PART I

Public Infrastructure Support for Industry: Common Property versus Collective Property*

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Abstract: Public infrastructure and other publicly provided services that benefit industry fall into two board categories: pure and congestible. This paper explores the implications of both types of public inputs. The differences between the two have considerable implications for the design of tax and spending policy. In particular, this paper highlights the effects of a congestible public input as being analogous to a free-access or common property resource. Efficiency requires that fundamentally different means of financing each type.

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1. Introduction

Traditionally, neo-classical analysis of public spending has focused on either public consumption goods or transfer payments. Considerably less attention has been paid to public expenditure on activities that are productivity enhancing. Such activities include "hard" items in the form of public infrastructure such as highways, water and waste systems and harbours, as well as "soft" activities such as literacy, research and development, and marine weather forecasts. Provision of such commodities, which we refer to as public inputs, generally increases the productivity of industries. Certainly, there are examples of spectacularly successful public inputs: literacy, agricultural research and the Panama Canal.

The purpose of this paper is to explore some of the theoretical issues that still surround these commodities. Some of the important issues are: how to classify and model public inputs; the question of returns to scale; the generation of and implications of economic rent; and financing. To these ends, this paper is organized as follows. Section 2 deals with the classification and modeling of public inputs. Section 3 considers the provision of each of the two main classes of public inputs and highlights how a common-property problem can arise. That common-property attribute has important implications for the literature. Then Section 4 turns to the issue of economic rent and how its existence can impede first-best provision. The issue of financing is taken up in Section 5, and Section 6 concludes.

2. Classification

The most useful place to start is with a classification system. In the modest literature on the theory of public inputs, there have been a number of different specifications of public inputs.

2.1 Firm-augmenting Public Inputs.

There are some instances in the literature where public inputs are specified as "firm-augmenting". The essential idea is that a public input provides services to firms regardless of their size, which is somewhat analogous to the relationship between public consumption goods and persons. However, a firm, unlike a person, can be divided into smaller firms, the effect of which is to create more benefit from the same level of G, a feature which calls this specification into question from the start.

More formally, and for simplicity, assume that each firm in an industry is of equal size so we may write

$$X/N = F(K/N, L/N, G) \qquad (1)$$

where K and L are total capital and labour services employed in the industry. X is industry output and N is the number of firms (or, more precisely, the number of production units). G denotes quantity of the public input. Under the firm-augmenting specification, it is assumed that F is homogeneous of degree one in all inputs. By implication, aggregate output may be written as:

$$X = F(K, L, NG), \qquad (2)$$

which suggests that by dividing firms into ever smaller sizes, total output can be increased without bound.

This unboundness outcome can be avoided with additional assumptions; e.g., a minimum size for firms, or a fixed number of firms as in Richter (1998), or a fixed cost of establishing each firm as in Matsumoto (2000). On the other hand, a lack of realistic examples of this type of commodity, and the idea that the input is public among firms, which are institutional entities, have led McMillan (1979) and Henderson (1974), amongst others, to conclude that this specification is of limited practical interest. Hence, this paper will focus on other types of public inputs.[1]

2.2 Pure Public Inputs

An appealing specification of a public input is one of the "factor-augmenting" or "atmospheric" or "pure" type. Here the public input acts as an input that is generally available to firms and industries and it not subject to congestion, either by the number of firms or the quantities of output or primary inputs. Two examples are information from research and development that improves productivity and basic literacy for workers. The effect of such inputs is akin to a shift in industry production functions.

In general, we may write the i-th industry's production function as:

$$X_i = F_i (K_i, L_i, G) \tag{3}$$

and take the industry production functions as linearly homogeneous in the primary factors only and twice continuously differentiable in all inputs. Thus, G is beneficial across industries (although there may be industries for which G is not useful) and is not subject to any form of congestion. A good example might be flood control; if dams are built to prevent floods in a country then all industries within the geographic area at risk benefit in that their output would be higher than had there been a lower level of flood control.

The assumption of constant returns to scale for these types of public inputs is very intuitive. G is akin to information or technology. If K and L are both doubled, then we would expect output to double since G is readily available in undiminished quantity.

2.3 Congestible Public Inputs

While the factor-augmenting variety of public input appears to encompass some forms of public input, it does not seem to capture all possibilities. Consider capital infrastructure such as highways and bridges. These are congestible inputs. As output within an industry increases one would expect that such facilities would be used more. Thus, we would expect diminishing returns to scale in primary factors. The possibility of replication suggests that there would be constant returns to scale in all factors. For example, consider an agricultural economy, and let H denote land, L denote workers and G denote a national irrigation system. A doubling of only H and L cannot be expected to lead to double output. Output would be less than doubled unless the irrigation system is similarly extended.

2.3.1 Unpaid Factor in a One-good Economy

For a one-good economy, then, the industry production function could be written as:

$$X = F(K, L,G), \tag{4}$$

which is taken to be linearly homogeneous and twice continuously differentiable in all inputs.

[1] However, once one adopts assumptions such as a fixed number of firms or fixed start-up costs for new firms, then a firm-augmenting public input might be classified as a special case of a congestible public input.

2.3.2 Unpaid-Factor in a Two-good Economy

In a two-or-more-good economy, one must consider than in general the amount of utilization across industries will also have an impact on the extent to which any one industry can benefit from a fixed amount of G. If many industries use a highway or harbour then the benefits to one industry will vary not only with than industry's use but also with the amount of utilization by other industries. To capture this cross-industry effect, we must make an appropriate modification of the one-good case discussed above.

More specifically, consider a two-good economy. The production functions of each industry will be expressed as:

$$X_1 = F_1(K_1, L_1, G_1) \tag{5}$$

and

$$X_2 = F_2(K_2, L_2, G_2) \tag{6}$$

where

$$G_i = G - u_i X_j; \quad i = 1, 2 \text{ and } i \neq j; u_i \geq 0, \tag{7}$$

and where F_i is again linearly homogeneous and twice continuously differentiable in all three inputs.

In this specification, u_i, a utilization coefficient, represents the degree to which output in the other industry in the economy diminishes the availability of the public input to other industries. We use a simple case where u_i is a constant, indicating a linear relationship between the availability of G to one industry and the extent of use by another industry. This is done for simplicity and could be generalized.

