Sources of the Great Recession: 
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Sources of the Great Recession:  
A Bayesian Approach of a Data-Rich DSGE model  
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Abstract

In order to investigate sources of the Great Recession (Dec. 2007 to Jun. 2009) of the US economy in the latter portion of the first decade of the 2000s, we modified the standard New Keynesian dynamic stochastic general equilibrium (DSGE) model by embedding financial frictions in both the banking and the corporate sectors. Furthermore, the structural shocks in the model are assumed to possess stochastic volatility (SV) with a leverage effect. Then, we estimated the model using a data-rich estimation method and utilized up to 40 macroeconomic time series in the estimation. In light of a DSGE model, we suggest the following three empirical evidences in the Great Recession: (1) the negative bank net-worth shock gradually spread before the corporate net worth shock burst; (2) the data-rich approach and the structural shocks with SV found the contribution of the corporate net worth shock to a substantial portion of the macroeconomic fluctuations after the Great Recession, which is unlike the standard DSGE model; and (3) the Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in the corporate sector would still not have stopped deteriorating. Incorporating time-varying volatilities of shocks into the DSGE model makes their credible bands narrower than half of the constant volatilities, which result implies that it is a realistic assumption based on the dynamics of the structural shocks. It is plausible that tiny volatilities (or uncertainty) in ordinary times change to an extraordinary magnitude at the turning points of business cycles.

Keywords: New Keynesian DSGE model, Data-rich approach, Bayesian estimation, financial friction, stochastic volatility, leverage effect.

JEL Classification: E32, E37, C32, C53.

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1 Introduction

The U.S. National Bureau of Economic Research (NBER) has reported that the Great Recession began in December 2007 and ended in June 2009. The emergence of sub-prime loan losses in 2007 began the recession and exposed other risky loans and over-inflated asset prices. With loan losses mounting and the collapse of Lehman Brothers on September 15, 2008, a major panic broke out in the inter-bank loan market. The financial crisis played a significant role in the failure of key businesses, in the declines in consumer wealth—estimated in trillions of US dollars—as, and a downturn in economic activity, all leading to the 2008–2012 global recession and contributing to the European sovereign-debt crisis. In The New York Times in January 2009, Paul Krugman commented on this crisis as seemingly being the beginning of "a second Great Depression." The central debate about the origin of the recession has been focused on the respective parts played by the public monetary policy and by the practices of private financial institutions. The U.S. Senate's Levin–Coburn Report asserted that the financial institution crisis, one of the causes, was the result of "high risk, complex financial products; undisclosed conflicts of interest; the failure of regulators, the credit rating agencies, and the market itself to rein in the excesses of Wall Street." In order to strengthen the financial sector, the Troubled Asset Relief Program (TARP), in which assets and equity are purchased from financial institutions by the U.S. government, was enforced and originally was used to authorize the expenditures of $700 billion in October 2008.

The purpose of this study is to analyze the mutual relationship among macroeconomic and financial endogenous variables in terms of business cycles and to identify what structural exogenous shocks contributed to the Great Recession using a dynamic stochastic general equilibrium (DSGE) model. Because we obtained a broad consensus that solvency and liquidity problems of the financial institutions are chief among the fundamental factors causing the recession itself, as described above, it is plausible to embed financial frictions in both the banking and the corporate sectors of a New Keynesian DSGE model. In fact, according to Ireland (2011), who was the first to attempt to analyze the impact of the recession using a New Keynesian DSGE model, there are three sets of considerations that are premature for existing DSGE models. First, banking failures and liquidity dry-ups should be endogenously explained with other fundamental macroeconomic variables for producing economic insights. Second, most recessions have been accompanied by an increase in bankruptcies among financial and nonfinancial firms alike. And recessions have featured systematic problems in the banking and loan industry. And third, declines in housing prices and problems in the credit markets might have played an independent and causal role in the Great Recession’s severity. By identifying the structural shocks generated from two financial frictions in both the financial and the nonfinancial sectors and integrating them into a DSGE model, our study will look into the former two exercises. In addition, we will focus on the extreme change of volatility in financial markets and across the economy as a whole in the recession, by estimating the time-varying volatility of these structural shocks.

To this end, we will follow Nishiyama et al. (2011) who have already studied the US economy using a New Keynesian DSGE model with these two financial frictions in a data-rich environment. In this model, with asymmetric information between borrowers and lenders, banks have two roles in generating two agency costs: one is as the lenders to the corporate sector and the other is as the borrowers from depositors. Decomposing the effects of the two kinds of agency costs on macroeconomic fluctuations might be important for finding out the origin of the recession as well as measuring the degree of damage to the US economy. The data-rich approach is a useful method, separating the coherence of en-
ogenous variables with a mutual relationship and measurement errors from a significant amount of macroeconomic panel data, indicating to identify more robustly the structural shocks. This study will extend the estimated sample period to 2012Q2 and incorporate a stochastic volatility (hereafter SV) model with a leverage effect, which has been recently developed for measuring the volatilities of stock returns in the financial market, as by explaining the dynamics of the structural shocks in the DSGE model described above. Because we can more efficiently extract structural shocks as well as model variables by adopting the data-rich approach, we will be able to relax the specifications of the structural shocks and measure the impact of financial shocks on the real economy both during the Great Recession and after it.

We will consider four alternative cases, based on the number of observation variables (11 vs. 40 observable variables) and the specification of the volatilities of the structural shocks (constant volatility vs. time-varying volatility). We expect that by adopting forty macroeconomic time series as observable variables, data-rich information will make the decomposition between the measurement errors and the model variables from data more robust, and that relaxation of specifying the volatilities makes the rapid change of the shocks more detailed. Comparing the four cases, we suggest the following three empirical evidences in the Great Recession: (1) The negative bank net-worth shock gradually spread before the corporate net-worth shock burst, (2) the data-rich approach and the structural shocks with SV found the contribution of the corporate net worth shock to a substantial portion of macroeconomic fluctuations after the Great Recession, which is unlike a standard DSGE model, and (3) The Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in the corporate sector could still not have stopped deteriorating. Incorporating time-varying-volatilities of shocks into the DSGE model makes their credible bands narrower than half of the constant volatilities, which result implies it is a realistic assumption of the dynamics of structural shocks. It is plausible that the tiny volatilities (or the uncertainty) in ordinary times change to an extraordinary magnitude at the turning points of the business cycles. We also estimate that monetary policy shock has an opposite leverage effect of SV, which effect implies that tightening policies makes interest rates more volatile.

Finally, we will compare the achievements of our study with earlier studies from three aspects: financial frictions, the time-varying volatilities of the structural shocks, and the data-rich approach. First, there are many previous studies that have estimated a large-scale DSGE model by adopting a New Keynesian framework with the nominal rigidities proposed by Chirstiano, Eichenbaum and Evans (CEE)(2005), e.g., Smets and Wouters (2007). And one financial friction between the bank and the corporate sectors was developed by Bernanke et al. (1999), and then Christensen and Dib (2008) incorporated it into a CEE model. Meanwhile, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) recently proposed the other financial friction between banks and depositors. To the best of our knowledge, there is no literature combining these two frictions with a CEE model except Nishiyama et al.’s (2011), whose model we are using. Second, some research studies have allowed for a time variation in the volatility of the structural disturbances. In this respect, Justiniano and Primiceri (2008) focused on the Great Moderation using a model with structural shocks, including SV models but neither the financial frictions nor the leverage effect. And Liu et al (2011) estimated a DSGE model without financial frictions by another approach, by regime-switching the volatilities for analysing sources of the Great Moderation. And third, a few studies have dealt with the data-rich approach proposed by Boivin and Giannoni (2006), but they excluded Kryshko (2011), Nishiyama et al. (2011) and Iiboshi et al. (2012). Our study is the first attempt to combine the data-rich approach with the time-varying volatilities of structural disturbances.
The paper is organized as follows: Section 2 provides the framework of the DSGE model, including the data-rich approach and the structural SV shock with the leverage effect. Section 3 illustrates two financial frictions of the DSGE model. Section 4 presents the estimation technique. In Section 5, preliminary setting of structural parameters and data description are dealt with. Section 6 discusses the estimation results and interpretation of the Great Recession in terms of the New Keynesian model. Section 7 concludes the paper.

2 Data Rich Approach with Stochastic Volatility Shocks

2.1 Stochastic Volatility with Leverage in DSGE models

A DSGE model is the system of equations that is a collection of constraints and first-order conditions derived from micro-founded models in which economic agents such as households and firms are assumed to solve the inter-temporal optimization problem based on their rational expectations under an economic frictional environment. A log-linearized model is derived in the neighbourhood of a steady state of the DSGE model by the first-order approximation. Using, for example, Sims’ (2002) method, the law of motion surrounding the steady state of the model solved from log-linear approximation is represented below.

\[ S_t = G(\theta) S_{t-1} + E(\theta) \varepsilon_t \]  \hspace{1cm} (2.1)

where \( S_t \) is a \( N \times 1 \) vector of endogenous variables referred to as model variables, whereas \( \varepsilon_t \) is a \( M \times 1 \) vector of exogenous disturbances represented by structural shocks. \( \theta \) represents the structural parameters derived from the DSGE models based on macroeconomic theory. In particular, its core parameters are referred to as deep parameters, which govern the rational behaviours of economic agents. Matrices \( G(\theta) \), and \( E(\theta) \) are the function of \( \theta \). So far, disturbance terms \( \varepsilon_t \) are assumed to be independent and identically distributed (i.i.d.), normal distributions in most DSGE models. This study extends this assumption and replaces them with time varying variances, as shown below.

\[ \varepsilon_t = \Sigma_t z_t \]  \hspace{1cm} (2.2)

\[ z_t \sim \text{i.i.d.} ~ N (0, I_M) \]

\[ \Sigma_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \cdots, \sigma_{M,t}) \]

where \( z_t \) is a \( M \times 1 \) vector with all elements following standard normal distribution. \( I_M \) is a \( M \times M \) identity matrix. And, \( \Sigma_t \) is the standard deviation or the volatility of the disturbance shocks \( \varepsilon_t \), and it is represented as a diagonal matrix with elements such as \( \sigma_{1,t}, \cdots, \sigma_{M,t} \), which move following a stochastic volatility model.

\[ \log \sigma_{i,t+1} = \mu_i + \phi_i (\log \sigma_{i,t} - \mu_i) + \eta_{i,t}, \quad i = 1, 2, \cdots, M \]  \hspace{1cm} (2.3)

\[ \begin{pmatrix} z_{i,t} \\ \eta_{i,t} \end{pmatrix} \sim \text{i.i.d.} ~ N (0, \Omega_i), \quad \Omega_i = \begin{bmatrix} 1 & \rho_{i} \omega_i \\ \rho_{i} \omega_i & \omega_i^2 \end{bmatrix} \]  \hspace{1cm} (2.4)

Because volatility \( \sigma_{i,t} \) necessarily has a positive value, it is converted into a logarithmic value in order to throw off the limitation of the sign. \( \mu_i \) denotes the mean of the volatility \( \sigma_{i,t} \) of i-th shock. And \( \phi_i \) is a coefficient of persistence of the i-th volatility. In this SV model, the leverage effect of the volatility \( \sigma_{i,t} \) is introduced as an equation (2.4), which measures...
the correlation between the sign of disturbance terms and the size of the volatility. Typically, the correlation \( \rho \) is negative, signifying that a negative stock return \( (z_{i,t} < 0) \) tends to increase the volatility of a stock price \( (\eta_{i,t} > 0) \). We tried to verify whether the volatility of the net-worth shock in firms and bank sectors increased when they were negative. Although Justiniano and Primiceri (2008) inserted a SV model into a DSGE model, they did not consider the leverage effect of the volatility. This extension is one of the advantages of our model. Note that assumption of the asymmetry of the size of volatility for the sign of \( z_{i,t} \) does not affect the rational expectations of endogenous variables \( S_t \) when solving the model, because the innovation \( \eta_{i,t} \) determines the volatility in the next period \( t + 1 \) but not in the current period \( t \) as shown in Eq.(2.3).

2.2 Data Rich DSGE Models

2.2.1 Significance of Data Rich DSGE models

A data-rich DSGE model was developed by Boivin and Giannoni (2006) and by Kryshko (2011)\(^1\) and they composed it from a combination of a DSGE model and a dynamic factor model (DFM). This combination is basically possible since their frameworks are based on the same state-space representation. As a result, it enjoys the advantages of two existing models, and compensates for the drawbacks of these two models. We will examine these advantages and the properties of the combination before explaining the two models.

(1) Drawbacks of a standard DSGE model, and overcome by a DFM

In general, it is difficult to identify the model’s variables \( S_t \) and the measurement error \( e_t \) from the observable variables \( X_t \) in a standard DSGE model. However, it is plausible that data \( X_t \) is composed of co-movement (or systematic) components and idiosyncratic components, indicating measurement errors or noise that are not correlated with systematic movements. That is,

\[
data = \text{common (or systematic) component} + \text{idiosyncratic component},
\]

where two components are unobservable. Accordingly, it is assumed that there are no idiosyncratic components or measurement errors and “data = systematic components” in a standard DSGE model. A separation approach of these two factors is a DFM. In this model, co-movement is likely to be a component explained from an economic system affected by multi-variables with their mutual impacts. That is, a dynamic equation of co-movement might be appropriate to be represented as a VAR model. On the other hand, idiosyncratic components should be expressed as a univariable AR process, since they are thought to independently fluctuate.

(2) Drawbacks of a DFM, and overcome by a standard DSGE model

A DFM today is focused on as a model for decomposing co-movement and idiosyncratic errors from data. However, there remains a question: Is it possible to decompose them only from a statistical method? Conventionally, we just focused on a generic correlation among macroeconomic variables but not on a causal association among them from the viewpoint of an economic model. But, according to Woodford (2009), new neo-classical synthesis providing the theoretical foundation for much of contemporary mainstream macroeconomics asserts that models built out of theory should be focused on instead of looking at more generic correlations among data. In a DFM, co-movement tends to be measured from a

\(^1\)Recently, Schorfheide et al. (2010) published an empirical study applying a data-rich DSGE approach to forecast economic fluctuations.
conventional VAR model in which it is difficult to interpret their coefficients from economic theory. Instead, a DSGE model expresses the co-movement of multi-variables from a causal association and a theoretical coherence, based on a micro-founded dynamic model, following the spirit of new neo-classical synthesis. That is, converting from a conventional VAR model to a DSGE model in a systematic component part of a DFM indicates that

\[
\text{comovement (systematic variation)} = \text{genetic correlation},
\]

\[\Rightarrow \text{comovement} = \text{causal association}\]

which induces a resolution of the drawback of a DFM.

(3) Synergetic effect of combination

According to Stock and Watson (2002a,b), the consistency of the DFM suggests that increasing the number of observations, series \(X_t\) is expected to increase the certainty that idiosyncratic components not explained by an economic system are removed from the data using a DFM. Accordingly, increasing the number of observations could improve the accuracy of measuring the co-movement \(S_t\) and the exogenous structural shocks \(\varepsilon_t\). If the estimation of the structural shocks \(\varepsilon_t\) successfully explains actual business cycles, the indication will be the validity of the structural shocks and, in addition, that of the DSGE model. And in a data-rich framework, the same data set is applicable even for DSGE models with different model variables, so the possibility of model selection among many alternative models emanates. This possibility implies that the data-rich approach is expected to contribute to the evaluation and selection among DSGE models from the point of view of the validity of the structural shocks and the marginal likelihood (or the Bayes factor).

2.2.2 Dynamic Factor Model (DFM)

Recently, the estimation method of DFMs was developed and applied to many fields of macroeconomics and finance. The DFM, which is a statistical model estimating the common factors of business cycles, was proposed by Sargent and Sims (1977) and empirically applied by Stock and Watson (1989), who extracted one unobserved common factor of the business fluctuation from many macroeconomic time series using the Kalman filter.

The DFMs are represented by state-space models composed of the following three linear equations. When \(F_t\) denotes the \(N \times 1\) vector of the unobserved common factor, and when \(X_t\) denotes the \(J \times 1\) vector of a massive panel of macroeconomic and financial data, note that \(J \gg N\)

\(^2\)Stock and Watson (2002a,b) developed approximate DFMs using principal component analysis, extracting several common factors from more than one hundred macroeconomic time series and verifying that these factors include useful information on forecasting of macroeconomic time series. Nowadays, there are many studies in the literature concerning theoretical and empirical studies of DFMs. For example, Boivin and Ng (2005, 2006), Stock and Watson (2002a, b, 2005). The survey of DFMS covering the latest studies is Stock and Watson (2006, 2010). Kose et al. (2003) tried to extract common factors of world-wide and regional business cycles using a Kalman filter and DFM.
\[ X_t \overset{J \times 1}{=} \Lambda_{N \times J} F_t + e_t, \]  \hfill (2.5) 

\[ F_t \overset{N \times 1}{=} G_{N \times N} F_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.} \ N(0, Q), \]  \hfill (2.6) 

\[ e_t \overset{J \times 1}{=} \Psi_{J \times J} e_{t-1} + \nu_t, \quad \nu_t \sim \text{i.i.d.} \ N(0, R), \]  \hfill (2.7)

where \( \Lambda \) is \( J \times N \) matrix of the factor loadings, \( e_t \) is the idiosyncratic components (or measurement errors), which are allowed to be serially correlated as the equation (2.7). \( G \) is \( N \times N \) matrix, and the common factor \( F_t \) follows the AR process (2.6). Matrices, \( \Psi \), \( Q \) and \( R \) are assumed to be diagonal in an exact DFM, as Stock and Watson (2005) showed. The equation (2.5) is a measurement equation, and equations (2.6) and (2.7) are transition equations. A state-space model is composed of the two kinds of equations (2.5), (2.6), and (2.7).

The property of the model is to decompose the common components, \( AF_t \) and the idiosyncratic component \( e_t \) from the massive panel of macroeconomic and financial data \( X_t \) in (2.5). Meanwhile, it is difficult to interpret the factor \( F_t \) in terms of economic theory, since the above equations are statistically estimated by a conventional VAR model (2.6), and the parameters are not derived from a structural model with a micro-foundation.

### 2.2.3 Data-Rich DSGE Model

The idea of the data-rich approach is to extract the common factor \( F_t \) from a massive panel of macroeconomic and the financial time series data \( X_t \) and to match the model variable \( S_t \) to the common factor \( F_t \). A virtue of this approach is that even if a model variable \( S_t \) and observed data \( X_t \) are slightly detached, one can estimate the DSGE model by matching model variables to the common factors extracted from the large panel data and expect improved efficiency in estimating the parameters and the structural shocks of the model.

The DSGE model is known to be a state-space model and to be estimated using a Kalman filter as well as a DFM. So we can apply the framework of the DFM to a DSGE model. But the big difference between a DFM and a DSGE model is the meaning of their parameters. Those of the latter are derived from the structural parameters \( \theta \). A data-rich DSGE model is given as

\[ X_t \overset{J \times 1}{=} \Lambda(\theta) S_t + e_t, \]  \hfill (2.8) 

\[ S_t \overset{N \times 1}{=} G(\theta) S_{t-1} + E(\theta) \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.} \ N(0, Q(\theta)), \]  \hfill (2.9) 

\[ e_t \overset{J \times 1}{=} \Psi_{J \times J} e_{t-1} + \nu_t, \quad \nu_t \sim \text{i.i.d.} \ N(0, R), \]  \hfill (2.10) 

where the observable variables \( X_t \) are a \( J \times 1 \) vector, the state variables \( S_t \) are a \( N \times 1 \) vector, and the structural shocks \( \varepsilon_t \) are a \( M \times 1 \) vector. In a data-rich DSGE model, there are many more observable variables than there are state variables (\( J \gg N \) ), and likewise for a DFM. (On the other hand, in a regular DSGE model \( J \leq N \) ) And idiosyncratic components \( e_t \), which are a \( J \times 1 \) vector, will cause measurement errors following the AR (1) process in (2.10). (2.8), which shows a measurement equation that splits off components of the common factors \( S_t \) and idiosyncratic components \( e_t \), from the massive panel of
macroeconomic indicators $X_t$ and which consists of common factors $S_t$ with economic concepts. A transition equation (2.9) indicates an AR(1) process of common factors (or model concepts) $S_t$ with structural shocks $\varepsilon_t$ and also dynamics converging to a rational expectation equilibrium determined by a macroeconomic model. From (2.8) and (2.9), we can see that the model variable $S_t$ is realized from the inter-correlation of the data indicators $X_t$ whose movement is coherent with each economic concept derived from the economic theory. In contrast, measurement errors $e_t$ fluctuate from only specific factors of each observable variable $X_t$ but do not depend on economic theory and other data indicators.  

