EXCHANGE-RATE AND EXTERNAL-BALANCE CONSEQUENCES OF ASYMMETRIC DEMOGRAPHIC TRANSITIONS:
KEY DETERMINANTS OF CROSS-BORDER SUBSTITUTABILITY

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Abstract

This paper extends our earlier analysis of the open-economy effects of asymmetric declines in fertility. It highlights sensitivity tests of behavioral parameters that are key determinants of cross-border goods substitutability. One motivation for the paper was a concern that the earlier analysis might have overstated the extent of currency appreciation for a country with a larger fertility decline and thereby distorted inferences about cross-border transmission and welfare consequences. In the event, the basic qualitative conclusions in the earlier papers are not overturned. The findings do merit significant revision, however, in quantitative terms. The incorporation in the analysis of varieties effects, allowing for higher values of relative-price elasticities for imports, and raising the value of the elasticity of intertemporal substitution in consumption each work in the direction of reducing the extent of the medium-run currency appreciation for the country experiencing the larger fertility decline, significantly modifying its external-sector imbalances resulting from the demographic shocks, diminishing the favorable cushioning effects for its economy, and mitigating the adverse effects abroad where the fertility decline is smaller. The paper emphasizes that empirical estimates of the determinants of cross-border substitutability play critical roles in determining the welfare consequences of demographic shocks (and probably other shocks as well). For example, for a country experiencing a larger fertility decline, per-capita consumption is cushioned most strongly when the degree of cross-border substitutability is weak. But the opposite is true elsewhere: the rest of the world prefers an outcome where the degree of cross-border goods substitutability is high! Such dramatic divergences across countries in welfare inferences suggest that future research should assign a high priority to obtaining more robust estimates of the parameters conditioning cross-border substitutability.
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1. **Introduction**

The sensitivity of a home economy to shocks originating abroad and the extent to which home-originating shocks buffet foreign economies can be usefully summarized in terms of cross-border “substitutabilities.” Households and firms substitute between home-produced and foreign-produced goods in response to changes in their relative prices and to other determinants of cross-border trade flows (“goods substitutability”). Savers and investors respond across national borders or across currency denominations to changes in the relative expected returns among financial assets and liabilities (“financial substitutability”). Workers or firms may move across borders, temporarily or permanently, in response to economic incentives (immigration and emigration of people, or “outsourcing” of production facilities).

This paper highlights behavioral parameters that are key determinants of cross-border goods substitutability. The analysis focuses on demographic shocks – in particular declines in fertility rates (birth rates) that are asymmetric across nations.

Asymmetric demographic shocks generate powerful cross-border macroeconomic spillovers transmitted through changes in exchange rates and external-sector balances. For example, countries with larger and more rapid declines in fertility – hence with youth-dependency ratios that fall more rapidly than in other nations and, subsequently, elderly-dependency ratios that rise faster and hence cause faster aging of the population – experience a substantial appreciation of their currencies and strengthening of their net foreign asset positions. Such changes cushion the more rapidly aging economies from the full effects that larger demographic shocks would otherwise produce in a closed economy. The openness of the economy fosters, in effect, a partial sharing of the large demographic shock with the rest of the world and thereby mitigates the negative consequences of population aging on domestic output and consumption. Countries that age more slowly, on the other hand, may experience adverse effects as a result of openness because the larger demographic shocks abroad spill over into their economies, requiring them to absorb some of the burden of adjustment.1

The conclusions just summarized are qualitatively robust. However, the quantitative sizes of the cross-border spillovers – for example the extent to which asymmetric fertility declines induce very large or only moderate appreciations of the currency of a faster aging country – depend critically on the behavioral parameters determining the empirical degree of cross-border goods substitutability.