Also, notice that if one of either of the two industries does not operate then there is no inter-industry congestion. Any intra-industry congestion is embodied in the industry's production function through the constant returns property. Having no such inter-industry effects is difficult to conceive of if intra-industry congestion is possible. However, Negishi (1974) uses a model based on this specification.[2] The Negishi model implicitly takes $u_1 = u_2 = 0$. Thus, the public input is, in effect, congestible within an industry but not so across industries. This is indeed an unusual type of public input but perhaps a few examples are possible: industries may use G at different times of the year in which case one industry's use may not affect the others.[3] Nevertheless, this case, which, as Feehan (1989) notes, has been referred to as semi-public, is quite special and unlikely to apply to most industries or to most public inputs.

Quite remarkably, there does not appear to be any treatment of inter-industry affects in the public input\public infrastructure literature. One can find examples of unpaid public inputs in the literature but in one-good models, so congestion is not considered. There are a few instances of the Negishi specification. Otherwise, the public input is modeled as factor-augmenting or as firm-augmenting.

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[2] Tawada and Okamoto (1983) use the Negishi model to explore the implications for international trade, and Yututake (1995) also employs a version of the Negishi model.

[3] Tawada (1982) explores whether such a "semi-public" input generates a non-convex production set and finds that it does not. However, with inter-industry congestion in use, non-convexity of the production set must be expected; see the discussion of production externalities in Bhagwati and Srinivasan (1983; pp.185-191).

2.3.3 Rationale for Public Financing

An examination of either (5) or (6), considering the assumption of constant returns to scale in either, explains the frequent use of the term "unpaid factor" to describe this type of public input; applying Euler's theorem to it suggests that G is not analytically different from the other factor inputs. As such, one might conclude that this input could be financed directly by the private sector and be sold to users just like a private intermediate input. Perhaps the main justification for public finance lies in the need for mandatory subscription. Consider an agricultural example. An irrigation system must pass through all or most farms. Similarly, an aerial spray program may be needed for farms in an area but it may not be feasible to exclude adjacent farms. Further, in the case of highways, it is often not practical, due to high costs, to collect fees from users according to their utilization and to ensure that non-payers do not use the facility.

One could argue that for this sort of "public" input, that the beneficiaries would negotiate some collective action and provide the input on a joint basis. However, high transactions costs, lack of enforcement powers, or a very large number of agents could make that impractical; i.e., the conditions for application of the Coase theorem do not hold. Thus, this type of public input is distinct from an intermediate input that is truly private, e.g., seeds.

2.4 Synopsis

Based on the preceding discussion, one can conclude that analysis of public inputs should allow for, and be careful to distinguish between, two types of public inputs: factor-augmenting public inputs, which might be better classified as "pure public inputs" and the unpaid-factor type, which might be better described as "congestible public inputs." Table 1 on the next page summarizes these results.

At the practical level, it seems that pure public inputs encompass knowledge-related inputs such as literacy, agricultural advice, marine weather forecasts, and basic R&D, which benefits an industry or industries. On the other hand, congestible public inputs seem to capture the basic features of other forms of public infrastructure, e.g., highways, docks, harbours, water systems, and waste management systems for industries.

Ultimately whether a public input fits into one category of another is an empirical question. If it is established that the returns to scale are constant in all inputs then the public input must fall into the congestible category. If the returns to scale in the primary inputs are constant then the public input must be a pure public input. It seems reasonable to expect some types of public infrastructure, e.g., highways and bridges, to fall into the former category and others, e.g., weather satellites, agricultural research laboratories, and lighthouses, to be of the latter form.

TABLE 1

A Classification System for Public Inputs

Type:	PURE (Factor-augmenting)	CONGESTIBLE (Unpaid Factor)
Characteristics:		
1. Returns to scale:	Private-goods production functions exhibit constant returns to scale in primary inputs. There are IRS in all inputs.	Private-goods production functions exhibit decreasing returns to scale in primary inputs. Constant in all inputs.
2. Amount of G available to an industry i	$G_i = G$ due to non-congestibility	$G_i < G$ due to intra- and inter-industry congestion, except in the special case of a semi-public input
3. Impact on industries	Affects at least one industry	Affects at least one industry
Examples:	Literacy Geological surveys Weather forecasts Lighthouses Flood control?	Highways Ports Locks and canals Bridges Flood control?

3. First-best Provision

An important policy issue is whether the provision of a public input is efficient. If too much or too little G is provided then the economy is worse off than otherwise.

3.1 Derivation of Efficiency Conditions

To derive the conditions for first-best provision of a public input, we follow the traditional analysis analogous to Samuelson's treatment of public consumption goods and Kaizuka's (1965) consideration of a generic public input.[4]

Consider a simple economy with a representative individual who obtains utility from consumption of two private goods. Thus, the utility function is:

$$U = U(X_1, X_2), \tag{8}$$

which is assumed to be quasiconcave, twice continuously differentiable and to exhibit diminishing marginal utility.

Next, assume that there are two factors of production, labour and capital, in fixed supply. So, the allocation of factors is given by:

$$L_1 + L_2 + L_G = L \tag{9}$$

and

$$K_1 + K_2 + K_G = K. \tag{10}$$

Production of the two private goods is given by:

$$X_1 = F_1(L_1, K_1, G - u_1X_2) \tag{11}$$

and

$$X_2 = F_2(L_2, K_2, G - u_2X_1). \tag{12}$$

Note that this formulation encompasses the case of the pure public input; simply take the u's as zeros and consider F_1 and F_2 as linearly homogeneous in capital and labour.

Next, assume that the public input itself it produced with labour and capital according to a CRS production function:

$$G = F_G(L_G, K_G). \tag{13}$$

Then the Lagrangean for a Samuelson-type planner is:

$$Z = U(X_1, X_2) + b_1[F_1(L_1, K_1, G - u_1X_2) - X_1] + b_2[F_2(L_2, K_2, G - u_2X_1) - X_2]$$
$$+ b_3[F_G(L_G, K_G) - G] + b_4[K_1 + K_2 + K_G - K] + b_5[L_1 + L_2 + L_G - L]. \tag{14}$$

where the b's denote the respective Lagrangean multipliers.