Structural shocks $\varepsilon_t$ and disturbance terms $\nu_t$ of measurement errors $e_t$ follow normal distributions, i.e. $\varepsilon_t \sim \text{i.i.d. } N(0, Q(\theta))$ and $\nu_t \sim \text{i.i.d. } N(0, R)$, respectively. And their variance covariance matrix $Q(\theta)$ and $R$ are positive, definite, and diagonal matrices. Coefficients $\Psi$ of the AR(1) process (2.10) are also diagonal matrices. These indicate that the measurement errors $e_t$ are independent from each other in terms of cross section but dependent with their lag variables in terms of time series restriction. Matrices $G(\theta)$, $E(\theta)$, and $Q(\theta)$ are functions of the structural parameters $\theta$.

### 2.3 Data Rich DSGE models with Stochastic Volatility

#### 2.3.1 Stochastic Volatility in a Data-Rich DSGE models

In the previous subsection, the structural shocks $\varepsilon_t$ of the DSGE model are assumed to follow i.i.d. normal distribution. Since the adoption of the data-rich approach is expected to improve the accuracy of the structural shocks, the next task of the approach is thought to specify the structural shocks. In this paper, we used a relaxation of the assumption of

$\Phi$Alternatively, representing matrix-base equation (2.8) as element-base equation below, the framework of a data-rich DSGE model might be more understandable. As can be seen from the second row of matrix $A(\theta)$: $[0 \cdots 0]$, observable variables are directly relation with only one specified model variable $S_i$. And in order to identify the magnitude of each linkage model variable $S_i$, the value of $\lambda$ of just one variable of data indicators $X_i$ is unity as the first $N$ rows of matrix $A$: $N \times N$ identity matrix. Meanwhile, the parameters of information indicators (the remaining part of $X_i$) can be represented as full elements in matrix $A$ such as $[\lambda_{11} \lambda_{12} \cdots \lambda_{1n}]$ which connects information indicators through many-to-many relation with all model variable $S_i$.

$$
\begin{bmatrix}
\text{Output Gap series #1} \\
\text{inflation series #1} \\
\vdots \\
\text{Output Gap series #2} \\
\text{inflation series #2} \\
\vdots \\
\text{Output Gap series #n} \\
\text{inflation series #n} \\
\vdots \\
\text{information series #1} \\
\vdots \\
X_t \ (J \times 1)
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nN} \\
\lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{m1} & \lambda_{m2} & \cdots & \lambda_{mN} \\
\end{bmatrix}
\begin{bmatrix}
\lambda(\theta) \ (J \times N)
\end{bmatrix}
\begin{bmatrix}
\hat{y}_t \\
\hat{\pi}_t \\
\vdots \\
\hat{S}_{N,t} \\
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{y1} \ t \\
\varepsilon_{x1} \ t \\
\vdots \\
\varepsilon_{y2} \ t \\
\varepsilon_{x2} \ t \\
\vdots \\
\varepsilon_{ym} \ t \\
\varepsilon_{xn} \ t \\
\vdots \\
\varepsilon_{en} \ t \\
\vdots \\
\varepsilon_{et} \ (J \times 1)
\end{bmatrix}
$$

where $\hat{y}_t$ is the model variable of output gap, $\hat{\pi}_t$ is the model variable of inflation.

$\Phi$Measurement errors play an important role since we could expect that they remove some degree of undesirable relation between observable variables and the model concept variable influenced by model misspecification and mismatch of model concepts into observable variables. In addition, stochastic singularities can be avoided with measurement errors in a data-rich environment.
the shocks by inserting an SV model with leverage effects into a data-rich DSGE model. If
the flexibility of the shocks changes to interpret sources of the Great Recession described in
Section 6, inserting this will be valuable for analysing the business cycles from the DSGE
model's views. Combining an SV model described in Section 2.1 and a data-rich DSGE
model, our model is represented as follows:

\[
X_t = \Lambda(\theta) S_t + e_t, \quad (2.11)
\]

\[
S_t = G(\theta) S_{t-1} + E(\theta) \varepsilon_t, \quad (2.12)
\]

\[
e_t = \Psi_t e_{t-1} + \nu_t, \quad \nu_t \sim \text{i.i.d. } N(0, R), \quad (2.13)
\]

\[
\varepsilon_t = \Sigma_t \mathbf{z}_t, \quad (2.14)
\]

\[
\mathbf{z}_t \sim \text{i.i.d. } N(0, \Omega_t), \quad \Sigma_t = \text{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots, \sigma_{M,t}), \quad (2.15)
\]

where the notations of the system of equations are the same as in the previous sections,
2.1 and 2.2, thus we are omitting their explanations here. This study extends Justiniano
and Primiceri’s (2008) model to a data-rich approach and adds a leverage effect to the
SV model. Additionally, we combined the SV model with the data-rich model, in contrast
to Boivin and Giannoni (2006)’s approach. In addition, these two models only take into
account the nominal rigidities of the goods and labour markets, whereas our model involves
two financial frictions of the bank sector as well as the nominal rigidities of the goods and
labour markets. These extensions in terms of both economic and econometric approaches
are thought to be appropriate for analysing the sources of the Great Recession.

2.3.2 Transformation into Estimated State Space Model

It is difficult to directly estimate the state-space representation (2.11), (2.12), and (2.13),
as shown above, for applying to large panel data set since the size of the matrix in the
transition equations (2.12) and (2.13) is equal to the total number of model variables \(S_t\)
and measurement errors \(e_t\). This framework induces a dramatic increase of the matrix
as the amount of data \(X_t\) is increasing. To avoid this situation, we tried to minimize the
transition equations, as follows. That is, we eliminated the AR process of measurement
errors of (2.10) and expressed from only \(\nu_t\) with the i.i.d. process for measurement errors.
Inserting Eq.(2.10) into Eq.(2.8), the measurement equation is transformed as follows:

\[
(I - \Psi L) \tilde{X}_t = (I - \Psi L) \Delta \tilde{S}_t + \nu_t, \quad \nu_t \sim \text{i.i.d. } N(0, R).
\]

where \(L\) is the lag operator. By using notations \(\tilde{X}_t = X_t - \Psi X_{t-1}\), and \(\tilde{S}_t = [S_t' \ S_{t-1}']\), this
equation can be rewritten as
\[
\begin{pmatrix}
S_t \\
S_{t-1} \\
S_t
\end{pmatrix}
= \begin{pmatrix}
G(\theta) & 0 & 0 \\
I & O & 0 \\
0 & 0 & E
\end{pmatrix}
\begin{pmatrix}
S_{t-1} \\
S_{t-2} \\
S_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
E(\theta) \\
0 \\
\varepsilon_t
\end{pmatrix},
\]  
(2.18)

where I is a \(N \times N\) identity matrix. The estimation method of the data-rich DSGE model is explained using the state-space model (2.17) and (2.18), and we estimated this model using the Bayesian Method via MCMC in Section 4. For convenience, we set the parameters of the measurement equation (2.17) as \(\Gamma = \{\Lambda, \Psi, R\}\). And the Bayesian estimation of the parameters \(\Gamma\) follow Chib and Greenberg (1994), as described in Appendix A3.

3 The DSGE model with Two Financial Frictions in Corporate and Banking Sectors

In order to model the balance sheets of the corporate and banking sectors in a DSGE framework, we combined the essence of Bernanke, Gertler, and Gilchrist (hereafter, BGG) (1999), Gertler and Karadi (2011), and Gertler and Kiyotaki (2010). We adopted the stylized DSGE model based on CEE (2005) and Smets and Wouters (2003, 2007), which focused on the nominal rigidities of price level and wage as well as the quadratic adjustment cost of investment, and embedded the financial frictions of the corporate and banking sectors to it. This section is the highlight, modelling the frictions in our DSGE model. The rest of our model is described in Appendix A4.

3.1 Financial Friction in Corporate Sector

3.1.1 Entrance and Exit of Entrepreneurs

Following BGG (1999), there is a continuum of entrepreneurs indexed by \(j \in [0, 1]\) where each entrepreneur is risk-neutral and has a finite expected horizon.\(^5\) Each entrepreneur faces an exogenous time-varying stochastic survival rate of \(\gamma^E_{t+1}\) from period \(t\) to \(t+1\), which is common across all entrepreneurs.\(^6\)

Between period \(t\) and \(t+1\), after \(1 - \gamma^E_{t+1}\), a fraction of entrepreneurs will exit the business, and exactly the same number of new entrepreneurs will enter the business, so the population of entrepreneurs in the economy remains the same (i.e., a fraction \(j^E\) of the total members of the household) from period \(t\) to \(t+1\). Each entering entrepreneur will receive a “start-up” transfer from the household and the total “start-up” transfer from the household will be equal to the constant fraction \(\xi^E\) of aggregate net worth available in the corporate sector, \(n^E_t\), i.e., \(\xi^E n^E_t\). For \(1 - \gamma^E_{t+1}\), a fraction of entrepreneurs who happen to exit the business will first sell the capital they purchased during the last period and will retire all of their debts before their maturity. Then, they will transfer their remaining net

\(^5\)These assumptions ensure that each entrepreneur will not accumulate enough net worth to self-finance his or her new capital.

\(^6\)We assume that the stochastic process of \(\gamma^E_{t+1}\) is uncorrelated with any other shocks in the economy and has its mean equal to \(\gamma^E\), i.e., \(E[\gamma^E_{t+1}] = \gamma^E\).
worth back to the household. The total number of transfers from exiting entrepreneurs to the household will be \((1 - \gamma_{t+1}^E) n_t^E\). Accordingly, the net transfer, \(\Xi_{t+1}^E\) that the household will receive from entrepreneurs at period \(t + 1\) is \((1 - \gamma_{t+1}^E - \xi^E) n_t^E\).

### 3.1.2 Individual Entrepreneur's Problem

Each entrepreneur produces homogenous intermediate goods, \(y_t(j)\), and they are perfectly competitive when selling their products to retailers. The production function for the intermediate goods is given by

\[
y_t(j) = \omega_t(j) A_t k_t(j)^\alpha l_t(j)^{1-\alpha}, \tag{3.1}
\]

where \(k_t(j)\) is capital inputs and \(l_t(j)\) is labour inputs. The total factor productivity shock (hereafter, TFP shock), \(A_t\), is common across all entrepreneurs. However, following Carlstrom and Fuerst (1997) and BGG (1999), we assume each entrepreneur is subject to an idiosyncratic shock, \(\omega_t(j)\), which is a private information to entrepreneur \(j\) and assumed to be an i.i.d. shock with a mean equal to one, i.e., \(E[\omega_t(j)] = 1\).

The balance sheet statement of each entrepreneur at the end of period \(t\) can be expressed as

\[
q_t k_{t+1}(j) = b_t^E(j) + n_t^E(j), \tag{3.2}
\]

where \(q_t\) is the real price of capital, \(k_{t+1}(j)\) is the capital that will be used for production in period \(t + 1\) but purchased at the end of period \(t\), \(b_t^E(j)\) is the real debt issued at period \(t\), and \(n_t^E(j)\) is the net worth at period \(t\). With the assumption of risk-neutrality and finite planning horizon, the net worth itself is never enough in financing the cost of the capital purchase, and, therefore, each entrepreneur will rely on external financing in equilibrium.

The income statement for entrepreneur \(j\) is specified as follows:

\[
n_t^E(j) = p_t^{mc}(j) y_t(j) - w_t l_t(j) - \frac{R_{t-1}^E(j)}{\pi_t} h_{t-1}(j) + q_t (1 - \delta) k_t(j) \tag{3.3}
\]

where \(p_t^{mc}(j)\) is the real price of intermediate goods \(j\), \(R_{t-1}^E(j)/\pi_t\) is the real rate of borrowing cost, \(R_{t-1}^E(j)\) is the nominal borrowing rate and \(\pi_t\) is the inflation rate, and \(\delta\) is the capital depreciation rate.

Each entrepreneur entering period \(t\) maximizes her discounted cash flow by choosing capital inputs, labour inputs, and debt issuance, subject to (3.1), (3.2), and (3.3).\(^7\) The FOCs for each entrepreneur \(j\) are given as follows:

\[
\frac{w_t}{l_t(j)} = (1 - \alpha)\frac{p_t^{mc}(j) y_t(j)}{l_t(j)} \tag{3.4}
\]

\[
E_t \left[ \gamma_{t+1}^E \frac{R_{t+1}^E(j)}{\pi_{t+1}} \right] = E_t \left[ \gamma_{t+1}^E \frac{\alpha p_t^{mc}(j) y_{t+1}(j)/k_{t+1}(j) + (1 - \delta) q_{t+1}}{q_t} \right]. \tag{3.5}
\]

\(^7\)Each entrepreneur is a price-taker in the labour market, the financial market, and the capital market. At the beginning of period \(t\), each entrepreneur will utilize capital, \(k_t(j)\), and labour input, \(l_t(j)\), to produce the intermediate goods, \(y_t(j)\). Then, they will sell the intermediate goods to retailers in a perfectly competitive manner and earn the revenue, \(p_t^{mc}(j) y_t(j)\). After earning the revenue, each entrepreneur will pay the labour cost and also repay the debt. Finally, each entrepreneur will sell a depreciated capital in the capital market. The net income after these activities are captured by \(n_t^E\), which will be a net worth for the entrepreneur \(j\) at the end of period \(t\). Given this net worth, each entrepreneur will plan for the next period and decide how much capital to purchase and how much debt to issue at the end of period \(t\), which appears in the balance sheet equation (3.2).
(3.4) equates the marginal cost of labour to marginal product of labour and, thus, can be thought of as the labour demand function by entrepreneur j. (3.5) equates the expected marginal cost of capital financed by debt to the expected marginal return of capital financed by debt and can be thought of as capital demand function by entrepreneur j. Since stochastic survival rate, $\gamma_{t+1}$, is uncorrelated to any other shocks in the economy, (3.5) can be further rearranged as follows:

$$E_t \left[ \frac{R^F_t(j)}{\pi_{t+1}} \right] = E_t \left[ \frac{\alpha p_{mt+1}(j) y_{t+1}(j) / k_{t+1}(j) + (1 - \delta) q_{t+1}}{q_t} \right]$$

(3.6)

Under the assumption of risk-neutrality, the introduction of the stochastic survival rate will not alter the capital demand equation for any entrepreneur $j$ compared to the case with a constant survival rate, as described by BGG (1999).

### 3.1.3 Debt Contract

Each period, entrepreneur j issues a debt and engages in a debt contract with an arbitrary chosen financial intermediary m, where m is an indexed number uniformly distributed from 0 to 1. The debt contract is for one period only, and if entrepreneur j needs to issue a debt again the next period, another arbitrary financial intermediary $m'$ will be chosen for the next period. Following BGG’s (1999) recommendations, the idiosyncratic TFP shock, $\omega_t(j)$, is the private information of entrepreneur j and there exists asymmetric information between entrepreneur j and financial intermediary m. Due to the costly state verification, financial intermediary m cannot observe entrepreneur j’s output without cost but needs to incur a monitoring cost to observe it. Entrepreneur j, after observing the project outcome, will decide whether to repay the debt or to default at the beginning of period t. If the entrepreneur decides to repay, the financial intermediary will receive repayment of $R^F_t(j) / \pi_t$ for each unit of outstanding credit, regardless of the realization of the idiosyncratic shock. Otherwise, the financial intermediary will pay a monitoring cost to observe $y_t(j)$ and to seize the project outcome from the entrepreneur.

Under the optimal debt contract, BGG (1999) showed the external finance premium, $s_t(j)$, to be an increasing function of the leverage ratio. For estimation purposes, we followed Christensen and Dib’s (2008) specification of the external finance premium as follows:

$$s_t(j) = \left( \frac{q_t k_{t+1}(j)}{n^e_t(j)} \right)^\varphi$$

(3.7)

where parameter $\varphi > 0$ can be interpreted as the elasticity of the external finance premium with respect to the leverage ratio. In addition, discounting the external finance premium from the borrowing rate $R^F_t(j)$, the expected risk-adjusted nominal return for financial intermediary m from the debt contract from period t to $t + 1$ can be expressed as follows:

$$E_t R^F_{t+1}(m) = \frac{R^F_t(j)}{s_t(j)}.$$ 

(3.8)

### 3.1.4 Aggregation

Since bankruptcy cost is constant-return-to-scale and since leverage ratios are equal for all entrepreneurs j, the external finance premium is equal across all solvent entrepreneurs in equilibrium, i.e., $s_t = s_t(j)$ for all $j$. Since (3.6) holds at the aggregate level, the nominal borrowing rates across all solvent entrepreneurs become equal, i.e.: $R^E_t = R^E_t(j)$ for all
j. Consequently, because \( R_t^E = R_t^E(j) \) and \( s_t = s_t(j) \) for all \( j \), the expected risk-adjusted nominal return for banker \( m \) becomes equal across all bankers, i.e.:

\[
E_t \left[ R_{t+1}^E(m) \right] = \frac{R_t^E}{s_t} \text{ for all } m.
\] (3.9)

Next, we derived the law of motion of the aggregate net worth of the corporate sector. As for notation, the aggregate variable is expressed by suppressing the argument \( j \). Aggregating over the income statement (3.3) and taking into account the entrance and exit of entrepreneurs from period \( t \) to \( t+1 \), we obtained the following aggregate net worth transition equation:

\[
n_t^{E+1} = \gamma_t^{E+1} \left[ r_{t+1}^k q_t k_{t+1} - \frac{R_t^E}{\pi_{t+1}} b_{t+1}^E \right] + \xi_t^{E} n_t^{E} \] (3.10)

where \( r_{t+1}^k \) is the realized gross return from capital investment at period \( t+1 \) and is defined as

\[
r_{t+1}^k \equiv \frac{\alpha_{pc}^{mc} y_{t+1}/k_{t+1} + (1-\delta) q_t}{q_t} . \] (3.11)

Here, \( y_{t+1} \) is the average of project outcomes, \( y_{t+1}(j) \), for all entrepreneurs. Thus, the idiosyncratic factor stemming from \( \omega_t(j) \) is averaged away, and \( r_{t+1}^k \) only reflects the aggregate factors in the economy. Using the entrepreneur’s balance sheet (balance_sheet), the aggregate net worth transition (3.10) can be rearranged as follows:

\[
n_t^{E+1} = \gamma_t^{E+1} \left[ (r_{t+1}^k - \frac{R_t^E}{\pi_{t+1}}) q_t k_{t+1} + \frac{R_t^E}{\pi_{t+1}} n_t^{E} \right] + \xi_t^{E} n_t^{E} . \] (3.12)

Notice how the realization of \( r_{t+1}^k \) can affect the aggregate net worth in the next period. Ex-ante, by the rational expectation equilibrium condition (3.6), the expected return from the capital investment and the borrowing cost are equalized. As Eq.(3.11) ex-post, however, the realized return from the capital investment can exceed or fall below the borrowing cost, depending on the realizations of the aggregate shocks, and the ex-post return \( r_{t+1}^k \) affects the evolution of the aggregate net worth as well as the ex-ante one as described in Eq.(3.12). This is a case where the forecast error has an actual effect on the economy. Another factor that affects the evolution of the aggregate net worth is the realization of the stochastic survival rate \( \gamma_{t+1}^{E} \). At the micro-level, \( \gamma_{t+1}^{E} \) has an interpretation of the stochastic survival rate of entrepreneur \( j \) from period \( t \) to \( t+1 \). At the aggregate level, \( \gamma_{t+1}^{E} \) is interpreted as an exogenous shock to the aggregate net worth in the corporate sector. In our paper, we interpret it as an aggregate corporate net-worth shock.

### 3.2 Financial Friction in Banking Sector

#### 3.2.1 Entrance and Exit of Bankers

According to Gertler and Karadi (2010; 2011), there is a continuum of bankers indexed by \( m \in [0, 1] \) where each banker is risk neutral and has a finite horizon. We assume that each banker faces an exogenous time-varying stochastic survival rate of \( \gamma_{t+1}^{F} \) from period \( t \) to \( t+1 \), which is common to all bankers. By the same token, as in the corporate sector, the stochastic process of \( \gamma_{t+1}^{F} \) is uncorrelated with any other shocks in the economy, and its mean is equal to \( \gamma^{F} \), i.e., \( E[\gamma_{t}^{F}] = \gamma^{F} \).

After \( 1 - \gamma_{t+1}^{F} \), a fraction of the bankers exit; between period \( t \) and \( t+1 \), exactly the same number of new bankers will enter the banking business from the household. Each banker entering the banking business will receive a ‘start-up” transfer from the household,
while each banker exiting the business will transfer his net worth back to the household. In aggregate, the “start-up” transfer is assumed to be the constant fraction $\xi_F$ of the aggregate net worth available in the banking sector, $n^F_t$, i.e., $\xi_F n^F_t$, and the aggregate transfer from the exiting bankers is equal to $\gamma^F_{t+1} n^F_t$. Thus, the net transfer from the banking sector to the household, $\Xi^F_t$, is equal to $(1 - \gamma^F_{t+1} - \xi_F) n^F_t$.