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1 Several earlier papers develop these themes: Bryant (2004b); Bryant (2004a); Bryant, Faruqee, Velculescu, and Arbatli (2004). See also Bryant and McKibbin (2004).
The analysis in this and earlier papers employs a simplified macroeconomic model dividing the world economy into two regions that are equal-sized, equivalently open, and identical in domestic structure. This framework, an abridgement of a larger empirical model containing many separate countries and regions, is a research environment that facilitates a systematic analysis of the cross-border macroeconomic consequences of demographic change. The simplifications of the framework limit its applicability. But the framework has a great overriding merit: it reveals, in a genuine general-equilibrium analysis, the endogenous determination of domestic and cross-border macroeconomic interactions.

The paper reviews the literature concerned with modeling cross-border trade flows, beginning with the standard “elasticities approach” which has up until now been the basis for the trade equations in our analytical model. The paper next examines recent alternative contributions to the literature that cast doubt on the traditional approach and suggests a method for incorporating the alternative formulations into our framework.

To highlight the sensitivity of empirical conclusions to alternative assumptions about cross-border substitutability, the remainder of the paper focuses on three key parameters in the revised analytical framework. The first is the sensitivity of import demand to the share of exporters’ economies in the world economy, interpreted as a reflection of “varieties” or “home-market” effects on trade flows; these effects are omitted from the traditional approach. The second is the elasticity of import demand with respect to relative prices (the price of imports relative to the price of home-produced goods). The third key parameter is the elasticity of intertemporal substitution in consumption, a behavioral characteristic of consumption that is not a direct reflection of cross-border goods substitutability but nonetheless has powerful effects on cross-border spillovers. We investigate these three dimensions of cross-border substitutability one at a time (in isolation from each other) and then finally in a combination of the three. A final summary section pulls together our main conclusions.

2. **Traditional Assumptions about Trade Flows in Macroeconomic Models**

The empirical modeling of goods trade across national borders traditionally follows an “elasticities approach.” The standard macroeconomic assumptions underlying this approach are that (i) domestically-produced tradable goods in a home economy are imperfect substitutes in demand for tradable goods produced abroad imported into the home economy, and (ii) the preferences underlying the imperfect substitutability between home- and foreign-produced goods are unchanged over time. Import demand equations are specified to depend on two key variables, some measure of
aggregate demand in the importing country and the price of imports relative to an index of the prices of domestically-produced goods. Most studies embody the assumption of unchanged underlying preferences by imposing the restriction that estimated values of the income and price elasticities of import demand remain constant through time. Armington (1969a, 1969b), building on earlier work, provided a clear exposition of the standard assumptions and how they may be applied to empirical models with many separate countries and numerous categories of goods.

The assumption of imperfect substitutability across national borders distinguishes between “goods” and “products.” Goods are identified only in one dimension, by the kinds of wants or needs they serve (automobiles, machine tools, wine). In contrast, products are identified in two dimensions, both by kind and by country of production (Japanese automobiles, U.S. automobiles, French wine, Australian wine). The contention that import demands are sensitive to the national origin of goods acquired increasing plausibility through time because international trade in homogenous commodities (with countries’ comparative advantages dominated by differences in resource endowments) declined in relative importance. Concurrently, manufactured goods similar in kind but differentiated in product detail by small differences in brands and functions accounted for a larger and larger proportion of cross-border trade.

Each of the two economies in our world model produces a single composite good. Because the composite goods are imperfect substitutes, each country exports some of its production to the other.

In keeping with the traditional elasticities approach to cross-border trade, the model prior to this paper assumed that imports were a function only of national aggregate activity and relative prices. The agents in each economy had identical and time-invariant preferences over foreign and domestic goods. The model thus focused on the substitutability margin between home-produced and foreign-produced tradable goods.2

More specifically, the underlying import-demand equations for each region in the model took the form:

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2 Because of the simplifying assumption that each country produces only a single composite good, the model cannot be used to examine the important substitutability margin between nontraded and tradable goods within national economies. Obstfeld and Rogoff (2000, 2004) do not exposit a full general-equilibrium model for production as well as consumption and trade flows, but their framework does allow for nontraded as well as tradable goods. They correctly emphasize that the substitutability between tradable and nontraded goods within countries has macroeconomic implications no less important than the substitutability between home-produced and foreign-produced tradable goods. For example, the higher is the “intranational” substitution between nontraded and tradable goods, other things being equal the smaller will be the effects of shocks on the real exchange rate and on other price variables.
\[ IM_t = K \left( IM_{t-1}^{\alpha} \right) \left( Y_t^{\beta_0} \right) \left( Y_{t-1}^{\beta_1} \right) \left( PR_t^{\gamma_0} \right) \left( PR_{t-1}^{\gamma_1} \right) U_t \]  

(1)

where \( IM \) is the volume of the region’s imports of the good produced in the other region; \( Y \) is a measure of aggregate real economic “activity” in the region; \( PR \) is an index of the relative price of the region’s imports (in practice, the ratio of the local-currency price of the imports, \( PIM \), to the region’s price deflator for gross domestic product, \( PGDP \)); \( U \) is a stochastic error term with mean unity; \( K \) is an equation constant term; and \( \alpha, \beta_0, \beta_1, \gamma_0, \gamma_1 \) are constant coefficients. In practice, the measure of aggregate real activity for each region was a weighted average of real domestic absorption expenditures (\( A \)) and real expenditures on goods produced in the region but exported to the other region (\( XM \)):

\[ Y_t = \left( A^\theta \right) \left( XM_t^{1-\theta} \right) \]  

(2)

with the weight on domestic absorption, \( \theta \), set at 0.87.3

Equation (1) was transformed in our model into a first-differenced log-linear form, often referred to as an equilibrium error-correction format:

\[ \Delta im_t = \Phi + \beta_0 \Delta y_t + \gamma_0 \Delta pr_t + (1 - \alpha_t) \left[ \frac{\beta_0 + \beta_1}{(1 - \alpha_t)} \Delta Y_{t-1} + \frac{(\gamma_0 + \gamma_1)}{(1 - \alpha_t)} \Delta PR_{t-1} - \Delta IM_{t-1} \right] + e_t \]  

(3)

The variables \( im, y, \) and \( pr \) are, respectively, the logarithms of \( IM, Y, \) and \( PR \); the additive disturbance term in (3) is \( e_t = \log(U_t) \); and the constant term, \( \Phi \), is \( \log(K) \). The expression in large square brackets in (3) is known as the error-correction deviation, with the coefficient \( (1 - \alpha_t) \) calibrating the speed of adjustment to the long-run equilibrium relationship.

The error-correction formulation of equation (3), with the coefficients either estimated or calibrated, yields constant values for both short-run (first period) and long-run elasticities with respect to the determinants of imports. The short-run elasticity of imports with respect to the importing region’s aggregate real activity is given by the coefficient \( \beta_0 \). The short-run elasticity of import demand with respect to relative prices is \( \gamma_0 \). The long-run elasticity of imports with respect to activity – labeled here \( \eta_y \) – is given by the composite coefficient \( \eta_y = \left( \frac{\beta_0 + \beta_1}{1 - \alpha_t} \right) \). The long-run

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3 The conclusions emphasized in this paper do not appear to be sensitively dependent on this weighted-average method of defining the importing region’s aggregate real activity. The paper therefore does not further discuss this aspect of the import equations in what follows.
relative-price elasticity is $\eta_{pr} = \left( \frac{\gamma_0 + \gamma_1}{1 - \alpha_t} \right)$.

Our two-region framework has so far not addressed the issue of “pricing to market” (exporters using price discrimination in how they invoice and set local-currency prices for their exports in various foreign markets). Nor have we focused on the issue of exchange-rate “pass-through” (how quickly or slowly exchange-rate changes get incorporated into the local-currency prices of imports). Rather, our simplified specification of import prices assumes no pricing-to-market behavior by exporters and it assumes an immediate (unity) pass-through of exchange-rate changes into the local-currency price of imports:

$$PIM_t = \mathbb{N}_t PXM^*_t$$  \hspace{1cm} (4)

where $PIM$ is the local-currency price of imports in the home region, $PXM^*$ is the price abroad of the exports from the foreign region measured in foreign currency (identical to the price for domestic sales abroad), and $\mathbb{N}$ is the nominal exchange rate (the foreign-currency price of a unit of the home region’s currency).