The first order conditions for X_1, X_2, K_1, K_2, L_1, L_2, G, K_G, and L_G are given below:

$$U_1 - b_1 - b_2u_2F_{2G} = 0 \tag{15}$$
$$U_2 - b_2 - b_1u_1F_{1G} = 0 \tag{16}$$

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[4] Generic in the sense that the distinction between the two fundamentally different types of public input was not made.

$$b_1F_{1K} + b_4 = 0 \quad (17)$$
$$b_2F_{2K} + b_4 = 0 \quad (18)$$
$$b_1F_{1L} + b_5 = 0 \quad (19)$$
$$b_2F_{2L} + b_5 = 0 \quad (20)$$
$$b_1F_{1G} + b_2F_{2G} - b_3 = 0 \quad (21)$$
$$b_3F_{GL} + b_5 = 0 \quad (22)$$
$$b_3F_{GK} + b_4 = 0. \quad (23)$$

Using equations (17) with (19), (18) with (20), and (22) with (23), we obtain

$$F_{1L}/F_{1K} = F_{2L}/F_{2K} = F_{GL}/F_{GK} \quad (24A)$$

i.e.,

$$MRTS_{KL}^{1} = MRTS_{KL}^{2} = MRTS_{KL}^{G} \quad (24B)$$

where $MRTS_{KL}^{i}$ is the marginal rate of technical substitution between capital and labour in the production of good i, i = 1, 2 and G. Condition (24) is the familiar production efficiency requirement.

Next, using (19), (20) and (22) to substitute for b_1, b_2 and b_3, respectively, in (21) gives

$$F_{1G}/F_{1L} + F_{2G}/F_{2L} = 1/F_{GL} \quad (25A)$$

and similarly use of (17), (18) and (23) in (21) gives

$$F_{1G}/F_{1K} + F_{2G}/F_{2K} = 1/F_{GK}. \quad (25B)$$

Results (25) provide the condition for efficient provision of G, and correspond to the conditions identified in Kaizuka (1965).

Finally, using (15) and (16), and substituting for b_1 and b_2 using (17) and (18) one can obtain:

$$U_2/U_1 = [1 + u_2F_{2G}(F_{1K}/F_{2K})]/[(F_{1K}/F_{2K}) + u_1F_{1G}] \quad (26)$$

which is the product-mix efficiency condition. Of course, in the case of a pure public input, or when a congestible public input is semi-public, i.e., $u_1 = u_2 = 0$, then the right-hand-side of (26) simplifies to the marginal rate of transformation. However, in general, for ocongestible public inputs, the more complex expression applies.[5] Remarkably, this result for a congestible public input does not seem to be contained anywhere in the public-inputs literature.

3.2 The Implied Role of the Public Sector

Having obtained these efficiency conditions we can now consider the implied role for government in a market economy. Interesting, that role will vary considerably depending on whether the public input is pure or congestible, as these terms have been defined herein.

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[5] If, by chance, $u_{1}/u_2 = (F_{2G}/F_{1G})[(F_{1K}/F_{2K})^2$ then equality of the MRS and MRT are again called for; this occurs when the marginal cross-industry congestion externalities are exactly offsetting.

3.2.1 Pure Public Input

As noted when the public input is atmospheric, condition (26) simplifies to the standard requirement that the consumer's marginal rate of substitution (MRS) between the two private goods equal the corresponding marginal rate of transformation (MRT). In a competitive market economy with utility-maximizing behaviour on the part of households and profit-maximizing behaviour on the part of firms, this condition would be satisfied for any given quantity of G.

Also, recall that with a pure public input, the production functions of firms are characterized by constant returns to scale in the primary inputs; here, labour and capital. Firms maximize profit by hiring factors up to where the value of the factors' respective marginal products equal their factor prices, (and by Euler's theorem there is no residual for excess profit) so in the case of a pure public input, condition (24B) would be satisfied.

In effect, if the authorities have access to non-distortionary taxation then the only role for the public sector is to provide the public input. All other conditions would be realized as a result of competitive general equilibrium. Moreover, since factors hired by firms that produce G would hire labour and capital up to where the values of their respective marginal product equals the factor cost, (25) simply becomes

$$P_1F_{1G} + P_2F_{2G} = P_G \qquad (27)$$

where P_i represent the respective market prices (equal to marginal costs) of the two private goods and the public good. This is the only condition that needs to be realized by policymakers.

A caveat here relates to returns to scale in this economy. Since the production function for any commodity that is affected by G exhibits increasing returns to scale in all inputs, with constant returns in the set of primary factors, the production possibilities frontier may be convex to the origin at least over some range. This can give rise to multiple equilibriums, so policy-makers must ensure that G not only satisfies (27) but that no other level of G also satisfies it at a higher level of utility for the representative household.

3.2.2 A Congestible Public Input

In the case of a congestible public input, meeting the efficiency conditions is more demanding. First, consider condition (26), the product-mix efficiency condition when more than one industry benefits from the public input. Producers do not consider the fact that their output decisions have an adverse effect on the other industry. In effect, there is a cross-industry output externality. Condition (26) indicates that an appropriate wedge between the MRS and the private MRT is needed in order to achieve product mix efficiency. In other words, some form of corrective policy is needed to realize the required equality; a production tax cum subsidy is one such corrective policy.[6]

Is such a tax cum subsidy policy combined with choosing the appropriate level of G and relying on a lump-sum taxation sufficient to achieve the optimum? The answer to this question depends crucially on whether the primary factors are allocated across industries according to equality of marginal rates of technical substitution, as in condition (24), i.e., on whether each factor's unit reward coincides with the value of its marginal product. However, a public input generates an economic rent. Can that rent lead to behaviour that causes a misallocation of factors? That is the subject matter of the next section.

[6] The obvious exceptions to this are: (i) when we have a single-good economy or (ii) when the values of the terms in the right-hand-side expression of (26) are such that, by chance, the expression equals the MRT.

4. Economic Rent

The existence of economic rent due to a public input is important for two reasons. First, if there is economic rent then it provides a source from which the authorities can raise revenue in a non-distortionary manner. This is important if other non-distorting revenue instruments are not available (e.g., if factors are in variable supply). Secondly, the rent may possibly induce rent-seeking behaviour, leading to the dissipation of the rent and therefore causing economic inefficiency. If so, then corrective measures are needed in order to offset rent dissipation.