### 3.2.2 Individual Banker’s Problem

We will now describe the individual banker’s problem. The treatment here basically follows that of Gertler and Karadi (2011) and the perfect inter-bank market version of Gertler and Kiyotaki (2010). The balance sheet equation of the individual banker $m$ is given by the following:

$$b^F_t(m) = n^F_t(m) + b^F_t(m)$$  \hspace{1cm} (3.13)

where $b^F_t(m)$ is the asset of banker $m$, which is lent to an arbitrarily chosen entrepreneur $j$ at period $t$; $n^F_t(m)$ is the net worth of banker $m$; and $b^F_t(m)$ is the liability of banker $m$, which is also a deposit made by the household at period $t$.

By receiving deposits $b^F_t(m)$ from the household at period $t$, banker $m$ pledges to pay the deposit rate of $R_t/\pi_{t+1}$ in real terms during the next period. As a result of the banking business, the net worth transition for banker $m$ at period $t+1$ is given by the following:

$$n^F_{t+1}(m) = r^F_{t+1}(m)b^F_t(m) - r_{t+1}n^F_t(m),$$

where $r^F_{t+1}(m) \equiv R^F_{t+1}(m)/\pi_{t+1}$ and $r_{t+1} \equiv R_t/\pi_{t+1}$. Using the balance sheet equation ([banker’s balance sheet]), the net worth transition equation can be reformulated as follows:

$$n^F_{t+1}(m) = (r^F_{t+1}(m) - r_{t+1}) b^F_t(m) + r_{t+1}n^F_t(m).$$  \hspace{1cm} (3.14)

As shown by Gertler and Kiyotaki (2010), with the agency cost present between banker $m$ and the depositor, the expected spread between $r^F_{t+1}(m)$ and the real deposit rate $r_{t+1}$ becomes strictly positive, i.e., $E_t [r^F_{t+1}(m) - r_{t+1}] > 0$. However, of course, whether the net worth of banker $m$ increases or decreases in the next period depends on the realization of $r^F_{t+1}(m)$.

Given the above net-worth transition equation, risk-neutral banker $m$ will maximize the net worth accumulation by maximizing the following objective function with respect to bank lending, $b^F_t(m)$:

$$V^F_t(m) = E_t \sum_{i=0}^{\infty} \beta^i (1 - \gamma^F_{t+1}) \gamma^F_{t+1,t+1+i} \left[ (r^F_{t+1+i}(m) - r_{t+1+i}) b^F_{t+i}(m) + r_{t+1+i}n^F_{t+i}(m) \right]$$  \hspace{1cm} (3.15)

where $\gamma^F_{t+1,t+1+i} \equiv \prod_{j=0}^{i} \gamma^F_{t+1+j}$. Now, since the expected spread between the risk-adjusted bank lending rate and the deposit rate is strictly positive, it is in the interest of banker $m$ to lend an infinite amount to an entrepreneur by accepting an infinite number of deposits from the depositor.

In order to avoid the infinite risk-taking by the banker, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) imposed a moral hazard/costly enforcement problem between the banker and the depositor. In each period, the banker has the technology to divert the fraction $\lambda$ of his asset holding to the household and exit from the banking business. However, by doing so, the banker is forced to file bankruptcy, and a fraction $(1 - \lambda)$ of his asset will be seized by the depositors. Thus, in order for the banker to continue business and for the depositors to safely deposit their funds to the banker, the following incentive constraint must be met each period:
\[ V_t^F(m) \geq \lambda b_t^E(m). \]  
\[ (3.16) \]

In other words, the net present value of the banking business needs to always exceed the reservation value retained by the banker.\(^8\)

Now, assuming the incentive constraint (3.16) to be binding each period and by maximizing the objective function (3.15) subject to the constraint (3.16), Gertler and Kiyotaki (2010) showed that the value function of the banker can be expressed as follows:

\[ V_t^F(m) = \nu_t b_t^E(m) + \eta_t n_t^F(m) \]  
\[ (3.17) \]

where

\[ \nu_t \equiv E_t \left[ (1 - \gamma_{t+1}^F) \beta (r_{t+1}^F - r_{t+1}) + \beta \gamma_{t+1}^F \frac{b_{t+1}^E(m)}{b_t^E(m)} \nu_{t+1} \right] \]  
\[ (3.18) \]

\[ \eta_t \equiv E_t \left[ (1 - \gamma_{t+1}^F) + \beta \gamma_{t+1}^F \frac{n_{t+1}^F(m)}{n_t^F(m)} \eta_{t+1} \right]. \]  
\[ (3.19) \]

Now, from the incentive constraint (3.16) and the value function (3.17), it follows that

\[ \frac{b_t^E(m)}{n_t^F(m)} \leq \frac{\eta_t}{\lambda - \nu_t} \equiv \phi_t \]  
\[ (3.20) \]

which states that the leverage ratio of banker \( m \) cannot exceed the (time-varying) threshold \( \phi_t \). By the assumption that the incentive constraint is binding for every period in equilibrium, the asset and the net worth by banker \( m \) have the following relationship:

\[ b_t^E(m) = \phi_t n_t^F(m). \]  
\[ (3.21) \]

### 3.2.3 Aggregation

Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) showed that time-varying threshold \( \phi_t \) does not depend on banker-specific factors and is common among all bankers. Consequently, from eq. (3.14), the aggregate asset and net worth in the banking sector can be expressed as follows:

\[ b_t^E = \phi_t n_t^F \]  
\[ (3.22) \]

where \( b_t^E \equiv \int_0^1 b_t^E(m) dm \) and \( n_t^F = \int_0^1 n_t^F(m) dm \). Now, from an individual banker’s net worth transition (3.14) and taking into account the entrance and exit of bankers, the aggregate net worth transition equation of banking sector is given by the following:

\[ n_{t+1}^F(t) = \gamma_{t+1}^F \left( (\tau_{t+1}^F - r_{t+1}) b_t^E + r_{t+1} n_t^F \right) + \xi_{t+1}^F n_{t+1}^F \]  
\[ (3.23) \]

where \( \tau_{t+1}^F \) stands for the average of the realized risk-adjusted returns, \( r_{t+1}^F(m) \), among all bankers. From the optimal debt contract specified in (3.8) and using the aggregate condition in (3.9), \( \tau_{t+1}^F \) is related to the borrowing rate, the external finance premium, and the inflation rate, as follows:

\[ \tau_{t+1}^F = \frac{R_t^E}{\pi_{t+1}s_t}. \]  
\[ (3.24) \]

---

\(^8\)To see how this constraint binds, consider the case where the banker increases the asset enormously. Then, the reservation value by the banker (the right side of the inequality (3.16)) will exceed the net present value of the banking business (the left side of the inequality (3.16)) such that the banker will decide to divert the assets to the household. As stakeholders, the depositors will not allow this reckless behaviour by the banker and will ask the banker to keep his asset, \( b_t^E(m) \), low enough (or, equivalently, by not supplying the deposits beyond the incentive constraint) to meet the incentive for the banker to remain in business.
As can be seen from the above equation, the idiosyncratic factor pertaining to banker \( m \) is averaged away and, thus, the realization of the risk-adjusted return of the banking sector (i.e., \( \tau_{m,t+1}^F \)) only depends on the aggregate factors in the economy. Now, by using (3.22), the aggregate net worth transition equation becomes

\[
\hat{n}_{t+1}^F = \gamma_{t+1}^F \left[ (\tau_{t+1}^F - r_{t+1}) \phi_t + r_{t+1} \right] n_t^F + \xi^F n_t^F. \tag{3.25}
\]

### 3.3 Incorporation of the two Frictions within the DSGE model

To incorporate the two financial frictions into a stylized DSGE model, we used twelve constraints and FOC equations, which consist of five and seven equations derived in the corporate and banking sectors, respectively.\(^9\) The five equations representing the financial friction in the corporate sector are (i) the balance sheet statement of the corporate sector (3.2), (ii) the capital demand function (3.6), (iii) the external financial premium (3.7), (iv) the realized gross return from the capital investment (3.11), and (v) the aggregate net worth transition equation of the corporate sector (3.12). On the other hand, the seven equations expressing the financial friction in the banking sector are (vi) the balance sheet statement of the banking sector (3.13), (vii) the dynamics of the weight on the lending volume for the value of the banking business, \( \nu_t \), (3.18), (viii) the dynamics of the weight on the bank’s net worth for the value of the banking business, \( \eta_t \), (3.19), (ix) the definition of the threshold, \( \phi_t \), (3.20), (x) the banker’s leverage ratio constraint (3.22), (xi) the relationship between the corporate nominal borrowing rate, and the risk-adjusted nominal lending rate of the banking sector (3.24), and (xii) the aggregate net worth transition equation of the banking sector (3.25).

To complete our model, we employed the CEE (2005)-type medium scale DSGE model described in Appendix A4 with the equations above, as well as the structural shocks. We set the following eight structural shocks, each of them having a specific economic interpretation; i.e., (1) TFP shock, (2) preference shock, (3) labour-supply shock, (4) investment specific technology shock, (5) government spending shock, (6) monetary policy shock, (7) corporate net-worth shock, and (8) bank net-worth shock. With the exception of the monetary policy shock, all of the structural shocks are assumed to follow AR(1) stochastic processes. We devoted the following two shocks out of the eight to identifying the fundamental factors causing the financial crisis. Corporate net-worth shock \( \varepsilon^E_t \) was inserted into the AR(1) process of the survival rate of the corporate sector \( \gamma^E_t \), which is a component of equation (3.12), while the bank net worth shock \( \varepsilon^F_t \) is put into the AR(1) process of the survival rate of the banking sector \( \gamma^F_t \), which is that of the equation (3.25). The two shocks are given as follows:

\[
\text{Corporate net worth shock} \quad : \quad \hat{\gamma}^E_t = \rho^E \hat{\gamma}^E_{t-1} + \varepsilon^E_t,
\]

\[
\text{Bank net worth shock} \quad : \quad \hat{\gamma}^F_t = \rho^F \hat{\gamma}^F_{t-1} + \varepsilon^F_t,
\]

where \( \rho \) is for the AR(1) coefficients for respective structural shocks. Both shocks indicating stochastic survival rate for entrepreneurs and bankers at the micro-level can be interpreted as net-worth shocks for the corporate and banking sectors at the aggregate level,

\(^9\)We have twelve model (or endogenous) variables corresponding to the twelve estimated equations pertaining to the financial frictions. These variables are (1) the capital, \( k_t \), (2) the real price of the capital, \( q_t \), (3) the asset of the corporate sector, \( b_t^E \), (4) the asset of the banking sector \( b_t^F \), (5) the corporate net worth \( n_t^E \), (6) the bank net worth \( n_t^F \), (7) the external financial premium, \( s_t \), (8) the gross return from capital investment, \( r_t \), (9) the time-varying weight of lending for the value of the banking business, \( \nu_t \), (10) the time-varying weight of the bank’s net worth for the value of the banking business, \( \eta_t \), (11) the corporate nominal borrowing rate, \( R_t^E \), and (12) the risk-adjusted lending rate of the banking sector, \( R_t^F \).
respectively. Notice that each stochastic disturbance $\varepsilon_t$ is assumed to follow a time-varying volatility using the SV model mentioned in Section 2.

4 Method of Estimation

In this study, a hybrid MCMC (also referred to as Metropolis-within-Gibbs) was employed as an estimation method of the data-rich DSGE model following Boivin and Giannoni (2006), and Kryshko (2011). The contribution of our study is to extend the data-rich DSGE to include SV shocks from i.i.d. shocks.

The benefit of employing a hybrid MCMC is not only to implement a sampling of the posterior of the model variables $S_t$, but also to implement a sampling of the posterior of the structural shocks $\varepsilon_t$. Using the sampling of the structural shocks, we composed historical decompositions. The MH algorithm, which is a general estimation method used for the regular DSGE model, has disadvantages for policy analysis because it cannot generate posterior structural shocks and induces the impossibility of generating a credible interval in policy simulation.

The objects of estimation in the state-space model, (2.17), and (2.18) are the structural parameters $\theta$—the parameters in the measurement equations: $\Gamma(= \{\Lambda, \Psi, R\})$, model variables $S^T(= S_1, S_2, \ldots, S_T)$, and stochastic volatilities $H^T(= h_1, h_2, \ldots h_T)$. For convenience, let log $\sigma_t$ denote $h_t$, hereafter. Notice that estimating $\theta, \Gamma, S^T, H^T$ is necessary and sufficient for estimating our model because matrices $G(\theta), E(\theta), Q(\theta)$ in the transition equation (2.18) are a function of the structural parameters $\theta$.

General speaking, the Bayesian estimation of parameters $\theta, \Gamma, H^T$ is implemented as described in the following steps.

Step I. We set priors of paramters $\theta, \Gamma, H^T$, i.e. $p(\theta, \Gamma, H^T)$ where $p(\theta, \Gamma, H^T) = p(\theta|\Gamma H^T)p(\Gamma|H^T)p(H^T)$, since $\theta, \Gamma, H^T$ are assumed to be independent.

Step II. Using Bayes theorem, posterior $p(\theta, \Gamma, H^T|X^T)$ is derived from prior $p(\theta, \Gamma, H^T)$ and likelihood function $p(X^T|\theta, \Gamma, H^T)$.

$$p(\theta, \Gamma, H^T|X^T) = \frac{p(X^T|\theta, \Gamma, H^T)p(\theta, \Gamma, H^T)}{\int p(X^T|\theta, \Gamma, H^T)p(\theta, \Gamma, H^T)d\theta d\Gamma dH^T}.$$

Step III. We obtain representative values (mean, median, credible band etc.) of parameters $\theta, \Gamma, H^T$ from posterior $p(\theta, \Gamma, H^T|X^T)$ using numerical technique.

However, it is troublesome to sample directly the joint posterior distribution $p(\theta, \Gamma, H^T|X^T)$ of a state-space model (2.17), (2.18) in step II. Instead, using Gibbs sampling, we obtained the joint posterior $p(\theta, \Gamma, H^T|X^T)$ from the conditional posterior $\theta, \Gamma$ and $H^T$ as described below.

$$p(\theta|\Gamma, H^T, X^T), \quad p(\Gamma|\theta, H^T, X^T), \quad p(H^T|\theta, \Gamma, X^T)$$

The algorithm of MCMC for estimating DSGE models was developed by Schorfheide (2000). Our method extends Schorfheide’s (2000) model to the hybrid MCMC. The algorithm of hybrid MCMC is described in Chapter Six of Gamerman and Lopes (2006) etc. We also adopted a speed-up algorithm of sampling state variables by Jungbacker and Koopman (2008), but found that it dropped the accuracy of the estimates of state variables, so we omitted the algorithm from the MCMC procedure.
In addition, since parameter $\Gamma$ is dependent on model variable $S_t$, we have to separate the two conditional posterior $p(S^T \mid \Gamma, \theta, H^T, X^T)$ and $p(\Gamma \mid S^T, \theta, H^T, X^T)$ from the above conditional posterior $p(\Gamma \mid \theta, H^T, X^T)$ and insert $S_t$ into the posterior. We also adopted a forward-backward recursion for sampling from $p(S^T \mid \Gamma, \theta, H^T, X^T)$ and from $p(H^T \mid \Gamma, \theta, S^T, X^T)$ as a data augmentation method, Gibbs sampling for sampling from $p(\Gamma \mid S^T, \theta, H^T, X^T)$, and from the MH algorithm for sampling from $p(\theta \mid \Gamma, H^T, X^T)$, respectively. In this way, different algorithms were employed for different parameters in a hybrid MCMC. In sum, we show six steps of the hybrid MCMC for estimating a data-rich DSGE model as follows.\textsuperscript{11}

\begin{enumerate}
  \item \textbf{Step 1.} Specify initial values of parameters $\theta^{(0)}$, $\Gamma^{(0)}$, and $H^{T(0)}$. And set iteration index $g = 1$.
  \item \textbf{Step 2.} Solve the DSGE model numerically at $\theta^{(g-1)}$ based on Sims' (2002) method and obtain matrices $G(\theta^{(g-1)})$, $E(\theta^{(g-1)})$, and $Q(\theta^{(g-1)})$ in equation (2.18).
  \item \textbf{Step 3.} Draw $\Gamma^{(g)}$ from $p(\Gamma \mid \theta^{(g-1)}, H^{T(g-1)}, X^T)$.
    \begin{enumerate}
      \item \textbf{Step 3.1} Generate model variables $S_t^{(g)}$ and structural shocks $\varepsilon_t^{(g)}$ from $p(S^T, \varepsilon^T \mid \Gamma^{(g-1)}, \theta^{(g-1)}, H^{T(g-1)}, X^T)$ using simulation smoother by de Jong and Shephard (1995).
      \item \textbf{Step 3.2} Generate parameters $\Gamma^{(g)}$ from $p(\Gamma \mid S^{T(g)}(t), \theta^{(g-1)}, H^{T(g-1)}, X^T)$ based on the sampled draw $S^{T(g)}(t)$ using Gibbs sampling by Chib and Greenberg(1994).
    \end{enumerate}
  \item \textbf{Step 4.} Draw $H^{T(g-1)}$ from $p(H^T \mid \theta^{(g-1)}, \Gamma^{(g)}, \varepsilon^{T(g)}, X^T)$.
    \begin{enumerate}
      \item \textbf{Step 4.1} Generate stochastic volatility $H^{T(g)}$ from $p(H^T \mid \Gamma^{(g)}, \theta^{(g-1)}, \varepsilon^{T(g)}, u^{T(g-1)}, \phi^{(g-1)}, X^T)$, using a draw of $\varepsilon^{T(g)}$ at Step 3.1, and the forward-backward recursion by Cater and Kohn (1994).
      \item \textbf{Step 4.2} Generate the indicators of the mixture approximation $u^{T(g)}$ using discrete density proposed by Omori et al. (2007).
      \item \textbf{Step 4.3} Generate the coefficients $\phi^{(g)}$ of stochastic volatility process using Metropolis step.
    \end{enumerate}
  \item \textbf{Step 5.} Draw deep parameters $\theta^{(g)}$ from $p(\theta \mid \Gamma^{(g)}, H^T(g), X^T)$ using Metropolis step:
    \begin{enumerate}
      \item \textbf{Step 5.1} Sample from proposal density $p(\theta \mid \theta^{(g-1)})$ and, using the sampled draw $\theta^{\text{proposa}}$, calculate the acceptance probability $q$ as follows.
        \[ q = \min \left[ \frac{p(\theta^{\text{proposa}} \mid \Gamma^{(g)}, H^T(g), X^T)}{p(\theta^{(g-1)} \mid \Gamma^{(g)}, H^T(g), X^T)} \frac{p(\theta^{(g-1)} \mid \theta^{\text{proposa}})}{p(\theta^{\text{proposa}} \mid \theta^{(g-1)}}) \right]. \]
      \item \textbf{Step 5.2} Accept $\theta^{\text{proposa}}$ with probability $q$ and reject it with probability $1 - q$. Set $\theta^{(g)} = \theta^{\text{proposa}}$ when accepted and $\theta^{(g)} = \theta^{(g-1)}$ when rejected.
    \end{enumerate}
  \item \textbf{Step 6.} Set iteration index $g = g + 1$ and return to Step 2 up to $g = G$.
\end{enumerate}

The algorithm of sampling stochastic volatilities $H^T$ in Step 4 is explained in Appendix A1. And a simulation smoother in Step 3.1 and of sampling parameters $\Gamma$ in Step 3.2 are in Appendix A2 and A3, respectively.\textsuperscript{12}

\textsuperscript{11}Bayesian estimations using MCMC for state space models are described in detail in textbooks such as Kim and Nelson (1999) and Bauwens et al. (1999).

\textsuperscript{12}Here, we supplement Steps 1 and 5. On setting of initial values of parameters $\theta^{(0)}$ and $\Gamma^{(0)}$ in Step 1, it is
5 Preliminary Settings and Data Description

5.1 Specifications of Four Alternative Cases

This study considered four alternative cases based on the number of observation variables (11 vs. 40 observable variables) and on the specification of the volatilities of the structural shocks (constant vs. time-varying volatility) as summarized in Table 1. The reason why four cases are considered is that we wanted to verify whether data-rich information decomposes the measurement errors and the model variables from more robust data, and whether relaxation of specifying the volatilities depicts a rapid change of the more detailed shocks in the data-rich approach. The first case (referred to as Case A) dealt with one of the standard DSGE models that used 11 observable variables in the measurement equation (2.11) and the structural shocks with i.i.d. Normal distribution in the transition equation (2.12). In Case A, each observable variable was connected with its specified model variable by one-to-one matching. The second case (Case B) was extended to the data-rich approach with i.i.d shocks, including 40 observable variables, which indicate more or less four observable variables corresponding to one specified model variable. The third case (Case C) extends to SV shocks from Case A. And the fourth case (Case D) extends to the data-rich approach with SV shocks from Case B. Nishiyama et al. (2011) have already studied the comparison of a standard DSGE approach (Case A) and a data rich approach (Case B). This study focused on the remaining two cases (C and D) with SV shock, using Case A as the reference model.