It would be wrong to ignore pricing-to-market behavior and incomplete pass-through of exchange-rate changes if one’s objective were accurately to analyze short-run dynamics. But our modeling framework focuses on medium and long-run horizons. So far, therefore, we have deemphasized or bypassed altogether issues that are predominantly short-run. Pass-through and pricing-to-market issues are probably of secondary importance for the longer run. Furthermore, as emphasized by Obstfeld and Rogoff (2000a, especially section 2), it is implausible – as some studies have done -- to assume extreme versions of local-currency pricing (in other words, that exporters actually invoice their exports to a home economy in the local currency and the local-currency prices are kept rigid). Campa and Goldberg (2002) show that over the long-run and across OECD countries, producer-currency pricing (PCP) is more prevalent than local-currency pricing (LCP) for many types of imported goods. They also report that “as a cross-country average, import prices in local currencies reflect 60 percent of exchange rate fluctuations in the short run, and nearly 80 percent over the long-run” (p. 23).  

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4 Obstfeld-Rogoff (2000a, p.127) write that there is a “prima facie case that the expenditure-switching effect of exchange rate changes is alive and well among industrial economies, and should be a central feature of open economy models. An argument to the contrary would have to contend that measured terms of trade are somehow irrelevant for the allocation of worldwide demand among countries.”

In the research reported in our prior papers on the global consequences of demographic change, we used values for the traditional trade-flow elasticities in equation (3) that had been estimated for the United States by the IMF staff. The value for each of two region’s long-run relative price elasticity was a bit greater than negative unity (\( \eta_{pr} = -1.097 \)). This estimate tended toward the high side of the range of conventional estimates; many estimates in the empirical literature of the relative-price elasticity have been in the range -0.5 to -1.0. Following the IMF staff and our own needs for a model with sensible long-run steady-state properties, we imposed the restriction that the long-run activity elasticity was unity (\( \eta_y = 1 \)). Without that restriction, empirical estimates tend to produce a value of \( \eta_y \) significantly greater than unity; values greater than unity would entail a permanently faster rise in imports (forever!) than in outputs (for both of the regions in our model).

3. Challenges to the Traditional Assumptions about Trade Flows

The traditional elasticities approach to modeling cross-border trade flows has been widely used. Its appeal stems in part from its simplified focus on the aggregate incomes (outputs) of national economies and on cross-border relative prices, widely believed to be the primary determinants of trade flows. The underlying assumptions of the approach and some of its main implications, however, have been challenged. The empirical estimates of the activity and relative-price elasticities obtained with the approach, moreover, are regarded as implausible by some analysts.

The implication of the traditional approach of greatest concern for this paper is its effects on exchange rates and external-sector balances generated by the assumption of unchanged goods preferences through time. The problem can be seen in its starkest form by imagining that every country in the world has identical elasticities of import demand with respect to home economic

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Gagnon (2003), and Frankel, Parsley, and Wei (2004).

6 As explained further below, the two regions in our modeling framework are based on empirical equations for the United States submodel in the MULTIMOD model of the IMF’s staff. For the trade equations and estimates of the trade elasticities, see Masson, Meredith, and Symansky (1990, pp. 6-11).

7 The restriction of unity for the long-run activity elasticity in turn requires that \( \beta_0 + \beta_1 = (1 - \alpha_i) \). In the simulations for this paper, we use a value of 0.75 for \( 1 - \alpha_i \) (so that \( \alpha_i = 0.25 \)).