4.1 A Pure Public Input and Economic Rent

With a pure public input, private-goods production functions in which G enters as an argument are linearly homogeneous in the primary factors. Industry profit is given by:

$$\prod i = P_iF_i(L_i, K_i, G) - wL_i - rK_i \tag{28}$$

were G is parametric, being provided by the public sector and not variable to the firms. Factor prices are denoted as w and r. In competitive market conditions, labour and capital are hired according to:

$$P_iF_{iL} = w \tag{29}$$

and

$$P_iF_{iK} = r. \tag{30}$$

Since, the production function contained in (28) is linearly homogeneous in capital and labour, by Euler's theorem

$$F_{1L}L_1 + F_{1K}K_1 = P_1F_1(L_1, K_1 G) \tag{31A}$$

$$F_{2L}L_2 + F_2K_2 = P_2F_2(L_2, K_2, G) \tag{31B}$$

and

$$F_{GL}L_G + F_{GK}K_G = P_GF_G(L_G, K_G). \tag{31C}$$

So, noting (29) and (30), we see from (31) that factor payments exactly exhaust revenues leaving no excess profit for firms in competitive equilibrium. Effectively, the gains from the public input to the private-goods producing industries are similar to the gains from a shift in the production function due to technological improvement. These gains accrue to the factors of production as their marginal products rise, at any given factor proportion.

That absence of excess profit means that industries hire factors so that the MRTS between the primary factors coincide with the respective factor-price ratio. Hence, in the case of a pure public input, and as alluded to previously, condition (24) is met. It also means that that a profits tax on firms would yield no revenue so other means of finance must be considered.

In summary, for the case of a pure public input, it is confirmed that if a lump-sum tax, or its equivalent, is available then the only role for the public sector in a competitive economy is to finance G in accordance with (27). The other required efficiency conditions, including the market-clearing conditions, will be achieved by market forces. The case of distortionary taxation will be considered in section 5.

4.2 A Congestible Public Input and Economic Rent

Since the production functions of industries that utilize a congestible public input are characterized by constant returns to scale in all inputs, Euler's equation indicates that rent will be generated. The crucial question at hand is whether, in equilibrium, this rent remains intact with the owners of firms, or be dissipated by rent-seeking behaviour, or accrue to the factors of production. The literature contains different opinions on this matter.

4.2.1 The Literature

In some recent contributions in which public inputs are congestible, i.e., where there are decreasing returns to scale in the primary factors, it is assumed that rent from the public input is an equilibrium phenomenon.[7] The rent, which is measured by the value of the public input's marginal product multiplied by its quantity, is assumed to accrue to the industry as excess profit. If so, then this rent would serve as an ideal source of public revenue, while at the same time not leading to any inefficiency.

The essential logic for the presumption of equilibrium excess profit can be illustrated in a one-good model. This avoids the complications of inter-industry effects. By Euler's theorem

$$F(K,L,G) = F_L(K, L, G)L + F_K(K, L, G)K + F_G(K, L, G)G \tag{32}$$

The private-good producing industry takes G as parametric, and maximizes industry profit. Normalizing the price of the private good to unity, the profit is given by:

$$\prod = F(K,L,G) - wL - rK. \tag{33}$$

The first-order conditions for maximization of that profit are:

$$F_L(K, L, G) = w \tag{34}$$

and

$$F_K(K, L, G) = r. \tag{35}$$

Substituting the first-order conditions back into the profit function, noting the Euler equation, gives $\prod = F_G(K, L, G)G$, which is positive.

This notion that positive excess profit can persist in a competitive equilibrium is somewhat unsettling since one might expect the existence of profit to act as an incentive for economic agents to employ resources in order to obtain some of the rent. It is also in contraction to other contributions to the literature. Consider Henderson (1974, p.324). With an industry production function that is CRS in all factors and for a given level of a public input, which he denotes as "R", he concludes:

> "…firms in the X industry will initially earn profits or rents according to the value of the marginal product of R. Following the usual analysis (Meade, 1955; Worcester, 1969), we assume that this rent accrues to

[7] Examples are Keen and Marchand (1997) and Dahlby and Wilson (2003).

some ownership factor such as capital or entrepreneurship. As with a private unpriced input, these profits will attract other firms into the X industry, leading at the limit to dissipation of the profits."

In addition, Gramlich (1994, pp.1185-1186) suggests that for the case in which the industry production function exhibits constant returns to scale in all inputs, i.e., there is decreasing returns to scale for capital and labour, then labour and capital are paid more than their respective marginal products, i.e., the rent accrues to factors not firms. Further, in a two-good model with CRS in all inputs including a semi-public input (but abstracting from inter-industry congestion), Negishi (1974) enforces a zero-excess-profit condition by having the rent dissipated through the excessive employment of capital. The implication of Negishi's analysis is that resource misallocation must occur since the rent per unit of capital will not in general be equal across industries at the optimal level of the public input.

Clearly then the literature contains strongly contradictory positions.

4.2.2 Resolution: Economic Rent does not Arise

If there are constant returns to scale in all inputs, and therefore decreasing returns to scale in the private inputs, then the provision of G without charge by government is analytically equivalent to a common-property situation. With an unrationed common property resource, it is well established that firms will not utilize the private inputs in quantities that accord with the equality of the values of their marginal products to their respective factor prices.

Consider the simple case of an economy with one primary factor and a public input which benefits the X_1-industry. The industry production function is given by

$$X_1 = F_1(L_1, G) \tag{36}$$

and the industry's profit is

$$P_1X_1 - wL_1. \tag{37}$$

With constant returns to scale in both inputs, but with G being provided free-of-charge by government, firms' profit-maximizing behaviour leads to the classic inefficiency result known as the Tragedy of the Commons, as in Varian (2003, p.618). Firms hire the private input until all the rent is dissipated. L_1 is hired up to where the value of the <u>average</u> product of labour in the X_1 industry equals the wage:

$$P_1(APL_1) = w \tag{38}$$

where $APL_1 = X_1/L_1$ is the average product of labour. And, with decreasing returns to scale in L_1, the average product of labour exceeds its marginal product, $APL_1 > F_{1L}$, so the amount of L_1 is excessive. In fact, in this one-primary-factor case, the gap between the average and marginal product of labour, is the average rent per unit of L, so labour is hired up to where:

$$P_1APL_1 = F_{L1} + R_1/L_1 = w \tag{39}$$

where $R_1 = P_1F_{1G}G$ is the rent generated by the amount of G provided by government, and open to dissipation by excessive employment of labour in this industry. [(39) is easy to confirm with a Cobb-Douglas numerical example]

The flaw in the presumption that excess profits persist in equilibrium when there is decreasing returns to scale in the primary inputs is due to a failure to look below the industry level at the behaviour of firms. The only ways that profit could be generated as an equilibrium phenomenon in these circumstances are:

* the government not only provides G but allocates G to a fixed number of chosen firms, possibly just one, in specific amounts, (this would be akin to issuing individual quotas to use a community pasture); or

* he conditions of the Coase theorem hold (i.e., existing firms would agree to share the common property resource in a manner that maximized industry profit), which is extremely unlikely since the product market is competitive, entailing free entry.