Known that arbitrary initial values are eventually converged in MCMC. However, we require a huge number of iterations in MCMC simulation to converge to target posterior distributions in the case of a considerable number of parameters. Accordingly, following Boivin and Giannoni (2006) we set initial values as below for converging effectively. First, the posterior mode of structural parameters \( \theta \) in a regular DSGE model without measurement errors is derived from numerical calculation and set as initial values \( \theta^{(0)} \). Second, implementing simulation smoother of state variables \( S_t \) using \( \theta^{(0)} \), we get initial value \( S_t^{(0)} \). Finally, initial values \( \Gamma^{(0)} \) of measurement equations are obtained by OLS using \( S_t^{(0)} \) and \( X_T \).

Next, on generating structural parameters \( \theta \) from proposal density in Step 5.1, we adopt a random walk MH algorithm following previous works. Proposal density \( \theta^{(\text{proposal})} \) is represented as

\[
\theta^{(\text{proposal})} = \theta^{(g-1)} + u_t, \quad u_t \sim N(0, c\Sigma),
\]

where \( \Sigma \) is variance covariance matrix of random walk process, and \( c \) is the adjustment coefficient. The matrix \( \Sigma \) is the Hessian \((-l^{(g-1)}(\theta)')\) of log posterior distribution \( l(\theta) = \ln p(\theta | \Gamma, X_T) \) when obtaining initial value \( \theta^{(0)} \). And the case of sampling in MH algorithm,

\[
p(\theta^{(g-1)} | \theta^{(\text{proposal})}) = p(\theta^{\text{proposai}} | \theta^{(g-1)})
\]

is held in a stationary state of the target distribution, so that acceptance rate \( q \) is reduced to the following equation.

\[
q = \min \left[ \frac{f(\theta^{(\text{proposal})})}{f(\theta^{(g-1)})}, 1 \right],
\]

In this equation, acceptance rate \( q \) is not dependent on proposal density \( p(\theta | \theta^{(g-1)}) \). As a result, it is the advantage of a random walk MH algorithm that we do not need to adopt a proposal density close to posterior density. However, when proposal value \( \theta^{\text{proposai}} \) departs from the previous sample \( \theta^{(g-1)} \), acceptance rate \( q \) becomes small and efficiency of MCMC worsens. To avoid this, the adjustment coefficient \( c \) should be small, but doing so narrows the range of sampling space of \( \theta^{\text{proposai}} \). Roberts et al. (1997) and Neal and Roberts (2008) reported that the optimal acceptance rate \( q \) of a random walk MH algorithm is around 25%. Accordingly, adjustment coefficient \( c \) of this study is set so that the acceptance rate is close to around 25 %.
5.2 Calibrations and Priors of Parameters

We calibrated the subset of the structural parameters in the model that are not identifiable (i.e., the parameters that are only used to pin down the steady states) or that are difficult to identify from the observed data. Calibrated parameters with their descriptions are reported in Table 2. We assumed the discount factor was \( \beta = 0.995 \) so as to make the steady state, real interest rate to be 2% (annual rate). We assumed the profit margin of the retailers to be 10% in the steady state and, thus, set the elasticity of substitution of intermediate goods as \( \epsilon = 11 \). Having no reliable information regarding the new entry rate of entrepreneurs (i.e., \( \xi^E \)), we simply set it as equal to the calibration for a new banker’s entry rate, as did Gertler and Kiyotaki (2011). The rest of the calibrated parameter values were borrowed from Smets and Wouters (2003), Christensen and Dib (2008), and Gertler and Kiyotaki (2011).

Most of the steady states were pinned down by equilibrium conditions of the model, but some others need to be calibrated. For the steady-state value of the external finance premium, we followed the calibration of Christensen and Dib (2008). For the steady state corporate borrowing rate (real, quarterly rate), we calculated the historical average of the yields of Moody’s Baa-rated corporate bonds and set it as the steady-state rate. In the same way, we calculated the historical average of the non-farm, non-financial business leverage ratio based on Flow of Funds and set it as the steady state of the corporate leverage ratio. Finally, the government expenditure-to-output ratio in the steady state is set to be 0.2 because of being borrowed from Gertler and Kiyotaki’s (2011) calibration.

Next, we turn to describe the prior distribution of interest as a preamble for the Bayesian estimation. The settings of priors are reported in Table 3. We set \( \varphi = 0.05 \) for the prior mean of this parameter, which controls the sensitivity of the external finance premium with respect to the corporate leverage ratio, following the calibration of BGG. For the AR(1) persistence parameters for the structural shocks, we set the prior mean equal to 0.5 for all of them. For standard errors of the structural shocks, we set the prior mean equal to 1% for each standard error, except for the monetary policy shock (where a change of policy rate for more than 25 basis point is rare). By the same token, we set the prior mean equal to 1% for most of the measurement errors, except for the data related to interest rates.

5.3 Data Description

For adopting the data-rich approach, a relatively large and quarterly panel dataset with as many as 40 observable variables was used, as described in detail in Data Appendix (Table A1). The sample period of the estimation was between 1985Q2 and 2012Q2, because we wanted to avoid the effect on the estimation result from the instability of the monetary policy regime changes, especially around the end of the 1970s and the early 1980s; i.e., pre- and post-regimes, by Volcker and Greenspan, (See Clarida et al. 2000, Lubik and Schorfheide 2004, and Boivin 2005 ) and from the structural change of the Great Moderation, which began in mid-1980s (See Bernanke 2004, Stock and Watson 2002, Kim and Nelson 1999, and McConnell and Perez-Quiros 2000). And another reason the sample period was determined as it was had to do with the availability of financial data, because charge-off rates for banks are available only from 1985Q1.

In Cases A and C, we looked at the following eleven series: (1) output, \( y_t \), (2) consumption, \( c_t \), (3) investment, \( i^k_t \), (4) inflation, \( \pi_t \), (5) real wage, \( w_t \), (6) labour input, \( l_t \), (7) the nominal interest rate, \( R_t \), (8) the nominal corporate borrowing rate, \( R^E_t \), (9) the external finance premium, \( s_t \), (10) the corporate leverage ratio, \( q_t k_t / n^E_t \), and (11) the bank leverage
ratio, $b_t^F/n_t^F$, as observable variables in the measurement equation (2.11). The first seven series are generally used in a large amount of literature estimating DSGE models (see, for instance, Smets and Wouters, 2003 and 2007). Using the four remaining financial time-series as observable variables is the featured difference of our DSGE model compared with existing models. These four actual series were selected for matching the model variables corresponding to the two financial frictions. The entrepreneur’s nominal borrowing rate (8), $R_t^E$, is the yield on Moody’s Baa-rated corporate bonds, which is de-trended via the Hodrick-Prescott filter for the same reason of inflation and the interest rate. To measure the external financial premium (9), $s_t$, we employed the charge-off rates for all banks’ credits and issuer loans, measured as an annualized percentage of uncollectible loans. The charge-off rate was deemed to be consistent with our model variable (10 and 11). The two leverage ratios, $q_t^k/n_t^E$ and $b_t^F/n_t^F$, were calculated as their total asset divided by their net worth, respectively. We took the natural logarithm for both leverage ratios, and then either demeaned for the entrepreneur’s leverage ratio, or detrended the banking sector leverage ratio with the Hodrick-Prescott filter, based on taking into account the Basel Capital Accord Revision.

In Cases B and D, which adopted the data-rich approach, indicating one model variable corresponding to four actual series, we employed an additional 29 series, which consisted of 18 series of key macroeconomics and 11 series of the banking sector, with the existing 11 series in Cases A and C. We selected 18 data indicators for the six key model variables used in a standard DSGE model such as (1) output, (2) consumption, (3) investment, (4) inflation, (5) real wage, and (6) labour input, but not the nominal interest rate, as mentioned in the Data Appendix, along the same lines of Boivin and Giannoni (2006). On the other hand, 11 series of the banking sector were selected along the lines concentrated on banking, which is a departure from previous work dealing with the data-rich framework. Three additional banking indicators include the following: (i) the core capital leverage ratio, (ii) the domestically chartered commercial banks’ leverage ratio, and (iii) the leverage ratio of brokers and dealers, and these were selected as data indicators, corresponding to the model variable, such as the banking sector leverage ratio. Notice that as an observable variable of the leverage ratio, we used the inverse of the commonly-used ratio, i.e., bank asset over bank equity. As data indicators for the external financial premium, we selected three kinds of loan charge-off rates based on different institutions, which were transformed as percentage deviations from trends using the same detrending methods described above.

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13 The output (1) is the real GDP less the net export. Consumption (2) and investment (3) are normalized, respectively, to personal consumption expenditures and fixed private domestic investment. Following Altig et al. (2003), Smets and Wouters (2003), and Boivin and Giannoni (2006), the nominal series for consumption and investment are deflated with the GDP deflator. The real wage (5) was normalized with the hourly compensation for the nonfarm business sector, divided by the GDP deflator. The labour input (6) corresponds to hours worked per person. The average number of hours of the non-farm business sector are multiplied by the number for civilian employment to represent the limited coverage of the non-farm business sector, compared to the GDP, as discussed by Smets and Wouters (2003) and by Boivin and Giannoni (2006). We express these six series as percentage deviations from steady states, consistently with the model concepts, taking the natural logarithm, extracting the linear trend by an OLS regression, and multiplying the resulting de-trended series by 100. Inflation measures (4) were obtained by taking the first difference of the natural logarithm of the GDP deflator, and then multiplying the result by 400 to express the annualized percentages. The nominal interest rate (7) is the effective federal funds rate. Both inflation and the interest rate were de-trended via the Hodrick-Prescott filter (the penalty parameter is 1600), indicating a time-varying targeting inflation rate.

14 The core capital leverage ratio represents tier 1 (core) capital as a percentage of the average total assets. Tier 1 capital consists largely of equity. We used the inverse of the core capital leverage ratio because of its corresponding to the ratio of banks’ assets to banks’ net worth, before taking the natural logarithm, and then we detrended with the Hodrick Prescott filter. Following Adrian and Shin (2010), we added the leverage ratio of brokers and dealers since investment banks are categorized as brokers and dealers in Flow of Funds (FOF), and the financial shock was caused mainly by the deterioration of investment bankers’ balance sheet conditions.
6 Empirical Results

In this section, we report the results of our estimation and especially focus on the estimates of several key structural parameters, those of the SV shocks, and historical decompositions of four principal model variables: (1) output, (2) investment, (3) bank leverage and (4) borrowing rate, playing a significant role in the recession and the financial crisis of 2007-2008. Then, we discuss and remark on the sources of the recession in light of the data-rich approach. Our estimation results are constructed from 300,000 draws of the hybrid MCMC algorithm as posterior distributions of interest for every case.

6.1 Key Structural Parameters

The estimates of the structural parameters of Cases A and B are summarized in Table 9, and those of Cases C and D are expressed in Table 10. Table 11 notes the estimates of the parameters of the SV models concerning the eight structural shocks. Because the New-Keynesian DSGE model with the two financial frictions was estimated, we focused on interpreting the seven key structural parameters, i.e., two financial frictions, two nominal rigidities, and three monetary policy parameters. Table 4 shows the collection of the parameters of the four cases for ease in comparing with one another. The parenthesis in the table indicates the 90% credible interval of the posterior distribution of the structural parameters.

First of all, we considered two estimated parameters involved in the financial friction of the corporate sector; \( \kappa \) and \( \varphi \). \( \kappa \) denotes the elasticity of the quadratic adjustment cost of investment in eq.(A20), as described in Appendix A4, while \( \varphi \) is the elasticity of the external financial premium in eq.(3.7). According to Table 4, the posterior mean of \( \kappa \) in Case B (the data-rich approach with constant-volatility shocks) is around 0.88, whereas those in the rest of the cases are between 0.56 and 0.63. The large elasticity, \( \kappa \), in Case B implies that the corporate net worth shock more strongly amplifies the fluctuation of business cycles via the channel of adjustment of the capital asset price (Tobin's marginal q). On the other hand, the posterior means of \( \varphi \) are less than 0.03 in Cases A and B (models with constant volatility shocks), whereas those on the counterpart cases with SV shocks are nearly 0.04. Because the elasticity of the external financial premium, \( \varphi \), which reflects the size of the agency costs in the corporate sector, is the coefficient of the logarithm of the corporate leverage ratio for explaining the aggregated level of external financial premium \( s_t \) as shown in eq.(3.7), the relatively large size of \( \varphi \) in Cases C and D, incorporating the effect of the SV shocks into the DSGE models, suggests that variation of the leverage ratio in the corporate sector is likely to be more influential in enlarging the external financial premium and to lead to a more severe decline of investment than would the results of Cases A and B with structural shocks following normal i.i.d. Notice that there are no parameters concerning the financial friction in the banking sector of our model, following Gertler and Kiyotaki (2010) and Gertler and Kradi (2011), so we cannot show the comparison among the cases in the banking sector. Instead, structural shock pertaining to banking net-worth will be compared among the cases in the following subsection.

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15 We adopted Sims’ (2002) method in solving for our DSGE model, and all estimation procedures were implemented using GAUSS.
16 300,000 iterations were implemented using the MH-within-Gibbs. We sampled one draw out of every 10 replicates to reduce the impact of auto-correlations between draws of interest on their target posterior distributions, and then we stored a total of 30,000 samples. Then, we discarded the first 10,000 samples, and the remaining 20,000 samples were used for calculating moments of the posterior distributions.
Next, the nominal rigidities of the price level and the wage are examined. In our study, these rigidities are governed by a Calvo-type model and are explained in Appendix A4. The Calvo price $\theta_P$ implies that the ratio of intermediate-goods firms face monopolistic competition and reconcile to maintain their price without optimally setting a new price. The value of $\theta_P$ is theoretically a real number between 0 and 1. As can be seen from Table 4, the posterior mean of $\theta_P$ in Case B is nearly half the size (0.37) of the other three cases as those have a relatively high normal rigidity (around 0.8), indicating that new prices are set at one-time per every five-quarter period. On the other hand, Calvo wage $\theta_W$ (the ratio of workers facing monopolistic competition and reconciling to maintain their wage without optimally setting a new wage) is nearly 0.43 in Case B and those in other cases are between 0.5 and 0.6. These values imply that wages are reset with a frequency of around one time per half year.

Finally, Table 4 also reports the three parameters of a Taylor-type monetary policy rule, eq.(A24) described in Appendix A4. $\rho^R$ denotes the size of the inertia of the policy interest rate, i.e., the federal fund rate. And, $\mu^\pi$ and $\mu^Y$ are Taylor coefficients in response to the inflation gap and the output gap, respectively. There are no big differences among the three parameters in the four cases. That is, $\rho^R$ is between 0.63 and 0.67, $\mu^\pi$ is between 2.8 and 3.0, and $\mu^Y$ is tiny, such as from 0.006 to 0.010. These results imply that the central bankers’ reactions for the inflation gap are aggressively implemented, while those for the output gap are not so. However, the volatilities of the monetary policy shocks are largely different among the four cases. We will see in Section 6.2 that time-varying volatilities of the monetary policy shocks rapidly increased in the period of the recession.

### 6.2 Structural Shocks and their Volatilities

Figure 1 shows the posterior mean and a 90% credible interval of the eight structural shocks in Cases A and B, which deal with models with constant volatility shocks, whereas Figure 2 shows those in Cases C and D, estimating models with time-varying volatility shocks. In panel (a) of each figure, the shocks of the model with the 11 observable variables are drawn with deep blue solid lines for posterior means and with a light blue shade for the 90% interval, while panel (b) shows the estimates of the shocks of the data-rich approach with deep red solid lines and with a light red shade in a similar way. In each panel, the lines and shadings of its counterpart are underlain because of their being compared to each other. Also Figure 3 depicts the posterior means and the 90% intervals of the time-varying volatility in Cases C and D. From Figures 1 and 2, two points are impressively observed. First, the fluctuation of each structural shock is different in the four cases based on the number of observable series and the specification of the shocks, despite using the same DSGE model. This induces different interpretations of economic analysis for the business cycle, despite adopting the same models. Second, the structural shocks (red shade) estimated from the data-rich approach seemed to fluctuate with bigger swings than those (blue shade) of the standard approach. In particular, the red shade is distinguished in the data-rich approach as covering almost the entire area of the blue shade in Case C in Figure 2, dealing with models with SV shocks.

[Insert Figure 1 and 2 around here]

Next, we focus on the two structural shocks pertaining to the financial frictions in the banking and corporate sectors. In Table 5, the timings of the peaks of the two shocks are described for the four cases. At first, the banking net-worth shocks have exactly the same peak at 2008Q3 for all cases. In this period, i.e., September and October 2008, several major financial institutions either failed, were acquired under duress, or were subject
to government takeover. These financial institutions included Lehman Brothers, Merrill Lynch, Fannie Mae, Freddie Mac, Washington Mutual, Wachovia, Citi group, and AIG. However, the timings of the peaks of the corporate net-worth shock are not consistent and are divided into two periods, i.e., 2009Q1 in Cases A and B, and 2009Q2 in Cases C and D. Notice that the corporate net-worth shocks have peaks after the banking sector shocks hit its peaks, whatever the case.

[Insert Table 5 around here]

We considered the accuracy of the estimation for the eight shocks using an average range of 90% to be a credible interval across all of the sample period, as shown in Table 6. If we observe that the 90% interval ranges are smaller, then we might think that the shocks are likely to be identified more precisely. Among the four cases, five average intervals of shocks out of eights are smaller in Cases C and D than in Cases A and B. These five shocks are (1) preference, (2) banking net worth, (3) labour supply, (4) government spending, and (5) monetary policy. In the two cases with time-varying volatility shocks, the intervals in the former three shocks are about half those of the other two cases with constant shocks. And those of government spending shocks shrank by one eighth to one tenth by adopting SV shocks. These statistics suggest that the constant volatilities of shocks might be mis-specified and that the shocks follow time-varying volatilities. In particular, the volatilities are expected to change to larger values at the turning points of the business cycles, as shown later.

[Insert Table 6 around here]

Figure 3 draws estimates of the time-varying volatilities of shocks for Cases C and D. Surprisingly, the seven shocks, minus the government spending shocks, are very similar in both cases. In Figure 3, the deep blue and the deep red solid lines denote the posterior means of Cases C and D, respectively. As seen from this figure, the six shocks, minus the preference and labour supply shocks, are very stable and level off between 1990Q1 and 2007Q3, while the preference and labour supply shocks might play an important role of the boom between 2003 and 2005. After August 2007, when the financial crisis of 2007 to 2009 began with the seizure in the banking system, precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialized in US mortgage debt, the volatilities of both the banking and the corporate net-worth, investment, and TFP rapidly increased. Although our DSGE model is not thought to perfectly capture the macroeconomic fluctuations in the period of the Great Recession, the estimates show extraordinary sizes of the volatilities in this period.

[Insert Figure 3 around here]

Table 7 indicates the average 90% intervals of the SV over the entire sample period in the two cases because of our verifying whether the data-rich approach contributes to the improvement of the estimates of the SV of the shocks. As seen from Table 7 as well as from Figure 3, there are no differences of means of the interval ranges between Cases C and D. However, it might be too hasty to conclude that the data-rich method does not improve the accuracy of the SV estimates. We would need to validate the further evidences in this question.

[Insert Table 7 around here]
Next, we turn to discuss the leverage effects of the SV shocks. Table 11 summarizes the estimation results of the parameters in the SV model defined in eq.(2.2) through eq.(2.4) and used in Cases C and D. The leverage effect is represented by the sign of the correlation coefficient $\rho_{\sigma}$ of each shock. If $\rho_{\sigma}$ is negative, the shock has a leverage effect, which implies that the negative shock at the present period amplifies its volatility at the next period, and vice versa. Table 8 sums up the sign of the correlation coefficient $\rho_{\sigma}$ of each shock in terms of the 90%-credible interval. The mark “-” indicates the negative of $\rho_{\sigma}$ (leverage effect) at the 90%-credible degree of the posterior probability, while the mark “+” indicates the positive of $\rho_{\sigma}$ (opposite to the leverage effect) in a similar way. The mark “0” implies that we do not judge the sign of $\rho_{\sigma}$ nor the leverage effect of each shock because zero is within the 90% interval of $\rho_{\sigma}$. According to many financial empirical studies, the leverage effect is often observed in financial time series such as in stock prices. Our question is whether banking and corporate net-worth shocks have the leverage effect, which would imply that a decline of net-worth shocks leads to extending its volatility or its uncertainty in the next period. However, we did not observe leverage effects for these two shocks; instead we observe the opposite leverage effect of the corporate net-worth shock in Case C, as shown in Table 8. This result might be derived from either the number of observations, the specification of our DSGE model, or something else. To answer this question, we need to continue development of the econometric method as well as to select and to accumulate data.

