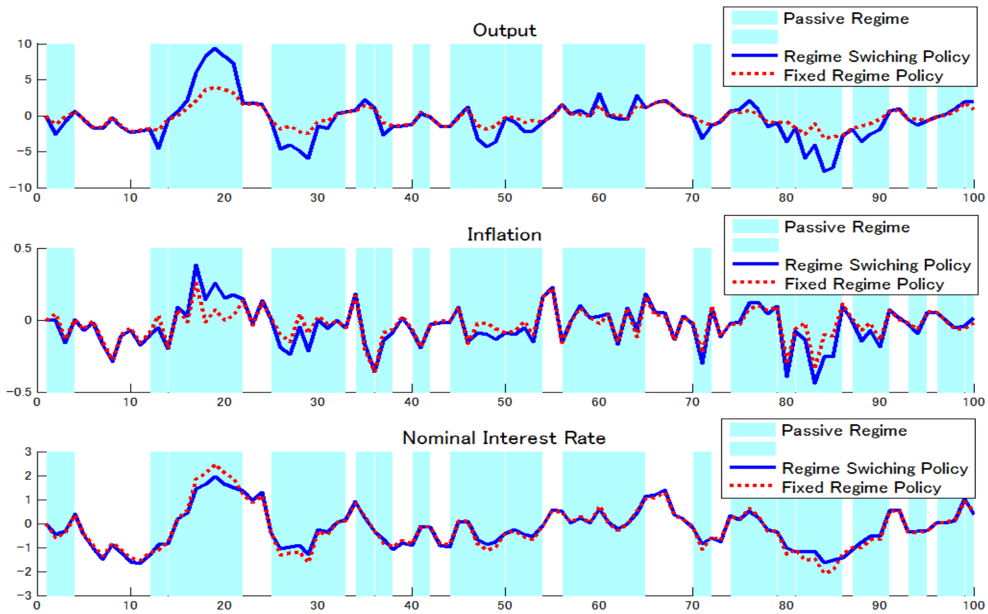
(b)the Case in presence of the ZLB constraint

Policy	variables	R.S. Policy		Fixed Policy		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
Stoc. Expect.	output	-0.71	3.60	-0.91	2.91	+0.20	+0.69
	inflation	-0.01	0.21	-0.02	0.18	+0.01	+0.03
	interest rate	0.12	0.73	0.12	0.74	0.00	-0.01
Perfect Fore.	output	-0.38	3.40	-0.51	2.64	+0.13	+0.76
	inflation	-0.01	0.20	-0.01	0.17	0.00	+0.03
	interest rate	0.08	0.70	0.09	0.69	-0.01	+0.01

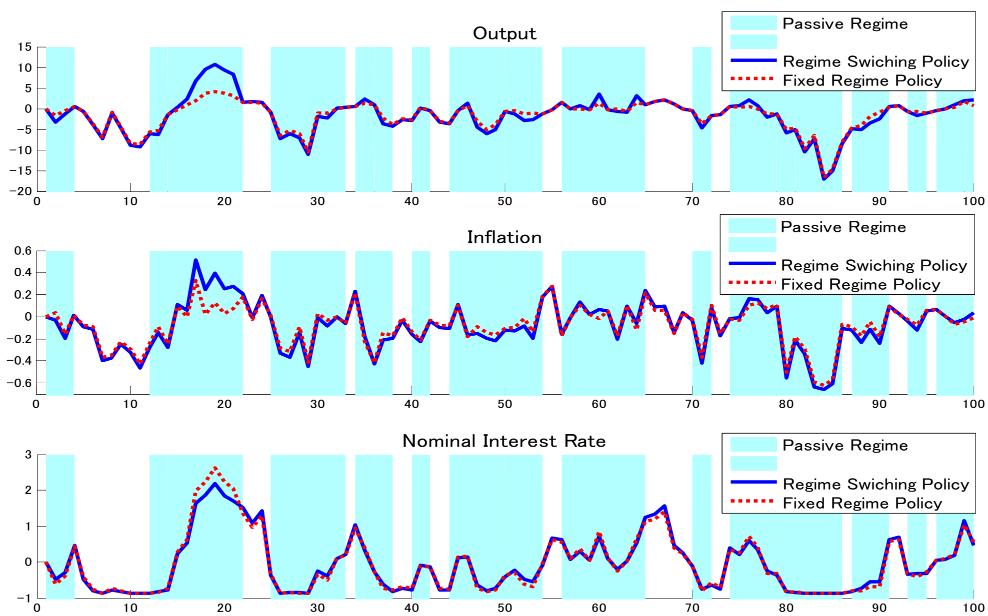
Notes: Means and standard deviations are calculated from 100,000 replications derived from policy functions of the three variables under each expectations form. R.S. Policy is a regime swiching policy in which the Taylor rule switches between the active and the passive regime based on setting paramers. Fixed Policy is a policy in which coefficients of the Taylor rule are fixed at the same values as active regime without regime switching. Stoc. Expect. and Perfect Fore. denote stochastic rational expectations and perfect foresight, respectively. Differences means subtraction of the value under the active regime from under the passive regime in each expectations form.