There are immediate implications here for the literature that presumes equilibrium excess profits, e.g., Keen and Marchand (1997) and Dahlby and Wilson (2003). First, there is no excess profit so components of their analyses that rest on positive revenues from profit taxation cannot likely be fully sustained without qualification. Secondly, it is necessary to recognize and incorporate both the misallocation of factors that occurs as a result of not charging levies for use of public inputs or having some other rationing mechanism. Otherwise, results based on equilibrium profits can be salvaged by adopting the assumption that the public input is rationed by government among firms or that the Coase theorem holds. However, since neither is very likely in practice, the results from analyses based on realization of profit must be considered as special or limiting cases.

4.2.3 Implications for the First-Best Outcome

The implications for achieving the first-best conditions are also far reaching. None of the three conditions, (24), (25) and (26), can be expected to hold when the public input is congestible. Whether (24) or (26) holds has already been discussed, so attention here can be devoted to (25). Recall that it is the requirement of equality of the marginal rates of technical substitution between primary factors across all industries, including the public-input producing industry. In the two-good two-factor model, this condition is $MRTS_{KL}^1 = MRTS_{KL}^2 = MRTS_{KL}^G$. (In the one-primary-factor model, the corresponding condition is equality of the values of the marginal products of that primary factor across industries.) Even if the first-best quantity of G is provided, we cannot expect factors to be allocated in a manner that satisfies this efficiency requirement. The common-property attribute of G leads to factors being drawn to industries until all rents are dissipated.

How do we model this rent-dissipating phenomenon in a model with more than one final good? In the two-factor two-good context, with CRS in all inputs and with G provided in a non-rationed manner then one would expect that firms in each industry would hire labour and capital until:

$$P_1F_{1L} + S_1R_1/L_1 = w = P_2F_{2L} + S_2R_2/L_2 \qquad (40A)$$

and

$$P_1F_{1K} + (1\text{-}S_1)R_1/K_1 = r = P_2F_{2K} + (1\text{-}S_2)R_2/K_2 \qquad (40B)$$

where S_i, for 1 = 1 and 2, is a fraction, and represents the share of the average rent per worker in industry i that firms obtain from hiring an additional worker; $(1\text{-}S_1)$ has an analogous interpretation for capital. Also, $R_i = P_iF_{iG}G$ is the rent in industry i arising from the supply of G that has been provided by the public sector. From inspection of these conditions, it follows that equality of marginal rates of technical substitution between labour and capital across industries cannot be expected. Hence, the rent-seeking behaviour arising from the free-access to G means that the economy is operating inside its production possibilities frontier.

Clearly, in the case of one factor (L), condition (40) simplifies to the previous and well established result given in (39) since then $S_1 = 1$. In the more general case of two (or more) factors, the value of S_i is not so readily determined. Negishi (1974), using a model with a semi-public input, assumes that the rent is captured through the use of more capital only, i.e., $S_i = 0$ and $(1-S_i) = 1$. However, this seems difficult to rationalize as a general proposition.[8] It requires some form of Leontif-type fixed-proportions technology. We adopt the more general neo-classical assumptions regarding production functions, e.g., strict-quasiconcavity in all arguments, positive continuous first-order partial derivatives with respect to all factors, and positive cross-partial derivatives. In these circumstances, what might determine the values of S_i?

To answer that question, suppose that the economy is in a competitive equilibrium situation, so all the rent from the current level of G has been dissipated. Now assume that G is increased by one unit, which distributes this equilibrium. Firms in an industry have an incentive to hire more factors to capture the rent. At the current wage-rental ratio, an individual firm will hire capital and labour at the current capital-labour ratio (assuming that the production function is characterized by CES-constant elasticity of substitution, of which the Cobb-Douglas formulation is a special case).[9] Hence, the shares of the rent accruing to capital and labour will be determined solely by the capital-labour ratio. Therefore, we have:

$$S_i = S_i(K_i/L_i) \text{ and } S_i = \quad S_i/ \quad (K_i/L_i) > 0; \; i = 1, 2 \ldots N \text{ firms}; \qquad (41)$$

for CES production functions.

For the limiting case of Cobb-Douglas, note that the right-hand-side of (41) is a constant. For instance, suppose $X = K^g L^d G^q$ where $g + d + q = 1$. Then g, d, and q would give the shares of revenue accruing to the respective factors if they are priced. Now, suppose that the price of G is zero and G is effectively unrationed common property. At given wage and rental one would not expect firms to alter their capital-labour ratios. Therefore the share of revenue going to labour would now be $g/(g + d)$ and to capital be $d/(g + d)$.

4.3 Summary

It is reasonable to believe that some public goods are pure and some are not. In the former case, it is readily established that the public sector, if it has access to non-distortionary taxes, need only purchase and make the first-best level available.

However, the preceding analysis demonstrates that for the latter form of public input, not only must government finance the appropriate level of G, i.e., meet (27) and impose a tax cum subsidy scheme on private output to handle cross-industry congestion, but must also either ration the use of G amongst firms or charge firms for the use of G in accordance with its marginal value at its first-best level. This last condition (charging for G) may be difficult to implement in many cases, e.g., a multi-access highway or city streets, since exclusion may be either impossible or highly costly. Without such rationing, rent-seeking factor allocation occurs, and the economy fails to achieve the first-best equilibrium even if lump-sum taxation

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[8] Yukutake (1995), also using a model with a semi-public input, appears to be aware of this problem and assumes complete complementarity between the semi-public input and one of the two primary inputs.

[9] The analysis may hold for other specifications of the production function but there remains to be explored.

is possible. Other rationing mechanisms, such as factor taxes or output taxes, are needed to halt the inefficiency arising from rent dissipation.