[Insert Table 8, 9, 10 and 11 around here]

Finally, we note the monetary policy in the period of the Great Recession, although we adopted a liner-type Taylor rule and estimated it for the sample period including QE1 (round 1 of quantitative easing by FRB, between 2008Q4 and 2010Q2) and QE2 (2010Q4 to 2011Q2). The monetary policy shocks in Figures 1 and 2 seem to have two big negative spikes after 2007. The first negative spike was observed at 2007Q4 when the BNP Paribas announcement impacted the global financial market. And the second one was observed at 2008Q3, immediately before the FRB conducted an unconventional monetary policy (QE1). In particular, the magnitudes of these two negative shocks are distinguished in the cases of time-varying volatility, as shown in Figure 2. Figure 3 also captures the rapid appreciation of these volatilities of policy shocks in the period between 2007Q4 and 2008Q3. Table 8 shows the monetary policy has the opposite leverage effect across all of the sample periods in Case D, even though FRB took tremendous monetary easing policies in the recession. That is, tightening the policy is likely to happen more boldly and without hesitation, while easing policy might be done more carefully, according to the results with the 90% credible degree of the posterior probability.

6.3 Historical Decompositions

To investigate the sources of the Great Recession, we focused on the historical decompositions of four observable variables: (1) the real GDP as an output gap, (2) the gross private domestic investment (fixed investment) as investment, (3) Moody’s bond index (corporate Baa) as the corporate borrowing rate, (4) the commercial banks’ leverage ratio as the bank leverage ratio, which is described in detail in the Data Appendix. Each of Figures 4 through 7 draws four decompositions of each observable variable based on the four cases for the periods between 2000Q1 and 2012Q2, and the light blue shade denotes the period of the Great Recession (2007Q3 to 2009Q2). To facilitate visualization and focus on the contributions of the two financial frictions, the technology and monetary policy shocks for the recession, we collected the remaining four miscellaneous shocks as one bundle in these figures.
First we will examine real activities. Figures 4 and 5 show the historical decompositions of the real GDP and the gross private domestic investment, respectively. Because the decompositions of these real activities’ variables have similar properties, we discuss them as a whole. Although the signs of the contribution of each shock are the same in every case of the two variables at each period, we can see that the sizes of the contribution of shocks are different depending on the cases. In Case A (standard DSGE model), the TFP shock accounted for a large portion of the sources of the Great Recession (light blue shade), while the decline of the bank net-worth impacted a small part of drops. And the positive corporate net-worth increased and contributed to raising these variables by a significant portion during the recession, in this case. On the other hand, the remaining three cases showed that the positive effect of the corporate net-worth shock were small, and that the bank net-worth shock accounted for a bigger portion of the downturn of these real activities in the period. Even in the period of the US economy's recovery, Cases A and C show a different picture from what Cases B and D show. The main source of blocking the recovery is derived from the negative TFP shocks in Cases A and C, whereas TARP worked and prominently improved the bank's balance sheet, so the positive bank net-worth shock contributed to raising the real activities in the remaining cases. In addition, decompositions of these cases suggest that a deterioration of the corporate sector’s balance sheet is likely to be the main source blocking a recovery after the recession.

In Figure 6, Moody's bond index (corporate Baa) is decomposed as a corporate borrowing rate. According to the figure, a sharp rise of the rate might be derived from the mainly negative bank net-worth shock as well as from a fall of the TFP shock, whereas the positive firm net-worth shock contributed to the downward rate in the recession. And then, the firm net-worth shock turned to be remarkably negative, seriously deteriorating its balance sheet and accounting for a large portion of the rise of the rate after the recession. On the other hand, TARP might work well and make the bank net-worth shock become positive, which would contribute to the downward borrowing rate after 2010Q1. In particular, we can see these findings in Cases B and D.

Figure 7 depicts the decomposition of the commercial banks’ leverage ratio, defined as the inverse of the commonly-used ratio, i.e., the bank’s asset over the bank’s net-worth. As

---

17 According to the Agency Financial Report by the U.S. Treasury (2009), TARP, including a schema aimed at the capital injection into financial institutions, had been implemented from 2008 Q4 to 2009Q4. As Figure 2, the direction of the bank net-worth independent shock turned to positive sign from the negative one in 2008 Q3, and this can be thought to have captured the effect of TARP on the intended financial institutions. This figure also shows the bank net-worth shocks had been negative before TARP has been done in 2008 Q3. Since the historical decomposition shows the effect of shocks accumulated by AR process derived from independent structural shocks on the endogenous variables, the positive independent shock in 2008 Q4 was offset by accumulation of the negative shocks sustained up to 2008 Q3. The positive bank net-worth shocks have contributed to an increase in real GDP after 2010 Q1 as can be seen from Figure 4.

18 TARP had also provided financial supports for GM and Chrysler after 2009 Q1. This effect is depicted as switch of the direction of the corporate net-worth shock from negative to positive in 2009 Q2 as Figure 2. However, the corporate net-worth shock was the main source of decreasing real GDP after 2009 Q3, than to the accumulated negative shocks sustained up to 2009 Q1 as Figure 4. The reasons why the positive independent shock by the financial supports could not make the AR shock change to positive are (i) the negative corporate net-worth shock had been so large up to 2009 Q1 relative to the financial support after 2009 Q1, (ii) the shocks was negatively amplified by the drop of the capital goods price due to the financial accelerator mechanism during the recession, and (iii) the persistency of the shocks was relatively high in terms of coefficient of AR process.
can be seen from this figure, the inverse ratio is observed to be countercyclical, and the contributions of the shocks to the fluctuations are explained. Both financial shocks in the banking and the corporate sectors with conflicting directions, i.e., a negative banking balance sheet shock and a positive corporate balance sheet shock, contributed to an increase in the ratio at almost the same proportion in the recession. Soon after that, an increase in bank equity by conducting TARP made its balance sheet improve, while the negative firm net-worth shock made the firm balance sheet much worse, leading to a sharp reduction of loans by the bank. Both of these things brought the ratio down, since the numerator of the ratio consists of both the loan and the equity in the banks, and the denominator is only its equity. These findings were observed in every case. However, the dynamics of the countercyclical inverse ratio were not generated from Gertler and Kiyotaki (2011). Recently, Adrian et al. (2012) tried to explain why the inverse ratio was countercyclical, following two conflicting movements of banking loans and bond financing of firms, i.e., loan declines and bond increases in the recession. Our findings about both conflicting financial shocks in the recession are consistent with Adrian et al.’s (2012) findings.

6.4 Observations and Interpretation

Overall, we can make three important observations based on our empirical results. First, as for the timing of the financial shocks during the period of the Great Recession shown in Figures 1 and 2, we observed that the bank’s net-worth shock occurred earlier than the corporate net-worth shock. Putting it differently, these two financial shocks did not occur concurrently, but the corporate net-worth shock occurred shortly after the bank’s net-worth shock. This timing pattern (not concurrent, but proximate timing) may point to the possibility of an endogenous relationship between the balance sheet conditions of the banking sector and the corporate sector. For instance, in reality, it is possible for the corporate sector to hold the financial sector’s equity as an asset, and the devaluation of the financial sector’s asset may affect the balance sheet condition of the corporate sector. Unfortunately, however, the model in this paper does not allow the corporate sector to hold the banking sector’s equity as an asset and further assumes the two financial shocks to be independent from each other. In this paper, the corporate sector is assumed to hold the asset fully in the form of physical capital. Thus, it is inappropriate to interpret the endogenous relationship between the two financial shocks in the context of the model assumed in this paper. Yet, the timing of the two financial shocks during the Great Recession is worth noting.

Second, through the historical decomposition results shown in Figure 4 through Figure 7, we observed the corporate net worth shock during the Great Recession to be relatively weak in Cases A and C compared to those in Cases B and D. This result may point to the possibility of an underestimation of the importance of the corporate net worth shock when the model is estimated by a plain Bayesian estimation method – i.e., without the data-rich estimation. Moreover, an accurate estimation of the corporate net-worth shock during the Great Recession is crucially important in accounting for the economic recovery of the U.S. economy in recent years. For instance, in Cases A and C, a slow recovery of output is mainly accounted for by the negative productivity shock, while in Cases B and D, it is mainly accounted for by a prolonged negative corporate net-worth shock. The slow recovery of the U.S. economy after the Great Recession remains an important puzzle, and a persuasive explanation of this puzzle calls for an accurate estimation of the structural shocks. For an accurate estimation of the structural shocks (especially for the corporate net-worth shock), a more accurate estimation of the structural shocks is required.

\[\text{In this paper, the corporate sector is assumed to hold the asset fully in the form of physical capital.}\]
worth shock), a data-rich estimation with stochastic volatility may be more reliable than an unenriched Bayesian estimation method.

Third, another important observation from the historical decomposition results is the behaviour of the bank’s net-worth shock. The bank’s net-worth shock declined sharply during the Great Recession and was the main source of the sharp decline in output and investment, as shown in Figures 4 and 5. But then, right after the Great Recession period, the bank’s net-worth shock quickly reversed its direction and contributed positively to output and investment. Considering the timing of this reversal, it is quite possible that the implementation of the TARP is behind this reversal. In other words, the implementation of TARP may have successfully countered the bank’s negative net-worth shock. Interpreting further, considering the positive contribution of the bank’s net-worth shock to the output and to the investment right after the Great Recession period, the implementation of TARP may be one of the major reasons behind the stopping of the Great Recession and contributing to the recovery (albeit weak) of the U.S. economy in recent years.

7 Conclusion

According to the NBER, the Great Recession, in which the financial crisis played a significant role in the failure of key businesses, declined consumer wealth—estimated in trillions of US dollars—and caused a downturn in economic activity leading to the 2008–2012 global recession and contributing to the European sovereign-debt crisis. This recession is reported to have begun in December 2007 and to have ended in June 2009. The purpose of this study is to analyze mutual relationship among macroeconomic and financial endogenous variables in terms of business cycles and to identify what structural exogenous shocks contributed to the Great Recession in light of a DSGE model. Because we obtained a broad consensus that solvency and liquidity problems of the financial institutions are chief among the fundamental factors causing the recession itself, it is plausible to embed financial frictions in both the banking and the corporate sectors of a New Keynesian DSGE model. To this end, we followed Nishiyama et al. (2011) who already studied the US economy using a New Keynesian DSGE model with these two financial frictions in a data-rich environment. In this model, with asymmetric information about borrowers and lenders, banks have two roles, generating two agency costs: one is as the lenders to the corporate sector and the other is as the borrowers from the depositors. Furthermore, the structural shocks in the model are assumed to possess SV with a leverage effect. Then, we estimated the model using the data-rich estimation method and utilized up to 40 macroeconomic time series in the estimation. Our study is the first attempt to combine the data-rich approach with the time-varying volatilities of structural disturbances.

We considered four alternative cases based on the number of observation variables (11 vs. 40 variables) and the specification of the volatilities of the structural shocks (constant volatility vs. time-varying-volatility). Comparing these four cases, we suggested the following three empirical evidences in the Great Recession: (1) the negative bank net worth shock gradually spread before the corporate net worth shock burst, (2) the data-rich approach and the structural shocks with SV evaluated the contribution of the corporate net worth shock to a substantial portion of the macroeconomic fluctuations after the Great Recession, in contrast to a standard DSGE model, and (3) the Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in the corporate sector could not yet have stopped deteriorating.

Incorporating time-varying volatilities of the shocks into the DSGE model made their credible bands narrower than half of the constant volatilities, implying that this is a realistic assumption of dynamics of the structural shocks. It is plausible that the tiny volatilities
(or the uncertainty) in ordinary times changed to an extraordinary magnitude at the turning points of the business cycles. We also estimated that the monetary policy shock had an opposite leverage effect of SV, which implies that tightening a policy makes interest rates more volatile.

**A Appendix**

**A.1 Sampling Stochastic Volatility with Leverage**

Step 4 of MCMC procedure described in Section 4 employs the algorithm of Omori et al. (2007) which is the extension of Kim et al. (1998) toward a SV model with leverage effect. This subsection is based on Justiniano and Primiceri (2008) who employed Kim et al. (1998) for drawing the stochastic volatilities.

According to Omori et al. (2007), the key idea of MCMC algorithm of a SV model with leverage effect is to obtain a draw from an approximate linear and Gaussian state space form such as

\[
\begin{pmatrix}
\sigma^*_t \\
h_{t+1} \\
z^*_t \\
\nu_t
\end{pmatrix} = \begin{pmatrix}
h_{t+1} \\
\mu + \phi(h_t - \mu) \\
z^*_t \\
\nu_t
\end{pmatrix} + \begin{pmatrix}
m_k + v_k\zeta_t \\
d_t \rho \omega (a_k + b_k v_k \zeta_t) \exp(m_k/2) + \omega \sqrt{1 - \rho^2} \zeta^*_t \\
\end{pmatrix},
\]

(A.1)

\[
\begin{pmatrix}
z^*_t \\
\nu_t
\end{pmatrix} | d_{i,t}, u_{it} = k, \rho_t, \omega_t = \begin{pmatrix}
d_t \rho \omega (a_k + b_k v_k \zeta_t) \exp(m_k/2) + \omega \sqrt{1 - \rho^2} \zeta^*_t \\
\end{pmatrix},
\]

(A.2)

where \( \sigma^*_{i,t} = \log \sigma_{i,t} = h_{i,t} + z^*_{i,t} \), \( h_{it} = \log \sigma_{i,t} \), and \( z^*_{i,t} = \log (z_{i,t}^2) \). And \( d_{i,t} \), and \( \eta_{i,t} \) are denoted as

\[
d_{i,t} = I(z_{i,t} \geq 0) - I(z_{i,t} < 0),
\]

\[
\eta_{i,t} = (h_{i,t} - \mu) - \phi(h_{i,t-1} - \mu),
\]

where, \( I(\cdot) \) is an indicator function which indicates \( d_{i,t} = 1 \) when \( z_{i,t} > 0 \), or otherwise:

\[
d_{i,t} = -1.
\]

Suppose that the MCMC algorithm has implemented iteration \( g \), generating samples \( \Phi_i^{(g)} = (\phi_i, \rho_i, \omega_i) \) and \( H_T^{(g)} \). In iteration \( g + 1 \), the following four steps are used to a set of new draws.

**Step 1:** Draw the structural shocks \( \varepsilon^{(g+1)}_i \).

In order to generate a new sample of stochastic volatilities, we need to obtain a new sample of structural shocks. This can be done using simulation smoother developed by de Jong and Shephard (1995) whose algorithm is described in Appendix A2. We obtain a new draw of structural shocks from eq.(A.12) of Appendix A2.

**Step 2:** Draw the stochastic volatilities \( H_T^{(g+1)} \) with leverage effect.

With a draw of Shocks in hand, nonlinear measurement equations (2.2) in Section 2.1, which is represented as eq.(A.3) for each structural shock, can be easily converted in linear one such as eq.(A.4) by squaring and taking logarithms of every elements. This induces the following approximating state space representation (A.4) and (A.5).

\[
\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}, \quad i=1,2,\cdots, M,
\]

(A.3)

\[
\tilde{\varepsilon}_{i,t} = 2h_{i,t} + z^*_{i,t},
\]

(A.4)
\[ h_{i,t} = \mu + \phi(h_{i,t-1} - \mu) + \nu_{i,t}, \nu_{i,t} \sim \text{i.i.d. N}(0, \omega^2_{i,t}) \]  
(A.5)

where \( \tilde{e}_{i,t} = \log[(\varepsilon_{i,t})^2 + \bar{c}] \); \( \bar{c} \) is the offset constant (set to 0.001); \( h_{it} = \log \sigma_{i,t} \) and \( z^*_{i,t} = \log(z_{i,t}^2) \). \( M \) is the number of structural shocks. Since the squared shocks \( \varepsilon_{i,t}^2 \) is very small, an offset constant is used to make the estimation procedure more robust. Eqs.(A.4) and (A.5) are linear, but non-Gaussian state space form, because \( z^*_{i,t} \) are distributed as a log \( \chi^2(1) \). In order to transform the system in a Gaussian state space form, a mixture of normals approximation of the log \( \chi^2(1) \) distribution is used, as described in Kim et al. (1998) and Omori et al. (2007). A draw of \( z^*_{i,t} \) is implemented from the mixture normal distribution given as

\[
f(z^*_{i,t}) = \sum_{k=1}^{K} q_k f_N(z^*_{i,t} | u_{i,t} = k), \quad i = 1, \ldots, M, \quad (A.6)
\]

where \( u_{i,t} \) is the indicator variable selecting which member of the mixture of normals has to be used at period \( t \) for shock \( i \). And \( q_k \) is the probability of \( u_{i,k} = k \); \( q_k = \Pr(u_{i,t} = k) \), and \( f_N(\cdot) \) denotes the probability density function of normal distribution. Omori et al (2007) select a mixture of ten normal densities \( (K = 10) \) with component probabilities \( q_k \), means \( m_k \), and variances \( \nu^2_k \), for \( k = 1, 2, \ldots, 10 \), chosen to match a number of moment of the log \( \chi^2(1) \) distribution. The constant \( (q_k, m_k, \nu^2_k) \) are reported as Table blow.

| Table of Selection Probability Function \((q_k, m_k, \nu^2_k, a_k, b_k)\) |
|-------|--------|-------|-------|-------|
| \( k \) | \( q_k \) | \( m_k \) | \( \nu^2_k \) | \( a_k \) | \( b_k \) |
| 1     | 0.00609 | 1.92677 | 0.11265 | 1.01418 | 0.50710 |
| 2     | 0.04775 | 1.34744 | 0.17788 | 1.02248 | 0.51124 |
| 3     | 0.13057 | 0.73504 | 0.26768 | 1.03403 | 0.51701 |
| 4     | 0.20674 | 0.02266 | 0.40611 | 1.05207 | 0.52604 |
| 5     | 0.22715 | -0.85173 | 0.62699 | 1.08153 | 0.54076 |
| 6     | 0.18842 | -1.97278 | 0.98583 | 1.13114 | 0.56557 |
| 7     | 0.12047 | -3.46788 | 1.57469 | 1.21754 | 0.60877 |
| 8     | 0.05591 | -5.55246 | 2.54498 | 1.37454 | 0.68728 |
| 9     | 0.01575 | -8.68384 | 4.16591 | 1.68327 | 0.84163 |
| 10    | 0.00115 | -14.65000 | 7.33342 | 2.50097 | 1.25049 |

Using generator of the mixture normal distribution above, the system has an approximate linear and Gaussian state space form. Therefore, a new draw of the stochastic volatilies \( H^T,(g+1) \) can be obtained recursively with standard Gibbs sampler for state space form using the algorithm of Carter and Kohn (1994).

**Step 3: Draw the indicators of the mixture approximation \( u^T,(g+1) \)**

In the case of SV with leverage effect, we need to modify the indicator \( u_{i,t} \) for the mixture normal described in Step 2, compared with Justiniano and Primiceri (2008). We follow the algorithm proposed by Omori et al. (2007), and obtain a new draw of indicators \( u_{i,t} \) which is generated conditional on \( z^*_{i,t}, H^T,(g+1) \) by independently sampling each from the discrete density defined by

\[
\pi(u_{i,t} = k | \varepsilon_{i,t}, h_{it}, \Phi) \propto \pi(u_{i,t} = k | \sigma^*_{i,t}, d_{it}, h_{it}, \Phi) \propto \pi(u_{i,t} = k | z^*_{i,t}, \eta_{i,t}, d_{it}, \Phi)
\]

\[
\propto q_k \frac{v^{-1}_k}{\nu^2_k} \exp \left\{ \frac{(z^*_{i,t} - m_k)^2}{2v^2_k} - \frac{\eta_{i,t} - d_{it} \rho_i \omega_i \exp(m_k/2) \{a_k + b_k (z^*_{i,t} - m_k)\}}{\omega^2_i (1 - \rho^2_i)} \right\} 
\]

(A.7)
Step 4: Draw the coefficients $\Phi_i^{(g+1)}(= (\phi_i, \rho_i, \omega_i))$ of stochastic volatility processes.

Having generated a sample $H^{T,(g+1)}$, we sample the elements of vector $\Phi_i^{(g+1)}$ from the density

$$p(\Phi_i | \sigma_{i,t}^*, d_{i,t}, u_{i,t}, \Phi_i) \propto p(\sigma_{i,t}^* | d_{i,t}, u_{i,t}) p(\Phi_i).$$

The density $p(\sigma_{i,t}^* | d_{i,t}, u_{i,t}, \Phi_i)$ is found from the output of Kalman filter recursion applied to the state space model (A.1) and (A.2). For the sampling we rely on the Metropolis-Hasting algorithm with a proposal density based on random walk such as

$$\theta^{(\text{proposal})} = \theta^{(g-1)} + u_t, \quad u_t \sim N(0, c\Sigma),$$

where $c$ is an adjustment constant.

### A.2 Simulation Smoother of Model Variable

Step 3.1 of algorithm of data-rich DSGE described in Section 4 employs simulation smoother (de Jong and Shephard, 1995) which generate sampling of model variables $S_t$ from conditional posterior distribution, $p(S_T | \Gamma^{(g-1)}, \theta, X_T)$. On the other hand, Boivin and Giannoni (2006), and Kryshko (2011) employ smoothing method proposed by Carter and Kohn (1994). But their method does not apply only to sample positive definite matrix as variance-covariance matrix of state variables so that their method discontinues on the way of sampling in MCMC pointed out by Chib (2001, p.3614). As a result, Kryshko (2011) transforms to ad hoc variance-covariance matrix of state variables. To avoid this problem, our algorithm employs simulation smoother instead of Carter and Kohn’s (1994) algorithm. Accordingly, our algorithm accomplishes generalization of estimating data-rich DSGE model.