Figure 10: Calibrations; Regime Switcthing Policy vs. Fixed Policy

(a) Case in Absence of the ZLB constraint

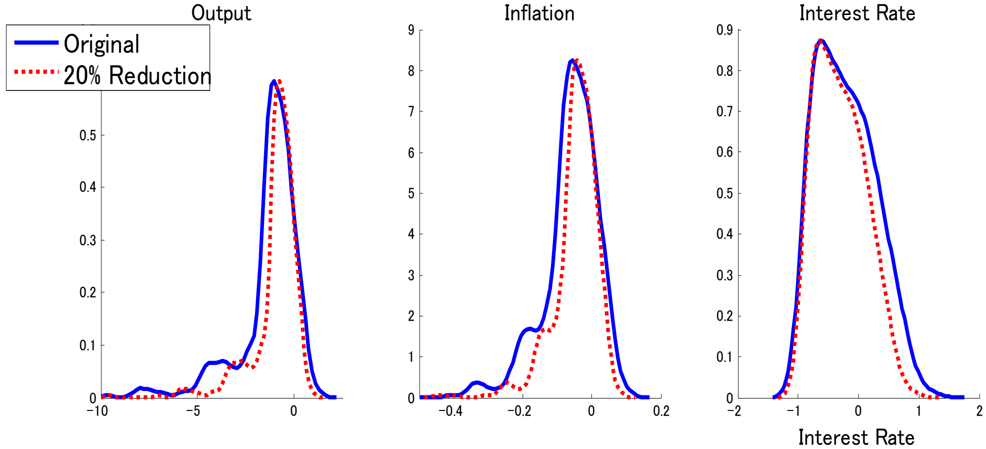


(b) Case in Presence of the ZLB constraint



Notes: Artificial data of regime switching and fixed policies generated from the 100 iterations of Monte Carlo simulation are depicted by distinguishing according to the presence or the absence of the ZLB, respectively. The same random seed is adopted in panels (a) and (b) for comparing the effect of the ZLB constraint, so that the same structural shocks would determine the endogenous variables in both graphs. A light shade indicates the passive regime. Solid line and dashed line represent the regime switching policy and the fixed policy, respectively.

Figure 11: 20% Reduction of St.D of Aggregate Demand Shock



Notes: These graphes show density functions of stochastic expectations of the three endogenous variables conditional on a -2% demand shock, generated from Monte Carlo simulation. Dashed line and solid line represent the case of a 20 % reduction of the volatility of aggregate demand shock and the case of original size of the shock, respectively.

density functions of the stochastic expectations of the three endogenous variables are conditional on a -2% demand shock. The ZLB constraint causes the functions to become unsymmetrical shapes (with fat tails in the left area), resulting in improvements of their expectation values by a 20% cut of the volatility of the demand shock. Since these density functions represent the belief of the agents in the model, shrinking the variance of belief by implementing a certain policy would work very well toward recovering an economy suffering from deflation at the zero interest rate.

Table 4 shows the calibration results of both policies from a 20% reduction in the standard deviation of the aggregate demand shocks by keeping those of aggregate supply shocks under stochastic rational expectations. The third column shows the effects of a 20% reduction, and the fourth column shows the values based on the original conditions, which are the same as those in Table 3. The sixth column shows the ratios of the effects resulting from a 20% reduction in the original. As in panel (a), the 20% reductions in the standard deviations of the shock induce only 20% reductions in those of the endogenous variables without any other changes in the case of the absence of the ZLB. In contrast, reducing the standard deviation of the aggregate demand shocks must be effective as a policy for stabilizing an economy by largely decreasing the volatilities of the endogenous variables in the case of the presence of the ZLB. For example, as shown in the second row of panel (b), the means and standard deviations of the output decline from -0.71 and 3.60 to -0.41 and 2.63 by two-thirds in the regime-switching policy, respectively. In the case of a fixed regime, the situation is similar. In this way, the policy reducing the variance of demand shocks might work more effectively for stabilizing an economy, but an active monetary policy regime does not work well in cases with the presence of the ZLB.