5. Financing Public Inputs

In this section, we consider the financing of both types of public goods in situations where the authorities do not have access to lump-sum taxation, and where labour and capital are no longer assumed in fixed supply, so factor taxation ceases to be equivalent to lump-sum taxation. Additionally, profit taxation is ruled out since it raises no revenue: (i) in the case of a congestible public input, in the absence of quotas on use, economic profit is zero in equilibrium, and (ii) in the case of pure public inputs, there are CRS in the primary inputs so factor payments exhaust revenue, leaving zero economic profit in equilibrium.

To facilitate the analyses, the section uses a one-good economy. This avoids the complications of cross-industry effects for a congestible public input, and, in the cases of both types of public inputs, facilitates the exposition in a model with variable factor supplies.

5.1 Congestible Public Inputs

Consider first the case of where the public input enters production functions that are CRS in all inputs. As discussed earlier, without appropriate charges for this input, it takes on the characteristics of a common property resource. To explore the implications for public financing, this section incorporates that common-property feature into a standard model, one similar to that employed by Keen and Marchand (1997).

5.1.1 The Model

Consider a one-good, two-factor, public-input economy in which there is a representative household. We assume a one-good economy for the convenience of abstracting from the inter-industry congestion effects that would otherwise occur.

The representative household consumes the private good subject to the following budget constraint:

$$C = (w-t)E + \rho K^{*} \tag{42}$$

where C denotes consumption of the private good, t is an employment tax, E is labour supply, K^{*} is the quantity of domestically owned capital which is assumed fixed but internationally mobile, and ρ is the after-tax return on capital that can be realized on world markets. Thus, $r = \rho + \tau$ is the gross return on capital in this economy, where τ is the domestic tax on capital. Note that the price of the private good has been normalized to unity.

The household maximizes utility over a function U(C, E) so, in light of (42), $C = C(w-t, \rho K^{*})$ and $E = E(w-t, \rho K^{*})$. Thus, the indirect utility function is given by

$$V = V(w-t, \rho K^{*}). \tag{43}$$

The instruments available to finance the public input are the capital tax and the employment tax. Assume that the cost of the public input, relative to the private good, is constant, and equal to q. Then the public sector budget constraint is:

$$tE + \tau K = qG \tag{44}$$

where K is the entire amount of capital within the economy, whether domestically owned or not. The supply of capital is perfectly elastic at the world net return.

Industry output of the private good, is given by F(L,K,G) where, reflecting Euler's theorem:

$$F(L,K,G) = F_L(K,L,G)L + F_K(K,L,G)K + F_G(K,L,G)G. \tag{45}$$

For future reference, note the partial derivatives of F(L,K,G) with respect to L, K, and G, can therefore be expressed, respectively, as

$$F_L = F_{LL}L + F_{KL}K + F_{GL}G + F_L \tag{46A}$$

$$F_K = F_{LK}L + F_{KK}K + F_{GK}G + F_K \tag{46B}$$

and

$$F_G = F_{LG}L + F_{KG}K + F_{GG}G + F_G \tag{46C}$$

so

$$F_{LL}L+F_{KL}K+F_{GL}G = F_{LK}L+F_{KK}K+F_{GK}G = F_{LG}L+F_{KG}K+F_{GG}G = 0. \tag{47}$$

Next, turn to the hiring decisions of firms. As discussed earlier, if the authorities do not charge for G according to its marginal value then rents are open to dissipation and there will be excessive use of the primary factors and inefficient factor proportions. In this one-good case, conditions given in (40) become:

$$F_L + SR/L = w \tag{48}$$

and

$$F_K + (1\text{-}S)R/K = r = \rho + \tau \tag{49}$$

where $R = F_G G$ is the rent generated by the public input.[10] It follows from these hiring conditions that the demand for labour and capital are functions of w, r and G; so K = K(w,r,G) and L(w,r,G). G is a policy variable, and r is determined by the world market and the domestic tax on capital. Therefore, a wage-determination mechanism is all that is necessary to complete the model. It is given as:

$$E(w\text{-}t,\ \rho K^*) = L(w,r,G). \tag{50}$$

5.1.2 Factor Demands.

As a preliminary to the policy analysis, it is necessary to elaborate on the nature of factor demands, K(w,r,G) and L(w,r,G). More specifically, the partial derivatives of each are needed, namely, L_w, L_r, L_G, K_w, K_r, and K_G. To obtain these, the hiring conditions, (48) and (49), must be totally differentiated with respect to w, r and G. That derivation is completed in the Appendix A, which shows that, using the following notation:

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[10] If the authorities charged p for each unit of G then the rent available for dissipation would be $R = (F_G\text{-}p)G$. If p is set equal to the marginal product of G, rent dissipation would not arise. The hiring conditions would be the standard marginal ones, and the first-best outcome could be achieved by financing G to the level at which p = q. If G were rationed according to a quota system then rents would accrue to firms but that would enable the authorities to apply a profits tax that actually yielded positive revenues, in which case a first-best outcome could again be realized.

$$A = -(1-S)F_{LL}L/K + SF_{KL} - S_L F_G G/K > 0;$$

$$B = (1-S)F_{LK} - SF_{KK}K/L - S_L F_G G/K > 0$$

$$J = F_{GK}K + (1-S)F_{GG}G - SF_G.$$

and

$$D = (F_G G/LK)[SB + (1-S)A + S(1-S)F_G G/LK] > 0,$$

the partial derivatives can be expressed as:

$$L_w = [(-L/K)B - (1-S)F_G G/K^2]/D < 0 \quad (51)$$

$$L_r = -B/D < 0 \quad (52)$$

$$K_w = -A/D < 0 \quad (53)$$

$$K_r = [(-K/L)A - SF_G G/L^2]/D < 0 \quad (54)$$

$$L_G = (F_G/LK^2)[BLK - (1-S)GJ]/D \quad (55)$$

and

$$K_G = (F_G/KL^2)[ALK + SGJ + SF_G G]/D. \quad (56)$$

Two observations are in order. First, L_w, L_r, K_w and K_r are all negative. Given the problem of the commons this makes sense. There is excessive factor allocation; an increase in a factor price tends to reduce the rents available for dissipation.

Secondly, note that if $J = -\ \ (F_L + SF_G G/L)/\ \ G$ is positive (negative) then K_G (L_G) is unambiguously positive (positive) but the sign of L_G (K_G) is ambiguous. Any increase in G, creates, initially, more rent and this will attract more factors to the industry, so at least one factor increases.