To simplify representation of algorithm of simulation smoother, we rewrite state space model of (2.17) and (2.18) described in Section 2.1 into (A.8), and (A.9) as below.

$$\tilde{X}_t = \tilde{A}\tilde{S}_t + \nu_t, \quad \nu_t \sim N(0, R), \quad (A.8)$$

$$\tilde{S}_t = \tilde{G}\tilde{S}_{t-1} + \tilde{E}\varepsilon_t, \quad \varepsilon_t \sim N(0, Q(\theta)), \quad (A.9)$$

The following four steps are conducted to generate a new draw of model variables.

**Step 1: Kalman filter for state space model is implemented.**

Kalman filter is represented as

$$\eta_t = \tilde{X}_t - \tilde{A}\tilde{S}_{t|t}, \quad F_t = \tilde{A}\tilde{P}_{t|t}\tilde{A}^\prime + R, \quad K_t = \tilde{G}\tilde{P}_{t|t}\tilde{A}^\prime F_t^{-1},$$

$$L_t = \tilde{G} - K_t\tilde{A}, \quad \tilde{S}_{t+1|t+1} = \tilde{G}\tilde{S}_{t|t} + K_t\eta_t, \quad \tilde{P}_{t+1|t+1} = \tilde{G}\tilde{P}_{t|t}L_t + \tilde{E}Q(\theta)\tilde{E}^\prime,$$
where \( \eta_t \) is forecasting errors, \( K_t \) is Kalman gain, \( \tilde{P}_t \) is variance covariance matrix of state variables \( S_t \). Filtering of \( S_{t|t}, \tilde{P}_{t|t} \) iterates forward for period \( t = 1, 2, \cdots, T \). And for initial value \( \tilde{S}_{1|1}, \tilde{P}_{1|1} \), we set \( \tilde{X}_1 = \hat{\Lambda} \tilde{S}_1 \), and \( \tilde{P}_{1|1} = \tilde{G} \tilde{P}_{1|1} \tilde{G}' + \tilde{E} \tilde{Q}(\theta) \tilde{E}' \), where subscript \( t|t \) of \( \tilde{S}_t \) denotes conditional expected value of \( S_t \) up to information on \( X_1, \cdots, X_t \) thus, \( E(\tilde{S}_t|X_1, X_2, \cdots, X_t) \).

**Step 2: Generate values of \( r_{t-1}, N_{t-1} \) by implementing simulation smoother.**

This algorithm is iterated backward from period: \( t = T, \cdots, 2, 1 \) using values obtained from Kalman filter, as following equations (A.10), (A.11).

\[
\begin{align*}
    r_{t-1} &= \tilde{A}' \tilde{F}_t^{-1} \eta_t - W_t'C_t^{-1} d_t + L_t'r_t, \\
    N_{t-1} &= \tilde{A}' \tilde{F}_t^{-1} \tilde{A} + W_t'C_t^{-1} W_t + L_t'N_t L_t,
\end{align*}
\]

where \( W_t \) and \( C_t \) are obtained from the equations such as

\[
W_t = Q(\theta) \tilde{E}'N_t L_t,
\]

\[
C_t = Q(\theta) - Q(\theta) \tilde{E}'N_t \tilde{E}Q(\theta),
\]

and random variable \( d_t \) is generated from \( N(0, C_t) \). Initial value \( r_T \) and \( N_T \) are set at \( r_T = 0 \), and \( N_T = 0 \).

**Step 3: Smoothing of structural shocks \( \tilde{\varepsilon}_{t|T} \), are implemented backward iteration using the equation (A.12).**

Subscript \( t|T \) of \( \tilde{\varepsilon}_{t|T} \) denotes expected value conditional on total sample period such as \( E(\tilde{\varepsilon}_t|X_1, X_2, \cdots, X_T) \).

\[
\tilde{\varepsilon}_{t|T} = Q(\theta) \tilde{E}' r_t + d_t \quad d_t \sim N(0, C_t), \quad t = T, \cdots, 2, 1
\]

**Step 4: Generate model variables \( \tilde{S}_t \) by forward iteration of the equation (A.13).**

\[
\tilde{S}_{t+1|T} = G \tilde{S}_t + E \tilde{\varepsilon}_{t|T}, \quad t = 1, 2, \cdots, T,
\]

where initial value \( \tilde{S}_{1|T} \) is obtained from \( \tilde{S}_{1|1} = \tilde{S}_{1|1} + \tilde{P}_{1|1} r_0 \).

The algorithm described above is procedure generating model variables \( S_t(t = 1, 2, \cdots, T) \) from conditional posterior distribution \( p(S^T|\Gamma^{(g-1)}, \theta, X^T) \) which is implemented in Step 3.1 of Section 4.

### A.3 Sampling of Parameters Set \( \Gamma \) of Measurement Equation (2.11)

In Step 3.2 of MCMC algorithm in Section 4, we sample parameters \( \Gamma = \{ \Lambda, R, \Psi \} \) of measurement equation obtained from (2.11) and (2.13). To do so, (2.11) is transformed by substituting (2.13) into it as

\[
(I - \Psi L) X_t = (I - \Psi L) \Lambda S_t + \nu_t, \quad \nu_t \sim i.i.d. N(0, R),
\]

where \( I \) denotes identity matrix. The sampling of parameters \( \Gamma = \{ \Lambda, R, \Psi \} \) from conditional posterior distribution \( p(\Gamma|S^T(\theta, \theta^{(g-1)}, X^T) \) given the unobserved model variables \( S^T \) and deep parameters \( \theta \), is conducted following the approach by Chib and Greenberg (1994) who proposed Bayesian estimation method of linear regression model with AR (1) errors such like (2.11) and (2.13).
For estimating above model, Chib and Greenberg (1994) divided it into two linear regression models. First, by using notations, \( X^*_k = X_{k,t} - \Psi_k S_{k,t-1} \) and \( S^*_k = S_{k,t} - \Psi_{kk} S_{k,t-1} \) where subscript \( k \) is \( k \)-th indicator of data set \( X_t \), above equation is represented as

\[
X^*_t = \Lambda S^*_t + \nu_t, \quad \nu_t \sim i.i.d. N(0, R),
\]

Second, by using notation \( e_{k,t} = X_{k,t} - \Lambda_k S_t \) which means measurement errors, the equation is also rewritten as

\[
e_k = \Psi_{kk} e_{k,-1} + \nu_k,
\]

where \( e_k = [e_{k,2}, \ldots, e_{k,T}]' \), \( e_{k,-1} = [e_{k,1}, \ldots, e_{k,T-1}]' \). We sample parameter \( (\Lambda, R) \) given parameter \( \Psi \) from the first equation, and parameter \( \Psi \) given \( (\Lambda, R) \) from the second equation sequentially based on the following two-step algorithm.

**Step 1.** Sampling \( (\Lambda_k, R_{kk}) \) from conditional posterior distribution \( p(\Lambda_{kk}, R_{kk} | \Psi_{kk}, S^T, \theta, X^T) \) for estimating equation

\[
X^*_t = \Lambda S^*_t + \nu_t, \quad \nu_t \sim i.i.d. N(0, R).
\]

The posterior density of \( (\Lambda, R_{kk}) \) given the unobserved state variables \( S^T \) and deep parameters \( \theta \) is represented as

\[
p(\Lambda_k, R_{kk} | \Psi_{kk}, S^T, X^T) \propto p(X^T | S^T, \Lambda_k, R_{kk}, \Psi_{kk}, \theta) p(\Lambda_{kk}, R_{kk}),
\]

where \( p(X^T | S^T, \Lambda_k, R_{kk}, \Psi_{kk}, \theta) \) is likelihood function and \( p(\Lambda_{kk}, R_{kk}) \) is prior density.

As shown by Chib and Greenberg (1994), the above likelihood function is proportional to a Normal -Inverse-Gamma density as

\[
p(X^T | S^T, \Lambda_k, R_{kk}, \Psi_{kk}, \theta) \propto p_{NIG}(\Lambda_k, R_{kk} | \hat{\Lambda}_k, (S''S^*)^{-1}, s, T - N - 2)
\]

where

\[
\hat{\Lambda}_k = (S''S^*)^{-1} S'' X^*_k
\]

\[
s = X_k' \left( I_T - S^* (S''S^*)^{-1} S'' \right) X_k^* = X_k' \left( X_k^* - S^* \hat{\Lambda}_k \right).
\]

Since above prior \( p(\Lambda_{kk}, R_{kk}) \) is assumed to be Normal -Inverse-Gamma \( p_{NIG}(\Lambda_k, R_{kk} | \Lambda_{k,0}, M_{k,0}, s_0, \nu_0) \), the resulting conditional posterior density is also Normal -Inverse-Gamma as following.

\[
p(\Lambda_k, R_{kk} | \Psi_{kk}, S^T, X^T) \propto p_{NIG}(\Lambda_k, R_{kk} | \hat{\Lambda}_k, (S''S^*)^{-1}, s, T - N - 2)
\]

\[
\times p_{NIG}(\Lambda_k, R_{kk} | \Lambda_{k,0}, M_{k,0}, s_0, \nu_0)
\]

\[
\times p_{NIG}(\Lambda_k, R_{kk} | \hat{\Lambda}_k, \bar{M}, \bar{s}, \bar{\nu})
\]
\[
M_k = M_{k,0} + \left( S^* S^* \right) \\
A_k = M_k^{-1} \left( M_{k,0} A_{k,0} + (S^* S^*) A_k \right) \\
\bar{s} = s_0 + s + (\lambda_{k,0} - \hat{\Lambda}_k) \left[ M_{k,0}^{-1} + (S^* S^*)^{-1} \right]^{-1} (\lambda_{k,0} - \hat{\Lambda}_k) \\
\bar{\nu} = \nu_0 + T
\]

and \( \lambda_{k,0}, M_{k,0}, s_0, \) and \( \nu_0 \) are parameters of the prior density.

We sample factor loading \( \Lambda_k \) and the variance of measurement error \( R_{kk} \) sequentially from

\[
R_{kk} | \Psi_{kk}, S^T, \theta, X^T \sim IG(\bar{s}, \bar{\nu}) \\
\Lambda_k | R_{kk}, \Psi_{kk}, S^T, \theta, X^T \sim N(\bar{\Lambda}_k, R_{kk}, \bar{M}_k^{-1})
\]

**Step 2. Sampling** \( \Psi_{kk} \) **from conditional posterior distribution** \( p(\Psi_{kk} | \Lambda, R_{kk}, S^T, \theta, X^T) \) for estimating equation of measurement errors

\[
e_k = \Psi_{kk} e_{k,-1} + \nu_k.
\]

The conditional posterior density : \( p(\Psi_{kk} | \Lambda, R_{kk}, S^T, \theta, X^T) \) is given as

\[
p(\Psi_{kk} | \Lambda, R_{kk}, S^T, \theta, X^T) \propto p(X_k^T | S^T, \Lambda_k, R_{kk}, \Psi_{kk}, \theta) p(\Psi_{kk}),
\]

where \( p(X_k^T | S^T, \Lambda_k, R_{kk}, \Psi_{kk}, \theta) \) is likelihood function and \( p(\Psi_{kk}) \) is prior density.

Then, above likelihood function is proportional to the normal density such as

\[
p(X_k^T | S^T, R_{kk}, \Psi_{kk}, \theta) \propto \exp \left[ -\frac{1}{2R_{kk}} (\Psi_{kk} - \hat{\Psi}_{kk})' e_{k,-1}' e_{k,-1} (\Psi_{kk} - \hat{\Psi}_{kk}) \right].
\]

And above prior density of coefficient of AR (1) errors \( \Psi_{kk} \) is also normal density but truncated at less than unity because dynamic of errors keep to be stationary. So, prior density is assumed to be such as

\[
p(\Psi_{kk}) \propto \exp \left[ -\frac{1}{2\sigma_{\Psi,0}^2} (\Psi_{kk} - \Psi_0)^2 \right] \times 1_{\{|\Psi_{kk}|<1\}},
\]

where \( 1_{\{|\Psi_{kk}|<1\}} \) denotes indicator function which is unity if \( |\Psi_{kk}| < 1 \), otherwise zero.

The conditional posterior density is proportional to a product of above two normal densities, and represented as

\[
p(\Psi_{kk} | R_{kk}, S^T, \theta, X^T) \propto \exp \left[ -\frac{1}{2R_{kk}} (\Psi_{kk} - \hat{\Psi}_{kk})' e_{k,-1}' e_{k,-1} (\Psi_{kk} - \hat{\Psi}_{kk}) \right] \\
\times \exp \left[ -\frac{1}{2\sigma_{\Psi,0}^2} (\Psi_{kk} - \Psi_0)^2 \right] \times 1_{\{|\Psi_{kk}|<1\}}.
\]
A.4 The Remaining Framework of the DSGE model

In this section, the remaining structure of our DSGE model described in Section 3 is dealt with.

A.4.1 Household Sector

There is a continuum of members in the household where the total population measures to one. Within the household, there are fractions of \( f^E \) entrepreneurs, \( f^F \) financial intermediaries (or “bankers”), and \( 1 - f^E - f^F \) workers. Entrepreneurs engage in a business where they produce intermediate goods and transfer the net worth back to the household when they exit from the business. Now, each financial intermediary manages a bank where it accepts the deposits from the household sector and lend to entrepreneurs. When financial intermediaries exit from their business, they also transfer their net worth back to the household sector. Finally, remaining fraction of the members of the household become workers. Workers supply labor input to earn wage and they transfer their wage earnings to the household each period. Within the household, each member shares the risk perfectly.

The representative household maximizes her expected discounted sum of utility over time and their objective function is specified as follow;

\[
\Psi_{kk} | R_{kk}, S^T, \theta, X^T \sim N(\tilde{\Psi}_{kk}; \tilde{V}_{kk}) \times 1_{\{|\Psi_{kk}| < 1\}},
\]

where \( \tilde{V}_{kk} = [(R_{kk}(e_{kk}-1)^{-1})^{-1} + (\sigma_{kk}^2)^{-1}]^{-1} \),

\[
\tilde{\Psi}_{kk} = \tilde{V}_{kk} [(R_{kk}(e_{kk}-1)^{-1})^{-1} \tilde{\Psi}_{kk} + (\sigma_{kk}^2)^{-1} \Psi_{kk}].
\]

Hence, we sample coefficient of AR (1) errors \( \Psi_{kk} \) from trancaeted normal such as

\[
\Psi_{kk} \mid R_{kk}, S^T, \theta, X^T \sim N(\tilde{\Psi}_{kk}; \tilde{V}_{kk}) \times 1_{\{|\Psi_{kk}| < 1\}},
\]

where \( \tilde{V}_{kk} = [(R_{kk}(e_{kk}-1)^{-1})^{-1} + (\sigma_{kk}^2)^{-1}]^{-1} \),

\[
\tilde{\Psi}_{kk} = \tilde{V}_{kk} [(R_{kk}(e_{kk}-1)^{-1})^{-1} \tilde{\Psi}_{kk} + (\sigma_{kk}^2)^{-1} \Psi_{kk}].
\]

\[
E_t \sum_{i=0}^{\infty} \beta^i \chi^c_{t+i} \left[ \frac{(c_{t+i} - hC_{t+i-1})^{1-\sigma^c}}{1-\sigma^c} - \chi^L_{t+i} \frac{(l_{t+i})^{1+\sigma^L}}{1+\sigma^L} \right] (A.14)
\]

where \( \beta \) is the discount rate, \( h \) is the habit persistence, \( \sigma^c \) is the inverse of intertemporal elasticity of substitution, \( c_t \) is final goods consumption, \( C_{t-1} \) represents the external habit formation, \( \sigma^L \) is the inverse of Frisch labor supply elasticity and \( l_t \) is the supply of aggregate labor by workers. Now, there are two structural shocks embedded in the function. \( \chi^c_t \) represents an intertemporal preference shock, while \( \chi^L_t \) represents labor disutility shock relative to consumption.

Next, turning to the budget constraint of the representative household, they make a deposit, \( b_t \), at period \( t \) and earn real interest rate, \( R_t/\pi_{t+1}, \) next period where \( R_t \) is risk-free gross nominal interest rate at period \( t \) and \( \pi_{t+1} \) is gross inflation rate at period \( t + 1 \). In addition, the household pays lump sum tax of \( \tau_t \) to the government. Now, they receive a lump-sum transfer of wage incomes from workers which is expressed as \( \int_0^{l_t} w_t(x) l_t(x) dx \), where \( w_t(x) \) and \( l_t(x) \) are real wage and labor supply by individual worker \( x \), respectively. Finally, the household earns the combined dividend of \( \Xi^E_{t} \) from retailers, earns the net transfer of \( \Xi^F_{t} \) from entrepreneurs, and the net transfer of \( \Xi^F_{t} \) from bankers each period. Thus, the representative household’s budget constraint at period \( t \) can be expressed as, in real terms, as follow, ,

\[21\)Here, the real wage set by worker \( x \) is defined as \( w_t(x) = W_t(x)/P_t \), where \( W_t(x) \) stands for the nominal wage set by worker \( x \) and \( P_t \) stands for the price index of final goods. The formulation of \( W_t(x) \) and \( P_t \) will be described later in this section.\]
\[ c_t + b_t = \frac{R_{t-1} b_{t-1} - \pi_t + \Xi_t^w + \Xi_t^E + \Xi_t^F}{\pi_t}. \] (A.15)

**Consumption and Deposit Decision**  The first-order conditions (hereafter, FOCs) of the household with respect to \( c_t \) and \( b_t \) as follows;

\[
\zeta_t^H = \chi_t^c (c_t - h c_{t-1})^{-\sigma_c} 
\]

(A.16)

\[
\zeta_t^H = \beta E_t \zeta_{t+1}^H \frac{R_t}{\pi_{t+1}}.
\]

(A.17)

where \( \zeta_t^H \) is Lagrangian multiplier associated with the budget constraint. (A.16) is the FOC of consumption which equates the marginal utility of consumption to the shadow price of the final goods. (A.17) is the FOC of deposit decision.

**Wage Setting Decision by Workers**  Following Erceg, Henderson, and Levin (2000) (hereafter, EHL), each worker indexed by \( x \in [0, 1] \) supplies differentiated labor input, \( l_t(x) \), monopolistically and sells this service to the labor union who is perfectly competitive.\(^{22}\)

Each worker sets his nominal wage according to Calvo style sticky price setting where fraction \( \theta^w \) of the entire workers cannot freely adjust the wages at their discretion. For fraction \( \theta^w \) of workers, the partial indexation of the nominal wage is assumed.\(^{23}\) Due to the perfect risk-sharing assumed in the model, each worker maximizes the objective function (A.14) by choosing the amount of individual labor supply, \( l_t(x) \), while taking the amount of consumption, \( c_t \), as given. Under this setting, \( (1 - \theta^w) \) fraction of workers maximize their objective function by setting the nominal wage, \( \tilde{W}_t \), such that

\[
E_t \sum_{i=0}^{\infty} \beta^t (\theta^w)^i \left[ \frac{\tilde{W}_t}{P_{t+i}} \left( \frac{P_{t+1+i}}{P_{t-1}} \right) \chi_t^c (c_t - h c_{t-1})^{-\sigma_c} - (1 + \psi^w) \chi_t^L l_t(x) \right] l_{t+i}(x) = 0. \] (A.18)

The law of motion of the aggregate wage index can be shown to be as follow,

\[
W_t^{-1/\psi^w} = \theta^w \left[ \frac{W_{t-1}}{W_{t-2}} \right]^{-1/\psi^w} + (1 - \theta^w)\tilde{W}_t^{-1/\psi^w}. \] (A.19)

Finally, the real wage index in the economy is defined as \( w_t \equiv W_t / P_t \).

**A.4.2 Capital Production Sector**

Capital producers are identical, perfectly competitive, and risk neutral. They purchase \( i_t^k \) units of final goods from the retailer, convert them to \( i_t^F \) units of capital goods, and combine them with existing capital stock, \( (1 - \delta) k_t \), to produce new capital stock, \( k_{t+1} \). Capital producers will, then, sell off new capital stock to entrepreneurs in a perfectly competitive manner. Capital producers have linear production technology in converting final goods to capital goods. In addition, they will incur quadratic investment adjustment cost when

\(^{22}\)The labor union transforms labor services to an aggregate labor input, \( l_t \) using the Dixit and Stiglitz type aggregator function. The factor demand function for \( l_t(x) \) is given by \( l_t(x) = (W_t(x) / W_t)^{-1/(1+\psi^w)/\psi^w} l_t \) where \( \psi^w \) is the wage markup, \( W_t(x) \) is the nominal wage set by worker \( x \) and \( W_t \) is the aggregate nominal wage index which is given as \( W_t \equiv \left[ \int_0^1 W_t(x)^{-1/(1+\psi^w)} dx \right]^{-1/\psi^w} \).