Summing up, there are two findings from the calibration results. First, there

Table 4: 20% Reduction of Std Dev of Aggregate Demand Shock under Regime Switching and Fixed Policies

(a) the Case in absence of the ZLB constraint

Policy	variables	20% reduction		Original		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
R.S. Policy	output	0.00	1.84	-0.01	2.22	0.01	83%
	inflation	0.00	0.14	0.00	0.15	0.00	93%
	interest rate	0.00	0.63	0.00	0.76	0.00	83%
Fixed Policy	output	0.00	1.00	0.00	1.21	0.00	83%
	inflation	0.00	0.12	0.00	0.12	0.00	100%
	interest rate	0.00	0.64	0.00	0.79	0.00	81%

(b) the Case in presence of the ZLB constraint

Policy	variables	20% reduction		Original		Differences	
		mean	Std Dev	mean	Std Dev	mean	Std Dev
R.S. Policy	output	-0.41	2.63	-0.71	3.60	57%	73%
	inflation	-0.01	0.18	-0.01	0.21	0%	85%
	interest rate	0.08	0.62	0.12	0.73	66%	84%
Fixed Policy	output	-0.57	1.84	-0.91	2.91	63%	63%
	inflation	-0.02	0.15	-0.02	0.18	0%	83%
	interest rate	0.09	0.61	0.12	0.74	75%	82%

Notes: Means and standard deviations are calculated from 100,000 replications derived from policy functions of the three variables under each expectations form. R.S. Policy is a regime swiching policy in which Taylor rule switches between the active and the passive regime based on setting paramers. Fixed Policy is a policy in which coefficients of the Taylor rule are fixed at the same values as active regime without regime switching. Differences means ratios of the value by 20 % reduction of standard deviation of aggregate demand shock to the original size in each expectations form under stochastic rational expectations.

is no gap in the fluctuations of output and inflation between stochastic expectations and perfect foresight due to linear policy functions, when an economy does not face the ZLB constraint. In contrast, this calibration shows that once-negative aggregate demand shocks make the nominal interest rate hit the ZLB under stochastic expectations, unlike perfect foresight; intensifying uncertainty measured by the volatilities of shocks would further deepen the declines of the endogenous variables even in response to the same shock, regardless of the monetary policy regime adopted. These results suggest that perfect foresight is biased upward so that the possibility exists that the steady states of a model in the absence of the ZLB are underestimated in periods of deflation, since the means often used as estimates of the steady states are biased downward from those. Second, the calibration shows the validity of a passive monetary policy under the ZLB, unlike the situation with no ZLB, in which Davig and Leeper (2007), dealing with the MS Taylor rule, showed that the passive policy always makes the output and inflation fluctuate more than the active policy. Since the passive policy regime makes the expected interest rate fall and induces both expected output and inflation to increase under the ZLB, it is suggested that the passive policy might be a candidate for an exit strategy from a zero interest rate policy. It is also suggested that a guidance policy to form expectations would play an important role in recovering an economy by contributing to mitigating the uncertainty of an aggregate demand shock, rather than retaining an active monetary policy regime, since the reduction of the volatility of the shock raises the means of output and inflation.[11]

6 Conclusion

Using the concept and algorithms of an SREE proposed by Billi (2013), I extend a simple new Keynesian model with the MS-type Taylor rule introduced by Davig and Leeper (2007) into one constrained by the ZLB. I also simulate and evaluate the effects of monetary policy regime shifts under the ZLB.

According to the calibration, there are two findings from the calibration results. First, there is no gap in the fluctuation of output and inflation between stochastic expectations and perfect foresight because of linear policy functions, when an economy does not face the ZLB constraint. In contrast, once negative aggregate demand shocks make the nominal interest rate hit the ZLB under

[11] I have conducted a robustness check by extending the model to low frequent regime changes under the ZLB with stochastic expectations. To this end, following Aruoba et al (2014), I have set transition probabilities such as $p_{11} = 0.99$ (transition probability from active to active), $p_{22} = 0.95$ (transition probability from passive to passive). Findings of the robust check are following: (1) Fixed points (equilibria) are found even in the low frequent regime changes environment. (2) The sizes of fluctuations of output and inflation are bigger under the passive regime with the low frequent regime changes than the high frequent ones. (3) Monte Carlo simulations show that standard deviations of the output, the inflation and the interest rate are smaller under the active regime with the low frequent regime changes than the high frequent ones, but vice versa under the passive regime. (4) Impulse responses under the active regime are almost the same as under the fixed regime for both of positive and negative shocks. On the other hand, impulse responses of output and inflation are bigger under the passive regime for both of positive and negative shocks than those of the high frequent regime changes.

stochastic expectations, unlike perfect foresight, intensifying uncertainty plays an important role in further declines of the output and price level even in response to the same shock, regardless of the monetary policy regime adopted. The calibration also indicates the possibility that the steady states of a model, in the absence of the ZLB, are underestimated in periods of deflation, since the means often used as estimates of the steady states are biased downward from these.

Second, the calibration shows the validity of a passive monetary policy under the ZLB, unlike the situation with no ZLB, in which Davig and Leeper (2007), dealing with the MS Taylor rule, showed that the passive policy always makes the output and inflation fluctuate more than the active policy. Since the passive policy regime lowers the expected interest rate and induces both expected output and inflation to increase under the ZLB, it is suggested that the passive policy might be a candidate for an exit strategy from a zero interest rate policy. It is also suggested that a guidance policy to form expectations would play an important role in recovering an economy by contributing to mitigating the uncertainty of an aggregate demand shock, rather than retaining an active monetary policy regime, since the reduction of the volatility of the shock raises the means of output and inflation.

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