5.1.3 Policy Analysis

The policy problem faced by the authorities may now be represented by the following Lagrangean:

$$\Gamma = V(w-t, \rho K^*) + \alpha[tL(w,r,G) + \tau K(w,r,G) - qG] + \beta[L(w,r,G) - E(w-t, \rho K^*)] \quad (57)$$

where α is the multiplier on the budget constraint and β is the one on the labour-market clearing condition.

The first-order conditions corresponding to G, t, τ and w are, respectively:

$$(\alpha t + \beta)L_G + \alpha(\tau K_G - q) = 0 \quad (58)$$

$$-V_w + \alpha L + \beta E_w = 0 \quad (59)$$

$$(\alpha t + \beta)L_r + \alpha(\tau K_r + K) = 0 \quad (60)$$

and

$$V_w + (\alpha t + \beta)L_w + \alpha\tau K_w - \beta E_w = 0. \quad (61)$$

Consider, first, the implications for the taxation of capital. To do this, substitute (59) into (61) and then ratio the resulting expression to the expression in (60) to obtain:

$$(\tau K_r + K)(L_w/L_r) = (\tau K_w + L). \tag{62}$$

Using the results of the previous subsection, this expression simplifies, as shown in Appendix B, to yield the following rule for the taxation of capital:

$$\tau = (1\text{-}S)F_G G/K. \tag{63}$$

The next step in this policy analysis is to establish the optimal spending rule. To do so, use (60) to obtain an expression for $(\alpha t + \beta)$ and substitute it into (58). Then using the expressions for L_G, K_G, L_r and K_r, and with some simplification, as in Appendix B, the result is:

$$F_G = q, \tag{64}$$

which is the Kaizuka efficiency condition for a one-good economy.

In light of (64), (63) and the budget constraint, it immediately follows that the tax on labour has the following form:

$$t = SF_G G/L. \tag{65}$$

These three key results – the labour tax, the capital tax, and the spending condition - indicate that the first-best optimum is achievable if factor taxes are available. The factor taxes serve dual purposes, raising revenue to finance the public spending, and correcting the distortion that otherwise occurs as a result of making the public services available to industry without charge. In short, adopting factor taxes in the correct proportions is a perfect substitute for charging directly for the use of the public input.[11]

Two remarks are now in order. First, the conditions given by (63), (64) and (65) not only are efficient but also are consistent with the equity criterion of the benefit principle. Secondly, the taxation of capital, despite complete mobility of capital, is required. This is in contrast to the conclusion of Keen and Marchand (1997) which suggests that taxing interjurisdictionally mobile capital to help finance this type of public input is inefficient. However, a zero-capital-tax prescription fails to recognize the "commons" problem: free access to public services leads to excessive use of capital as well as labour so a capital tax is needed to stem an inefficiently high inflow of capital.

5.2 Pure Public Inputs

Pure public inputs are said to be factor-augmenting or, in the terminology of Meade (1952), atmospheric. In these circumstances, there are constant returns to scale in primary inputs and therefore, as discussed in section 4.1, there is no rent accruing to firms. Consequently, the problems of the commons or free-access do not arise. That makes the analysis of financing far more straightforward. Indeed, much progress has been made in the literature regarding the financing of pure public inputs. Therefore, this section addresses that issue only briefly and largely by reporting some key results of that literature which allows comparison with the results derived above.

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[11] The factor taxes are of course benefit taxes; each factor being taxed according to the share of the rent attributable to it.

5.2.1 Factor Taxation

Under the assumption of variable factors, Feehan and Matsumoto (2002) demonstrates that the appropriate policy for provision of a pure public input is to follow the first-best spending rule, as in (64), and tax each factor according to the standard optimal-tax conditions (i.e., the inverse elasticity rule). In the setting considered earlier in which capital was completely mobile, the tax rate on capital would be zero and labour would bear the entire financing burden. Also, unlike the congestible public-input case, adhering to these conditions leads to a second-best outcome. Factor taxes are now distortionary; they do not serve a Pigovian function since there is constant return to scale in the primary inputs, which means that excessive use of factors cannot capture any rent.[12] Thus, even though the first-best rule for spending remains applicable, that rule does not in general lead to same level of G as if lump-sum taxation were available.[13]

In short, factor taxation no longer supports the first-best outcome and the structure of those taxes do not relate to relative shares of rent as is the case with a congestible public input.

5.2.2 Direct Charges

An efficient policy for financing a congestible public input is to charge for its use in accordance with its marginal product. If such pricing is not possible or is costly to implement, then, as demonstrated above, factor taxes, as in (63) and (65), can be configured to have the equivalent effect and support the efficient outcome.

In the case of a pure public input, the use of direct charges is no longer desirable. A direct fee does not serve as a rationing device nor does it apply to rents; therefore it must cause some distortion. Indeed, Manning, Markusen and McMillan (1985) reached the rather startling and extreme conclusion regarding the extent of that distortion; they argue that a practice of Lindahl pricing for a pure public input in a competitive economy implies a zero-output general equilibrium. The basic element of the zero-output proposition is that, by Euler's Theorem, payments to the primary factors exhaust revenue when there are constant returns to scale in those inputs. Therefore, imposing fees on firms causes losses.

Whether firms actually realize these losses as in Manning et al. (1985) or whether, alternatively, the fees are absorbed by factors depends on whether the fees vary in accordance with firms' output or are fixed. If the fees do vary in such a manner then the hypothesis of Manning et al. does not appear to be valid because the fee becomes equivalent to a sales or output tax.[14] On the other hand, if the fees are designed to share the costs of the public input among all firms in some exogenous fashion then, as Manning et al. (1985) maintain, firms cannot avoid losses except by ceasing to exist. While Manning et al. (1985) stop at their zero-output proposition, the ultimate outcome for the economy is likely to be a monopolization of industry. As firms make losses they exit the industry, so eventually only one firm is left in

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[12] The rent in this case is analogous to the gains arising from technological change rather than from free access to a third factor of production.

[13] If factors are in perfectly inelastic supply then the analysis is more straightforward. Manning, Markusen and McMillan (1985) derive the structure of a uniform factor-income tax that supports the first-best outcome. Feehan and Matsumoto (2000) obtain the factor tax rates if policy is based on the benefit-taxation principle.