\(^{23}\)The lagged inflation indexation is specified as \( W_t(x) = (P_{t-1}/P_{t-2})^{\psi^w} W_{t-1}(x) \) where \( \psi^w \) controls the degree of nominal wage indexation to past inflation rate.
they change the production capacity of capital goods from previous period. Each capital producer maximizes the expected discounted cash flow with respect to \( i_t^k \). \(^{24}\) The FOC is given by

\[
q_t = \frac{1}{A_t} \left[ 1 + \kappa \left( \frac{i_t^k}{i_{t-1}^k} - 1 \right) \frac{i_t^k}{i_{t-1}^k} + \frac{\kappa}{2} \left( \frac{i_t^k}{i_{t-1}^k} - 1 \right)^2 \right] - \beta \kappa \frac{A_t^k}{A_{t+1}^k} \left( \frac{i_{t+1}^k}{i_t^k} - 1 \right) \left( \frac{i_{t+1}^k}{i_t^k} \right)^2. \tag{A.20}
\]

where \( A_t^k \) is the investment-specific technology shock common across all capital producers and \( \kappa \) is the investment adjustment cost parameter. Finally, aggregate capital accumulation equation is given by

\[
k_{t+1} = i_t^k + (1 - \delta)k_t. \tag{A.21}
\]

\subsection{A.4.3 Retailing Sector}

Retailers \( z \in [0, 1] \) purchase intermediate goods from the entrepreneur at perfectly competitive price and resell them monopolistically in the retail market.\(^{25}\) We assume Calvo type sticky price setting for the retailer where, for any given period \( t \), fraction \( \theta^p \) of the entire retailers cannot freely revise their prices. Further, \( \theta^p \) fraction of the retailers who did not receive a 'signal of price change' will partially index their nominal prices to lagged inflation rate of price index.\(^{26}\) Under this setting, for \((1 - \theta^p)\) fraction of the retailers who received a 'price changing signal' at period \( t \), they maximize their expected discounted sum of profits by setting the nominal price, \( p_t \), such that

\[
E_t \sum_{i=0}^{\infty} \beta^i (\theta^p)^i \left\{ \tilde{p}_t \left( \frac{P_{t-1+i}}{P_{t-1}} \right)^{\theta^p} \left( \frac{\epsilon}{\epsilon - 1} \right) p_t^{mc} \right\} y_{t+i}(z) = 0. \tag{A.22}
\]

From the definition of aggregate price index, the law of motion of \( P_t \) can be shown to be as follow;

\[
(P_t)^{1-\epsilon} = \theta^p \left( P_{t-1-1} \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\theta^p} \right)^{1-\epsilon} + (1 - \theta^p)\tilde{p}_t^{1-\epsilon}. \tag{A.23}
\]

\subsection{A.4.4 The Rest of the Economy}

In closing the model, we describe the rest of the model structure here. The central bank is assumed to follow a standard Taylor-type monetary policy rule,

\[
\hat{R}_t = \rho^R \hat{R}_{t-1} + (1 - \rho^R) \left[ \mu^\pi \hat{\pi}_t + \mu^y \hat{Y}_t \right] + \epsilon^R. \tag{A.24}
\]

\(^{24}\)The profit function for each capital producer at period \( t \) can be expressed as follows,

\[
E_t \sum_{i=0}^{\infty} \beta^i \left\{ \tilde{q}_{t+i}^k + \frac{1}{A_t^k} \left[ \kappa \left( \frac{i_{t+i}^k}{i_{t+i-1}^k} - 1 \right) \frac{i_{t+i}^k}{i_{t+i-1}^k} \frac{\kappa}{2} \left( \frac{i_{t+i}^k}{i_{t+i-1}^k} - 1 \right)^2 \right] \right\}
\]

\(^{25}\)The demand function for retail goods sold by retailer \( z \) is given by \( y_t(z) = (P_t(z)/P_t)^{-\epsilon} Y_t \), where \( Y_t \) is aggregated final goods, \( P_t(z) \) is nominal price of retail goods \( y_t(z) \), \( P_t \) is aggregate price index of final goods, and \( \epsilon \) is the price elasticity of retail goods. Specifically, aggregated final goods, \( Y_t' \), and the aggregate price index, \( P_t \), are given as follows; \( Y_t' = \int y_t(z)^{(\epsilon - 1)/\epsilon} dz \) and \( P_t = \int y_t(z)^{1/(\epsilon - 1)} dz \).

\(^{26}\)The lagged inflation indexation is specified as \( p_t(z) = (P_{t-1}/P_{t-2})^\phi p_{t-1}(z) \) where \( \phi \) controls for the magnitude of price indexation to past inflation rate.
where $\rho^R$ controls the magnitude of interest smoothing, $\mu^\pi$ is Taylor coefficient in response to inflation gap, $\mu^y$ is Taylor coefficient in response to output gap, and $\varepsilon_t^R$ is i.i.d. monetary policy shock.

The government budget constraint is simply specified as

$$g_t = \tau_t.$$  \hfill (A.25)

The government expenditure, $g_t$, is financed solely by lump-sum tax, $\tau_t$. In our model, we simply assume that the government expenditure to follow stochastic AR(1) process.

Finally, the market clearing condition for final goods is given as follow,

$$Y_t = c_t + i^E_t + g_t.$$  \hfill (A.26)

### A.4.5 Structural Shocks in the Model

There are eight structural shocks in the model, each of them having a specific economic interpretation as below. Except for monetary policy shock, all of the structural shocks are assumed to follow AR(1) stochastic processes where $\rho$ is for the AR(1) coefficients for respective structural shocks.

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Shock Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>$\hat{A}<em>t = \rho^A \hat{A}</em>{t-1} + \varepsilon_t^A$</td>
</tr>
<tr>
<td>Preference shock</td>
<td>$\hat{\chi}^c_t = \rho^c \hat{\chi}^c_{t-1} + \varepsilon_t^c$</td>
</tr>
<tr>
<td>Labor supply shock</td>
<td>$\hat{\chi}^L_t = \rho^L \hat{\chi}^L_{t-1} + \varepsilon_t^L$</td>
</tr>
<tr>
<td>Investment specific technology shock</td>
<td>$\hat{A}^K_t = \rho^K \hat{A}^K_{t-1} + \varepsilon_t^K$</td>
</tr>
<tr>
<td>Government spending shock</td>
<td>$\hat{g}<em>t = \rho^G \hat{g}</em>{t-1} + \varepsilon_t^G$</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>$\varepsilon_t^R$</td>
</tr>
<tr>
<td>Corporate net worth shock</td>
<td>$\hat{\gamma}^E_t = \rho^E \hat{\gamma}^E_{t-1} + \varepsilon_t^E$</td>
</tr>
<tr>
<td>Bank net worth shock</td>
<td>$\hat{\gamma}^F_t = \rho^F \hat{\gamma}^F_{t-1} + \varepsilon_t^F$</td>
</tr>
</tbody>
</table>

Notice that each stochastic disturbance $\varepsilon_t$ including monetary policy shock is assumed to follow time varying volatility using SV model as mentioned in Section 2.

### References


[18] Ireland, P.N. (2011) “A New Keynesian Perspective on the Great Recession,” Journal of Money, Credit and Banking, 43 (1) 31-54.


### Data Appendix

<table>
<thead>
<tr>
<th>No.</th>
<th>Variables</th>
<th>Code</th>
<th>Series description</th>
<th>Unit of data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$R$</td>
<td>6</td>
<td>Interest rate: Federal Funds Effective Rate</td>
<td>% per annum</td>
<td>FRB</td>
</tr>
<tr>
<td>2</td>
<td>$Y_1$</td>
<td>5</td>
<td>Real gross domestic product (excluding net export)</td>
<td>Billion of chained 2000</td>
<td>BEA</td>
</tr>
<tr>
<td>3</td>
<td>$C_{1*}$</td>
<td>5</td>
<td>Gross personal consumption expenditures</td>
<td>Billion dollars</td>
<td>BEA</td>
</tr>
<tr>
<td>4</td>
<td>$I_1$</td>
<td>5</td>
<td>Gross private domestic investment - Fixed investment</td>
<td>Billion dollars</td>
<td>BEA</td>
</tr>
<tr>
<td>5</td>
<td>$\pi_1$</td>
<td>8</td>
<td>Price deflator: Gross domestic product</td>
<td>2005Q1 = 100</td>
<td>BEA</td>
</tr>
<tr>
<td>6</td>
<td>$w_1$</td>
<td>2</td>
<td>Real Wage (Smets and Wouters)</td>
<td>1992Q3 = 0</td>
<td>SW (2007)</td>
</tr>
<tr>
<td>7</td>
<td>$L_1$</td>
<td>1</td>
<td>Hours Worked (Smets and Wouters)</td>
<td>1992Q3 = 0</td>
<td>SW (2007)</td>
</tr>
<tr>
<td>8</td>
<td>$RE_1$</td>
<td>6</td>
<td>Moody’s bond indices - corporate Baa</td>
<td>% per annum</td>
<td>Bloomberg</td>
</tr>
<tr>
<td>9</td>
<td>$Lev_F$</td>
<td>7</td>
<td>Commercial banks leverage ratio</td>
<td>Total asset/net worth ratio</td>
<td>FRB</td>
</tr>
<tr>
<td>10</td>
<td>$Lev_E$</td>
<td>3</td>
<td>Nonfarm nonfin corp business leverage ratio</td>
<td>Total asset/net worth ratio</td>
<td>FRB</td>
</tr>
</tbody>
</table>

### Case A and Case D: The standard one-to-one matching estimation method

1. $Y_1$: Real gross domestic product
2. $C_{1*}$: Gross personal consumption expenditures
3. $I_1$: Gross private domestic investment
4. $\pi_1$: Price deflator
5. $w_1$: Real wage
6. $L_1$: Hours worked
7. $RE_1$: Moody’s bond indices - corporate Baa
8. $Lev_F$: Commercial banks leverage ratio
9. $Lev_E$: Nonfarm nonfinancial non-corporate business leverage ratio

### Case B and Case D: The data-rich estimation method

12. $Y_2$: Industrial production index - final products
13. $Y_3$: Industrial production index: total index
14. $Y_4$: Industrial production index: products
15. $C_2$: Real PCE excluding food and energy
16. $C_5$: Real PCE, quality indexes; nondurable goods
17. $C_4$: Real PCE, quality indexes; services
18. $I_2$: Real gross private domestic investment
19. $I_3$: Gross private domestic investment: fixed nonresidential
20. $I_4$: Manufactures’ new orders: nondurable capital goods
21. $\pi_2$: Core CPI excluding food and energy
22. $\pi_3$: Price index - PCE excluding food and energy
23. $\pi_4$: Price index - PCE - Service
24. $w_2$: Average hourly earnings: manufacturing
25. $w_3$: Average hourly earnings: construction
26. $w_4$: Average hourly earnings: service
27. $L_2$: Civilian Labor Force: Employed Total
28. $L_3$: Employees, nonfarm: total private
29. $L_4$: Employees, nonfarm: goods-producing
30. $RE_2$: Bond yield: Moody’s A corporate
31. $RE_3$: Bond yield: Moody’s A industrial
32. $Lev_F^*: $: Core capital leverage ratio PCA all insured institutions
33. $Lev_E^*: $: Nonfarm nonfinancial non-corporate leverage ratio
34. $Lev_F*: $: Domestically chartered commercial banks leverage ratio
35. $Lev_E*: $: Brokers and dealers leverage ratio
36. $Lev_F$: Nonfarm corporate leverage ratio
37. $Lev_E$: Total asset/net worth
38. $s_2$: Charge-off rate on all loans and leases all commercial banks
39. $s_3$: Charge-off rate on all loans all commercial banks
40. $s_4$: Charge-off rate on all loans banks 1st to 100th largest by assets

Note: The format is: series number; transformation code; series description; unit of data and data source. The transformation codes are: 1 - demeaned; 2 - linear detrended; 3 - logarithm and demeaned; 4 - logarithm, linear detrend, and multiplied by 100; 5 - log per capita, linear detrended and multiplied by 100; 6 - detrended via HP filter; 7 - logarithm, detrended via HP filter, and multiplied by 100; 8 - first difference logarithm, detrended via HP filter, and multiplied by 400; 9- the reciprocal number, logarithm, detrended via HP filter, and multiplied 100. A * indicates a series that is deflated with the GDP deflator. “PCE” and “SW (2007)” in this table denote personal consumption expenditure and Smets and Wouters (2007), respectively.
Table 1: Specifications of Four Alternative Cases

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Observation</td>
<td>11</td>
<td>40</td>
<td>11</td>
<td>40</td>
</tr>
<tr>
<td>Model Variable to Obs.</td>
<td>1 to 1</td>
<td>1 to 4</td>
<td>1 to 1</td>
<td>1 to 4</td>
</tr>
<tr>
<td>Structural Shock</td>
<td>i.i.d. Normal</td>
<td>i.i.d. Normal</td>
<td>SV with Leverage</td>
<td>SV with Leverage</td>
</tr>
</tbody>
</table>

Note: Item Number of Observation in the first column denotes the number of data indicators used for estimating the model of each case. Item Model Variable to Obs denote the ratio what number of observations per one model variable are adopted. In the case of a standard DSGE model, we adopt one to one matching between model variables and observations. In data rich approach, one to many matching are adopted between model variables and observations. Item Structural Shock denotes specification of stochastic process of shocks. SV is abbreviation of stochastic volatility.

Table 2: Calibrated Parameters and Key Steady States

<table>
<thead>
<tr>
<th>Calibrated Param.</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>Discount factor</td>
<td>0.995</td>
<td>Our setting</td>
</tr>
<tr>
<td>δ</td>
<td>Depreciation rate</td>
<td>0.025</td>
<td>Christensen and Dib (2008)</td>
</tr>
<tr>
<td>α</td>
<td>Capital share</td>
<td>0.33</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>s_{E}^{ss}</td>
<td>Survival rate of entrepreneur</td>
<td>0.972</td>
<td>Christensen and Dib (2008)</td>
</tr>
<tr>
<td>γ_{E}^{ss}</td>
<td>Survival rate of banker in steady state</td>
<td>0.972</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>λ</td>
<td>Bank’s participation constraint parameter</td>
<td>0.383</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>ψ_{w}</td>
<td>Wage markup</td>
<td>0.05</td>
<td>Smets and Wouters (2003)</td>
</tr>
<tr>
<td>ϵ</td>
<td>Elasticity Substitution of intermediate goods</td>
<td>11</td>
<td>Our setting</td>
</tr>
<tr>
<td>ξ_{E}</td>
<td>New entrepreneur entry rate</td>
<td>0.003</td>
<td>Our setting</td>
</tr>
<tr>
<td>ξ_{F}</td>
<td>New banker entry rate</td>
<td>0.003</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key Steady State</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{E}^{ss}</td>
<td>Steady state marginal cost</td>
<td>≤1</td>
<td>Christensen and Dib (2008)</td>
</tr>
<tr>
<td>S_{E}^{ss}</td>
<td>Steady state external financial premium</td>
<td>1.0075</td>
<td>From data (1980Q1-2010Q2)</td>
</tr>
<tr>
<td>r_{E}^{ss}</td>
<td>Steady state corp. borrowing rate (real, QPR)</td>
<td>1.0152</td>
<td></td>
</tr>
<tr>
<td>r_{F}^{ss}</td>
<td>Steady state bank lending rate (real, QPR, ex-premium)</td>
<td>1/β</td>
<td></td>
</tr>
<tr>
<td>r_{ss}</td>
<td>Steady state real interest</td>
<td>1/β</td>
<td></td>
</tr>
<tr>
<td>ν_{ss}</td>
<td>Steady state Nu</td>
<td>(1−γ_{F}^{ss})β(r_{E}^{ss}/r_{ss})</td>
<td></td>
</tr>
<tr>
<td>η_{ss}</td>
<td>Steady state Eta</td>
<td>(1−γ_{F}^{ss})β</td>
<td></td>
</tr>
<tr>
<td>Lev_{ss}</td>
<td>Steady state leverage ratio of banker</td>
<td>1.919</td>
<td>From data (1980Q1-2010Q2)</td>
</tr>
<tr>
<td>K_{E}^{ss}/N_{E}^{ss}</td>
<td>Steady state leverage ratio of entrepreneur</td>
<td>1.919</td>
<td></td>
</tr>
<tr>
<td>K_{E}^{ss}/Y_{ss}</td>
<td>Steady state capital/output ratio</td>
<td>0.2</td>
<td>Gertler and Kiyotaki (2010)</td>
</tr>
<tr>
<td>I_{E}^{ss}/Y_{ss}</td>
<td>Steady state investment/output ratio</td>
<td>1/β</td>
<td></td>
</tr>
<tr>
<td>G_{E}^{ss}/Y_{ss}</td>
<td>Steady state government expenditure/output ratio</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>C_{E}^{ss}/Y_{ss}</td>
<td>Steady state consumption/output ratio</td>
<td>1−I_{E}^{ss}/Y_{ss}−G_{E}^{ss}/Y_{ss}</td>
<td></td>
</tr>
</tbody>
</table>
Table 3: Prior Settings of Structural Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Density</th>
<th>Prior Mean</th>
<th>Prior SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Investment adjustment cost</td>
<td>Gamma</td>
<td>1.000</td>
<td>0.500</td>
</tr>
<tr>
<td>$h$</td>
<td>Habit formation</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\sigma^C$</td>
<td>IES of consumption</td>
<td>Gamma</td>
<td>1.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\sigma^L$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>Gamma</td>
<td>1.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Elasticity of premium to leverage ratio</td>
<td>Inv. Gamma</td>
<td>0.050</td>
<td>4.000</td>
</tr>
<tr>
<td>$i_P$</td>
<td>Price indexation</td>
<td>Beta</td>
<td>0.500</td>
<td>0.100</td>
</tr>
<tr>
<td>$i_W$</td>
<td>Wage indexation</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\theta_P$</td>
<td>Calvo parameter for goods pricing</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\theta_W$</td>
<td>Calvo parameter for wage setting</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Monetary policy persist. param.</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>Taylor coefficient for inflation</td>
<td>Gamma</td>
<td>1.500</td>
<td>0.500</td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>Taylor coefficient for output gap</td>
<td>Gamma</td>
<td>0.500</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Persistence Parameters for Structural Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Density</th>
<th>Prior Mean</th>
<th>Prior SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_A$</td>
<td>Persistent parameter for TFP shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>Persistent parameter for preference shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_K$</td>
<td>Persistent parameter for investment tech. shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>Persistent parameter for entrepreneur net worth shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_F$</td>
<td>Persistent parameter for banking sector net worth shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>Persistent parameter for government expenditure shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>Persistent parameter for labor supply shock</td>
<td>Beta</td>
<td>0.500</td>
<td>0.250</td>
</tr>
</tbody>
</table>

Standard Errors for Structural Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Density</th>
<th>Prior Mean</th>
<th>Prior SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_A$</td>
<td>SE of TFP shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_C$</td>
<td>SE of preference shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_E$</td>
<td>SE of entrepreneur net worth shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_F$</td>
<td>SE of banking sector net worth shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_G$</td>
<td>SE of government expenditure shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_K$</td>
<td>SE of investment specific technology shock</td>
<td>Inv. Gamma</td>
<td>1.000</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_L$</td>
<td>SE of labor supply shock</td>
<td>Inv. Gamma</td>
<td>0.707</td>
<td>4.000</td>
</tr>
<tr>
<td>$e_R$</td>
<td>SE or monetary policy shock</td>
<td>Inv. Gamma</td>
<td>0.224</td>
<td>4.000</td>
</tr>
</tbody>
</table>
Table 4: Posterior Estimates of Key Structural Parameters

<table>
<thead>
<tr>
<th>Parameters for Financial Friction in Corporate Section</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.614</td>
<td>0.877</td>
<td>0.564</td>
<td>0.562</td>
</tr>
<tr>
<td></td>
<td>[0.547, 0.689]</td>
<td>[0.818, 0.938]</td>
<td>[0.498, 0.632]</td>
<td>[0.470, 0.661]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.027</td>
<td>0.025</td>
<td>0.039</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>[0.024, 0.030]</td>
<td>[0.023, 0.026]</td>
<td>[0.032, 0.045]</td>
<td>[0.036, 0.046]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters for Nominal Rigidities</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_P$</td>
<td>0.854</td>
<td>0.374</td>
<td>0.804</td>
<td>0.760</td>
</tr>
<tr>
<td></td>
<td>[0.811, 0.895]</td>
<td>[0.305, 0.440]</td>
<td>[0.763, 0.846]</td>
<td>[0.697, 0.822]</td>
</tr>
<tr>
<td>$\theta_W$</td>
<td>0.589</td>
<td>0.428</td>
<td>0.623</td>
<td>0.516</td>
</tr>
<tr>
<td></td>
<td>[0.531, 0.649]</td>
<td>[0.351, 0.500]</td>
<td>[0.544 0.703]</td>
<td>[0.452, 0.580]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters for Monetary Policy Rule</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_R$</td>
<td>0.670</td>
<td>0.643</td>
<td>0.653</td>
<td>0.632</td>
</tr>
<tr>
<td></td>
<td>[0.581, 0.758]</td>
<td>[0.582, 0.707]</td>
<td>[0.605, 0.698]</td>
<td>[0.590, 0.675]</td>
</tr>
<tr>
<td>$\mu_\pi$</td>
<td>2.805</td>
<td>2.820</td>
<td>2.989</td>
<td>2.986</td>
</tr>
<tr>
<td></td>
<td>[2.767, 2.842]</td>
<td>[2.790, 2.848]</td>
<td>[2.979, 2.998]</td>
<td>[2.977, 2.995]</td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>0.006</td>
<td>0.010</td>
<td>0.006</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>[0.000, 0.014]</td>
<td>[0.000, 0.020]</td>
<td>[0.000, 0.013]</td>
<td>[0.000, 0.018]</td>
</tr>
</tbody>
</table>

Note: The parenthesis in the table indicates 90% credible interval of structural parameters. 300,000 iterations are implemented using algorithm of MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions.