8 Bayoumi (1999) writes “Numerous standard trade equations relating the volume of exports or imports to relative prices and to levels of activity have been estimated over the last 20 years or so with notable empirical success, so much so that they are now an accepted part of most policy and applied academic work in international economics.”
activity and to the relative price of imports. Consider a home country that grows faster than the rest of the world and hence that accounts for a larger share of world output over time. Given the traditional assumptions, that country will experience, other things equal, a faster increase in its imports than for its exports. The incipient imbalance in its trade with the rest of the world will then have to be associated with a real depreciation of its currency. With unchanged preferences at home and abroad for the imperfectly substitutable home-produced and foreign-produced goods, such depreciation is required to induce foreign and home residents to buy the now relatively less scarce output of the rapidly growing economy and to prevent its actual trade deficit from growing larger and larger. The opposite conclusion, exports growing faster than imports and hence a real appreciation of the currency, would apply to a home country growing more slowly than the rest of the world.\footnote{This analytical point was emphasized many years ago by Harry Johnson (1958) and others. More recently it has been stressed in contributions to international trade theory – for example Helpman and Krugman (1985) – that focus on monopolistic competition, increasing returns to scale, and differentiated goods in international trade. See below for further discussion.}

Essentially the same analytical problem is evident in our earlier papers on asymmetric demographic transitions in which we employed the traditional approach with its assumption of unchanged goods preferences. The region in our model with a larger and faster fertility decline experienced a real appreciation of its currency over the medium and longer runs. The underlying explanation for this real-exchange-rate effect is the slower rate of growth, and hence relative scarcity of the output, of the economy with the larger demographic shock.

Note that the traditional approach with its assumption of unchanged goods preferences tends to require continuing changes in real exchange rates even when countries have differing activity and relative-price elasticities of import demand (and would do so even if analysts had “correct” estimates of the income and relative-price elasticities for countries). Houthakker and Magee (1969) stimulated an entire literature about the sustainability of a U.S. current-account deficit after they obtained estimates of the activity elasticity of U.S. import demand an order of magnitude larger than the activity elasticity of foreigners’ demand for U.S. exports. Given an asymmetry in the estimates of U.S. activity elasticities, a result that has persisted in many empirical studies, the implication is that the U.S. trade and current-account deficits will widen indefinitely or, alternatively, that either relative prices (and the exchange value of the U.S. dollar) or U.S. growth relative to growth in foreign nations will have to persistently adjust through time to inhibit this widening.

The exchange-rate conclusions stemming from the traditional approach to trade equations...
are problematic. Rapidly growing economies in the world do not appear to have required a real depreciation in their currencies to induce the rest of the world to absorb part of their faster-growing output. Nor have slower growing economies invariably experienced real appreciations of their currencies. Rather, there seems in practice to have been a strong positive relationship between the elasticities in export equations with respect to foreign economic activity and the rate of growth of the exporters’ output. High output growth rates for an exporter country are highly correlated with high rates of export expansion.\textsuperscript{10} Thus one can legitimately question the implication of the traditional specification of trade equations that the substitutability in demand between domestic and foreign goods is unchanged through time, and in particular is independent of large changes in the relative sizes of economies.

The traditional approach raises a second set of problems. The activity and relative-price elasticities estimated in empirical research often have puzzling, if not outright implausible, values. Estimated relative-price elasticities for aggregated trade flows (a country’s total imports and exports) often have surprisingly “low” values (for example, markedly smaller than negative unity). Estimated activity elasticities from the standard specifications have “high” values (sometimes as high as 2 or 3) and may be biased upwards. Cross-border trade flows in the world economy have grown much faster than national outputs. As has been recognized for several decades, the standard approach to estimating relative-price and activity elasticities has difficulty in adequately accounting for this faster observed growth in cross-border trade.

Some puzzling features of estimated elasticities are likely due to mis-measurement of the data for prices and quantities of international trade. The official data for import types and import prices, for example, fail to incorporate the increased variety of new products and the effects of the new varieties on indexes of import prices. Measured import prices have often fallen less than “true” import prices, for example, so that the measured decline in the relative price of imports is too small.\textsuperscript{11} Another problem may be the failure of measured import prices adequately to reflect

\textsuperscript{10} Krugman (1989) emphasized this point about the presence or absence of long-run changes in real exchange rates and coined the phrase “45-degree rule” to summarize the correlation between high rates of export expansion and high rates of output growth.