[14] Feehan and Matsumoto (2002) deal with the use of sales and factor taxes to finance a pure public good.

each industry.[15] Then, through the use of monopoly power any fee for the public input can be passed on to the consumer.

In summary, charging a fee for an atmospheric public input is inefficient in general and may even lead to the monopolization of industry. This is in sharp contrast to the case of congestible public inputs where a fee regime, or its equivalent, is needed in order to achieve the efficient outcome.

6. Conclusion

Whether a public input is pure or not is largely an empirical question but it does seem reasonable, given our current state of knowledge, to expect that some public inputs are of one type and some are of the other. Therefore it is important that empirical investigation establish an accurate mapping.

This paper has demonstrated that the scope to achieve an efficient outcome and the appropriate tax regime are considerably different depending on which category a public input belongs. In particular, public services and infrastructure that are supportive of industry in the atmospheric sense, e.g., marine weather forecasts, and agricultural research laboratories, should be financed by general revenues. Congestible services and infrastructure, on the other hand, should be financed by pricing according to marginal benefits to the industries, or equivalently, by benefit taxation applied to the employment of primary factors across industries. Indeed, in the absence of such financing, it would be difficult to assess whether congested services to industries means inefficiently low levels of such services.

[15] For instance, suppose that a certain level of the public input costs $10,000 and that there are 200 firms. If each firm must pay a $50 then its average cost curve becomes downward sloping. This gives rise to natural monopoly.

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APPENDIX A

Start with

$$F_L + SR/L = w \qquad \text{(A1)}$$

and

$$F_K + (1\text{-}S)R/K = r. \qquad \text{(A2)}$$

Totally differentiate (A1) and (A2) with to w, r and G. The results are:

$$[F_{LL}+SF_{GL}/L+S_LF_GG/L-SF_GG/L^2]dL +[F_{LK}+SF_{GK}G/L+S_KF_GG/L]dK$$

$$= dw – [F_{GL}+SF_{GG}G/L + SF_G/L]dG \qquad \text{(A3)}$$

$$[F_{KL}+(1\text{-}S)F_{GL}G/K-S_LF_GG/K]dL +[F_{KK}+(1\text{-}S)F_{GK}G/K-S_KF_GG/K-(1\text{-}S)F_GG/K^2]dK$$

$$= dr – [F_{GK}+(1\text{-}S)F_{GG}G/K +(1\text{-}S)F_G/K]dG \qquad \text{(A4)}$$

Next, note that $S/ \ K = S_K = -S_L(L/K)$. Also, recall from (47) that

$$F_{LL}L+F_{KL}K+F_{GL}G = 0 \qquad \text{(A5)}$$

$$F_{LK}L+F_{KK}K+F_{GK}G = 0 \qquad \text{(A6)}$$

and

$$F_{LG}L+F_{KG}K+F_{GG}G = 0. \qquad \text{(A7)}$$

Substitute for S_K in (A3) and (A4). Use (A5) to substitute for $F_{GL}G$ on both sides of (A3) and use (A6) to substitute for $F_{GK}G$ on the left-hand-side of (A4). Those substitutions and some simplification allow (A3) and (A4) to be expressed as

$$[(-K/L)(A + SF_{GL}G/LK]dL + [B]dK = dw – [J/L]dG \qquad \text{(A8)}$$

$$[A]dL + [(-L/K)(B+(1\text{-}S)F_GG/LK]dK = dr – [J/K +F_G/K]dG \qquad \text{(A9)}$$

where $A = -(1\text{-}S)F_{LL}L/K + SF_{KL} – S_LF_GG/K > 0$

$B = (1\text{-}S)F_{LK} - SF_{KK}K/L– S_LF_GG/K > 0$

$J = F_{GK}K + (1\text{-}S)F_{GG}G -SF_G.$

Next, the determinant of the matrix formed by the left-hand-sides of (A8) and (A9) is

$$D = (F_GG/LK)[SB + (1\text{-}S)A + S(1\text{-}S)F_GG/LK] > 0. \qquad \text{(A10)}$$

The expressions for L_w, L_r, L_G, K_w, K_r, and L_G as given in the text follow accordingly from Cramer’s rule applied to the two-equation system given by (A8) and (A9).

APPENDIX B

The Optimal Capital Tax:

Following from (62), one can write:

$$[-K(L_w/L_r) + L] + t\,[K_w - K_r(L_w/L_r)] = 0 \qquad \text{(B1)}$$

Next, substituting for (L_w/L_r) gives

$$[-(1-S)F_G G/BK] + t\,[K_w - K_r(L_w/L_r)] = 0 \qquad \text{(B2)}$$

or

$$[-(1-S)/K] + t\,(B/F_G G)[K_w - K_r(L_w/L_r)] = 0. \qquad \text{(B3)}$$

However, it can be shown that $B[K_w - K_r(L_w/L_r)] = 1$. Hence, the optimal tax on capital is simply:

$$t = (1-S)F_G G/K. \qquad \text{(B4)}$$

The Spending Condition

Using the first-order condition (60) to obtain an expression for (át + â) and substituting it into (58) gives

$$(K + t\,K_r)(L_G/L_r) - t\,K_G = -q. \qquad \text{(B5)}$$

Substituting for L_G/L_r yields

$$(K + t\,K_r)\{-1 + [(1-S)GJ/KL]/B\}(F_G/K) - t\,K_G = -q, \qquad \text{(B6)}$$

where, as in the text, $B=(1-S)F_{LK}-SF_{KK}K/L-S_L F_G G/K$ and $J = F_{GK}K+(1-S)F_{GG}G - SF_G$.

Re-write (B6) as

$$F_G\{-1+[(1-S)GJ/KL]/B\} + t\,F_G(K_r/K)\{-1+[(1-S)GJ/KL]/B\} - t\,K_G = -q. \qquad \text{(B7)}$$

Next, it can be shown that

$$t\,F_G(K_r/K)\{-1+[(1-S)GJ/KL]/B\} - t\,K_G = -t\,J/BL. \qquad \text{(B8)}$$

Substituting (B8) into (B7) then gives

$$F_G\{-1+[(1-S)GJ/KL]/B\} - t\,J/BL = -q. \qquad \text{(B9)}$$

Finally, using the expression in (B4) for t in (B9) yields the spending rule:

$$F_G = q. \qquad \text{(B10)}$$