Table 5: Timings of Peaks of the Financial Shocks

<table>
<thead>
<tr>
<th>Structural Shock</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corp. Net Worth</td>
<td>2009Q1</td>
<td>2009Q1</td>
<td>2009Q2</td>
<td>2009Q2</td>
</tr>
<tr>
<td>Bank Net Worth</td>
<td>2008Q3</td>
<td>2008Q3</td>
<td>2008Q3</td>
<td>2008Q3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stochastic Volatilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corp. Net Worth</td>
</tr>
<tr>
<td>Bank Net Worth</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
### Table 6: Average Ranges of 90% Credible Interval of Structural Shocks over the entire sample periods

<table>
<thead>
<tr>
<th>Structural Shocks</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.635</td>
<td>0.353</td>
<td>0.528</td>
<td>0.539</td>
</tr>
<tr>
<td>Preference</td>
<td>1.593</td>
<td>1.633</td>
<td>1.058</td>
<td>0.824</td>
</tr>
<tr>
<td>Corp. Net Worth</td>
<td>0.141</td>
<td>0.148</td>
<td>0.246</td>
<td>0.216</td>
</tr>
<tr>
<td>Bank Net Worth</td>
<td>1.902</td>
<td>1.433</td>
<td>0.886</td>
<td>0.907</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td>2.207</td>
<td>2.018</td>
<td>0.417</td>
<td>0.322</td>
</tr>
<tr>
<td>Investment</td>
<td>0.983</td>
<td>0.236</td>
<td>0.575</td>
<td>1.107</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>2.516</td>
<td>3.133</td>
<td>1.447</td>
<td>1.430</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>0.121</td>
<td>0.178</td>
<td>0.127</td>
<td>0.126</td>
</tr>
</tbody>
</table>

Note: This table reports the average of the difference between the upper and the lower bounds of 90% credible interval of the structural shock over the entire sample periods (1985Q2-2012Q2), depicted in Figures 1 and 2.

### Table 7: Average Ranges of 90% Credible Interval of Stochastic Volatilities in the entire sample periods

<table>
<thead>
<tr>
<th>Structural Shocks</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.498</td>
<td>0.384</td>
</tr>
<tr>
<td>Preference</td>
<td>0.906</td>
<td>0.857</td>
</tr>
<tr>
<td>Corp. Net Worth</td>
<td>0.243</td>
<td>0.219</td>
</tr>
<tr>
<td>Bank Net Worth</td>
<td>1.043</td>
<td>0.908</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td>0.709</td>
<td>0.769</td>
</tr>
<tr>
<td>Investment</td>
<td>0.604</td>
<td>0.592</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>1.743</td>
<td>1.378</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>0.095</td>
<td>0.095</td>
</tr>
</tbody>
</table>

Note: This table reports the average value in the entire sample periods (1985Q2-2012Q2) of the difference between the upper bound and the lower bound of 90% credible interval on the stochastic volatility for the structural shock depicted in Figure 3.

### Table 8: Leverage Effect of Structural Shocks: Correlation between the Sign of Shock and its Volatility

<table>
<thead>
<tr>
<th>Structural Shocks</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Preference</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Corp. Net Worth</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Bank Net Worth</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Government Expenditure</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Monetary Policy</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: The mark “-” indicates negative of $\rho_\sigma$ (leverage effect) at 90% credible degree of posterior probability, while the mark “+” does positive of $\rho_\sigma$ (opposite leverage effect) in similar way. The mark “0” implies that we do not judge the sign of $\rho_\sigma$ and leverage effect of each shock because zero is within 90% credible interval of $\rho_\sigma$. 
Table 9: Posterior Estimates: Case A and Case B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A</th>
<th></th>
<th></th>
<th>Case B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Structural Parameters</td>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.614</td>
<td>0.043</td>
<td>[0.547, 0.689]</td>
<td>0.877</td>
<td>0.038</td>
<td>[0.818, 0.938]</td>
</tr>
<tr>
<td>$h$</td>
<td>0.464</td>
<td>0.045</td>
<td>[0.396, 0.537]</td>
<td>0.597</td>
<td>0.040</td>
<td>[0.535, 0.661]</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>1.628</td>
<td>0.036</td>
<td>[1.578, 1.688]</td>
<td>1.404</td>
<td>0.032</td>
<td>[1.356, 1.451]</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.939</td>
<td>0.071</td>
<td>[0.819, 1.052]</td>
<td>0.417</td>
<td>0.072</td>
<td>[0.323, 0.524]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.027</td>
<td>0.002</td>
<td>[0.024, 0.030]</td>
<td>0.025</td>
<td>0.001</td>
<td>[0.023, 0.026]</td>
</tr>
<tr>
<td>$\iota_P$</td>
<td>0.521</td>
<td>0.027</td>
<td>[0.478, 0.566]</td>
<td>0.358</td>
<td>0.017</td>
<td>[0.330, 0.386]</td>
</tr>
<tr>
<td>$\iota_W$</td>
<td>0.422</td>
<td>0.009</td>
<td>[0.408, 0.437]</td>
<td>0.450</td>
<td>0.007</td>
<td>[0.440, 0.459]</td>
</tr>
<tr>
<td>$\theta_P$</td>
<td>0.854</td>
<td>0.026</td>
<td>[0.811, 0.895]</td>
<td>0.374</td>
<td>0.041</td>
<td>[0.305, 0.440]</td>
</tr>
<tr>
<td>$\theta_W$</td>
<td>0.589</td>
<td>0.037</td>
<td>[0.531, 0.649]</td>
<td>0.428</td>
<td>0.048</td>
<td>[0.351, 0.500]</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>0.670</td>
<td>0.055</td>
<td>[0.581, 0.758]</td>
<td>0.643</td>
<td>0.038</td>
<td>[0.582, 0.707]</td>
</tr>
<tr>
<td>$\mu_P$</td>
<td>2.805</td>
<td>0.025</td>
<td>[2.767, 2.842]</td>
<td>2.820</td>
<td>0.018</td>
<td>[2.790, 2.848]</td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>0.006</td>
<td>0.005</td>
<td>[0.000, 0.014]</td>
<td>0.010</td>
<td>0.007</td>
<td>[0.000, 0.020]</td>
</tr>
</tbody>
</table>

Persistence Parameters for Structural Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A</th>
<th></th>
<th></th>
<th>Case B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
<td></td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.975</td>
<td>0.007</td>
<td>[0.964, 0.986]</td>
<td>0.975</td>
<td>0.005</td>
<td>[0.966, 0.983]</td>
</tr>
<tr>
<td>$\rho_C$</td>
<td>0.636</td>
<td>0.093</td>
<td>[0.504, 0.788]</td>
<td>0.088</td>
<td>0.053</td>
<td>[0.004, 0.166]</td>
</tr>
<tr>
<td>$\rho_K$</td>
<td>0.391</td>
<td>0.044</td>
<td>[0.323, 0.462]</td>
<td>0.998</td>
<td>0.001</td>
<td>[0.996, 0.999]</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>0.907</td>
<td>0.022</td>
<td>[0.873, 0.944]</td>
<td>0.976</td>
<td>0.012</td>
<td>[0.959, 0.996]</td>
</tr>
<tr>
<td>$\rho_F$</td>
<td>0.031</td>
<td>0.024</td>
<td>[0.000, 0.064]</td>
<td>0.016</td>
<td>0.011</td>
<td>[0.000, 0.031]</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.798</td>
<td>0.047</td>
<td>[0.733, 0.864]</td>
<td>0.671</td>
<td>0.012</td>
<td>[0.652, 0.686]</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>0.933</td>
<td>0.041</td>
<td>[0.876, 0.995]</td>
<td>0.967</td>
<td>0.009</td>
<td>[0.953, 0.982]</td>
</tr>
</tbody>
</table>

Standard Errors for Structural Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A</th>
<th></th>
<th></th>
<th>Case B</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
<td>Mean</td>
<td>SD</td>
<td>90% CI</td>
<td></td>
</tr>
<tr>
<td>$e_A$</td>
<td>0.564</td>
<td>0.043</td>
<td>[0.492, 0.629]</td>
<td>0.398</td>
<td>0.030</td>
<td>[0.347, 0.447]</td>
</tr>
<tr>
<td>$e_C$</td>
<td>1.475</td>
<td>0.161</td>
<td>[1.242, 1.716]</td>
<td>1.729</td>
<td>0.189</td>
<td>[1.441, 1.986]</td>
</tr>
<tr>
<td>$e_K$</td>
<td>0.787</td>
<td>0.072</td>
<td>[0.689, 0.898]</td>
<td>1.423</td>
<td>0.042</td>
<td>[1.358, 1.491]</td>
</tr>
<tr>
<td>$e_E$</td>
<td>0.238</td>
<td>0.016</td>
<td>[0.212, 0.265]</td>
<td>0.286</td>
<td>0.020</td>
<td>[0.254, 0.318]</td>
</tr>
<tr>
<td>$e_F$</td>
<td>0.757</td>
<td>0.057</td>
<td>[0.690, 0.843]</td>
<td>0.890</td>
<td>0.058</td>
<td>[0.811, 0.979]</td>
</tr>
<tr>
<td>$e_G$</td>
<td>0.520</td>
<td>0.050</td>
<td>[0.439, 0.603]</td>
<td>0.895</td>
<td>0.119</td>
<td>[0.751, 1.102]</td>
</tr>
<tr>
<td>$e_L$</td>
<td>0.881</td>
<td>0.110</td>
<td>[0.722, 1.060]</td>
<td>1.383</td>
<td>0.040</td>
<td>[1.325, 1.448]</td>
</tr>
<tr>
<td>$e_R$</td>
<td>0.228</td>
<td>0.016</td>
<td>[0.201, 0.255]</td>
<td>0.245</td>
<td>0.019</td>
<td>[0.215, 0.274]</td>
</tr>
</tbody>
</table>

Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items SD and 90% CI denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.
Table 10: Posterior Estimates: Case C and Case D

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>0.564</td>
<td>0.562</td>
</tr>
<tr>
<td>( \bar{h} )</td>
<td>0.334</td>
<td>0.221</td>
</tr>
<tr>
<td>( \sigma_C )</td>
<td>1.630</td>
<td>1.605</td>
</tr>
<tr>
<td>( \sigma_L )</td>
<td>0.819</td>
<td>0.597</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.039</td>
<td>0.041</td>
</tr>
<tr>
<td>( \iota_P )</td>
<td>0.397</td>
<td>0.503</td>
</tr>
<tr>
<td>( \iota_W )</td>
<td>0.475</td>
<td>0.489</td>
</tr>
<tr>
<td>( \theta_P )</td>
<td>0.804</td>
<td>0.760</td>
</tr>
<tr>
<td>( \theta_W )</td>
<td>0.623</td>
<td>0.516</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>0.653</td>
<td>0.632</td>
</tr>
<tr>
<td>( \nu_C )</td>
<td>2.989</td>
<td>2.986</td>
</tr>
<tr>
<td>( \nu_Y )</td>
<td>0.006</td>
<td>0.008</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_A )</td>
<td>0.989</td>
<td>0.956</td>
</tr>
<tr>
<td>( \rho_C )</td>
<td>0.819</td>
<td>0.909</td>
</tr>
<tr>
<td>( \rho_E )</td>
<td>0.333</td>
<td>0.918</td>
</tr>
<tr>
<td>( \rho_F )</td>
<td>0.192</td>
<td>0.167</td>
</tr>
<tr>
<td>( \rho_G )</td>
<td>0.655</td>
<td>0.619</td>
</tr>
<tr>
<td>( \rho_L )</td>
<td>0.924</td>
<td>0.982</td>
</tr>
</tbody>
</table>

Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items SD and 90% CI denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.
Table 11: Posterior Estimates of Parameters of SVs: Case C and Case D

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>90% CI</th>
<th>Mean</th>
<th>SD</th>
<th>90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_A$</td>
<td>0.373</td>
<td>0.099</td>
<td>[0.215, 0.500]</td>
<td>0.338</td>
<td>0.120</td>
<td>[0.158, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_A}$</td>
<td>0.059</td>
<td>0.307</td>
<td>[-0.490, 0.507]</td>
<td>0.347</td>
<td>0.390</td>
<td>[-0.186, 0.989]</td>
</tr>
<tr>
<td>$\phi_A$</td>
<td>0.737</td>
<td>0.168</td>
<td>[0.530, 0.986]</td>
<td>0.509</td>
<td>0.184</td>
<td>[0.213, 0.810]</td>
</tr>
<tr>
<td>$\mu_A$</td>
<td>0.442</td>
<td>0.035</td>
<td>[0.378, 0.491]</td>
<td>0.429</td>
<td>0.049</td>
<td>[0.349, 0.501]</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.479</td>
<td>0.027</td>
<td>[0.456, 0.500]</td>
<td>0.476</td>
<td>0.023</td>
<td>[0.447, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_C}$</td>
<td>0.357</td>
<td>0.190</td>
<td>[0.043, 0.658]</td>
<td>0.481</td>
<td>0.141</td>
<td>[0.226, 0.696]</td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>0.934</td>
<td>0.059</td>
<td>[0.854, 0.990]</td>
<td>0.958</td>
<td>0.037</td>
<td>[0.919, 0.990]</td>
</tr>
<tr>
<td>$\mu_C$</td>
<td>0.656</td>
<td>0.075</td>
<td>[0.544, 0.764]</td>
<td>0.933</td>
<td>0.055</td>
<td>[0.844, 1.026]</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>0.434</td>
<td>0.053</td>
<td>[0.357, 0.500]</td>
<td>0.412</td>
<td>0.073</td>
<td>[0.303, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_E}$</td>
<td>0.161</td>
<td>0.221</td>
<td>[0.066, 0.785]</td>
<td>0.280</td>
<td>0.329</td>
<td>[-0.217, 0.869]</td>
</tr>
<tr>
<td>$\phi_E$</td>
<td>0.854</td>
<td>0.124</td>
<td>[0.627, 0.990]</td>
<td>0.758</td>
<td>0.186</td>
<td>[0.493, 0.990]</td>
</tr>
<tr>
<td>$\mu_E$</td>
<td>0.665</td>
<td>0.051</td>
<td>[0.139, 0.162]</td>
<td>0.194</td>
<td>0.013</td>
<td>[0.173, 0.212]</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.266</td>
<td>0.050</td>
<td>[0.182, 0.327]</td>
<td>0.440</td>
<td>0.048</td>
<td>[0.373, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_G}$</td>
<td>0.384</td>
<td>0.350</td>
<td>[-0.228, 0.990]</td>
<td>0.044</td>
<td>0.367</td>
<td>[-0.536, 0.670]</td>
</tr>
<tr>
<td>$\phi_G$</td>
<td>0.663</td>
<td>0.286</td>
<td>[0.178, 0.990]</td>
<td>0.517</td>
<td>0.246</td>
<td>[0.071, 0.891]</td>
</tr>
<tr>
<td>$\mu_G$</td>
<td>0.505</td>
<td>0.028</td>
<td>[0.461, 0.548]</td>
<td>0.570</td>
<td>0.031</td>
<td>[0.519, 0.627]</td>
</tr>
<tr>
<td>$\sigma_K$</td>
<td>0.449</td>
<td>0.038</td>
<td>[0.399, 0.500]</td>
<td>0.452</td>
<td>0.063</td>
<td>[0.335, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_K}$</td>
<td>0.304</td>
<td>0.318</td>
<td>[-0.228, 0.841]</td>
<td>0.128</td>
<td>0.246</td>
<td>[-0.215, 0.540]</td>
</tr>
<tr>
<td>$\phi_K$</td>
<td>0.450</td>
<td>0.257</td>
<td>[0.001, 0.838]</td>
<td>0.219</td>
<td>0.214</td>
<td>[0.000, 0.548]</td>
</tr>
<tr>
<td>$\mu_K$</td>
<td>0.496</td>
<td>0.023</td>
<td>[0.457, 0.528]</td>
<td>0.406</td>
<td>0.049</td>
<td>[0.324, 0.476]</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.446</td>
<td>0.043</td>
<td>[0.386, 0.500]</td>
<td>0.482</td>
<td>0.016</td>
<td>[0.458, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_L}$</td>
<td>-0.163</td>
<td>0.308</td>
<td>[-0.781, 0.254]</td>
<td>0.232</td>
<td>0.178</td>
<td>[-0.071, 0.517]</td>
</tr>
<tr>
<td>$\phi_L$</td>
<td>0.779</td>
<td>0.263</td>
<td>[0.291, 0.990]</td>
<td>0.903</td>
<td>0.084</td>
<td>[0.813, 0.990]</td>
</tr>
<tr>
<td>$\mu_L$</td>
<td>1.010</td>
<td>0.116</td>
<td>[0.870, 1.207]</td>
<td>1.461</td>
<td>0.078</td>
<td>[1.351, 1.580]</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.422</td>
<td>0.045</td>
<td>[0.355, 0.493]</td>
<td>0.464</td>
<td>0.034</td>
<td>[0.407, 0.500]</td>
</tr>
<tr>
<td>$\rho_{\sigma_R}$</td>
<td>0.156</td>
<td>0.238</td>
<td>[-0.268, 0.520]</td>
<td>0.479</td>
<td>0.211</td>
<td>[0.122, 0.797]</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>0.763</td>
<td>0.109</td>
<td>[0.589, 0.948]</td>
<td>0.727</td>
<td>0.122</td>
<td>[0.540, 0.941]</td>
</tr>
<tr>
<td>$\mu_R$</td>
<td>0.099</td>
<td>0.006</td>
<td>[0.089, 0.106]</td>
<td>0.112</td>
<td>0.013</td>
<td>[0.092, 0.131]</td>
</tr>
</tbody>
</table>

Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items SD and 90% CI denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.
Figure 1: Structural Shocks with i.i.d. Normal in Cases A and B
Note: Deep blue line and blue shaded area are posterior mean and 90% credible interval of structural shocks in Case A, respectively. And deep red line and red shaded area are posterior mean and 90% credible interval of structural shocks in Case B.
Figure 2: Structural Shocks with SV in Cases C and D

Note: Deep blue line and blue shaded area are posterior mean and 90% credible interval of structural shocks in Case C, respectively. And deep red line and red shaded area are posterior mean and 90% credible interval of structural shocks in Case D.
Figure 3: Stochastic Volatilities of Structural Shocks in Cases C and D

Note: Deep blue line and blue shaded area are posterior mean and 90% credible interval of structural shocks in Case C, respectively. And deep red line and red shaded area are posterior mean and 90% credible interval of structural shocks in Case D. Black dashed lines are constant volatilities estimated in Case A and B.
Figure 4: Historical Decomposition of Real GDP

Note: Case A: 11 observable variables and structural shocks with i.i.d.  Case B: 41 observable variables and structural shocks with i.i.d.  Case C: 11 observable variables and structural shocks with SV.  Case D: 40 observable variables and structural shocks with SV.
Figure 5: Historical Decomposition of Gross Private Domestic Investment
Note: Case A: 11 observable variables and structural shocks with i.i.d.  Case B: 41 observable variables and structural shocks with i.i.d.  Case C: 11 observable variables and structural shocks with SV.  Case D: 40 observable variables and structural shocks with SV.
Figure 6: Historical Decomposition of Moody’s Bond Index (Corporate Baa)
Note: Case A: 11 observable variables and structural shocks with i.i.d.  Case B: 41 observable variables and structural shocks with i.i.d.  Case C: 11 observable variables and structural shocks with SV.  Case D: 40 observable variables and structural shocks with SV.
Figure 7: Historical Decomposition of Commercial Bank Leverage Ratio
Note: Case A: 11 observable variables and structural shocks with i.i.d.  Case B: 41 observable variables and structural shocks with i.i.d.  Case C: 11 observable variables and structural shocks with SV.  Case D: 40 observable variables and structural shocks with SV.