\textsuperscript{11} See, for example, Helkie and Hooper (1988), Feenstra (1994), Feenstra and Shiells (1994), Feenstra and Kee (2004a, 2004b), and Broda and Weinstein (2004a, 2004b). Broda and Weinstein (2004a, p.144) report that “our calculations suggest a pervasive and potentially large upward bias in import prices, particularly for liberalizing countries.” “Over the last thirty years, if one adjusts for new varieties, import prices [for aggregate U.S. imports] have been falling 1.2 percent per year faster than one would surmise from official statistics. In aggregate terms this means that the aggregate price index that takes variety changes into account has fallen by 28.1 percent relative to the conventionally measured import price index” (Broda and Weinstein, 2004b, pp. 2-3).
declines in the relative costs of transportation and communication. If import prices were measured correctly, estimates of the values of the relative-price elasticities of imports would be increased. Because correctly-measured import prices would give prices a larger role to play in explaining the variation in import volumes, they would also correspondingly diminish the role of economic activity in explaining that variation (in other words, would reduce the estimated values of activity elasticities). With more accurate price data, therefore, the standard approach would yield more accurate estimates of activity elasticities and do a better job in explaining the upward trend in the ratios of imports to GDP.

Possible problems with the standard estimates of activity elasticities have been extensively discussed. In addition to the problem of general upward bias in income elasticities, attention has been focused on apparent differences in income elasticities across countries. One result of the numerous papers sparked by the Houthakker and Magee (1969) finding of asymmetric activity elasticities for U.S. imports and U.S. exports has been efforts to improve the data for trade prices and to refine the econometric techniques with which activity elasticities are estimated.

4. Supplementing the Traditional Approach to Goods Preferences and Trade Flows

The problems with the traditional approach to goods preferences and trade flows create a strong presumption that empirical analysis should consider amendments to the traditional specifications of trade equations. This section provides some further background and suggests a way, within the limitations of our analytical framework, to supplement the traditional approach.

A. Introducing Varieties Effects. Early development of theory emphasized differences across countries in factor endowments as the underlying explanation for cross-border trade in goods. However, as an ever rising proportion of trade took the form of exchanges of manufactured goods and services differentiated in product detail by differences in styles and brands and, often, by only small differences in function, trade theorists sought to modify the emphasis on differences in factor endowments – or even to substitute an altogether different explanation. The increasing prominence

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12 Anderson and van Wincoop (2004)


14 For surveys and analysis of alternative estimates of activity and price elasticities, see among others Goldstein and Khan (1985), Helmke and Hooper (1988); Hooper, Johnson, and Marquez (2000); Marquez (1999, 2002); Caporale and Chui (1999); and Mann (2004).
of “intraindustry trade” in differentiated manufactured goods stimulated even further the interest in developing an alternative theoretical perspective.

The primary modification was to base theoretical models on monopolistic competition and increasing returns to scale in production. Theorists used the differentiated-products and love-for-“variety” framework developed by Spence (1976) and Dixit and Stiglitz (1977), for example, to develop models in which monopolistically competitive firms produce slightly differentiated final goods under increasing returns to scale. In such models, the numbers of goods produced differ across countries of differing economic size and depends endogenously on the taste for variety and fixed costs in production.\(^{15}\) In the older trade theory, the gains from trade arise from conventional adjustments in relative prices. In the newer theories, gains from trade can occur merely from changes in the number of goods (“varieties”). Ethier (1982) and Helpmann and Krugman (1985) provide overviews of the modified theory.\(^{16}\)

The newer theory accords a significant role to the relative sizes of countries. If all trade were in homogenous products, relative country size would not be an independent influence on the volume of trade after country differences in factor endowments were taken into account. The existence of trade in differentiated products driven by consumers’ tastes for variety and innovations that alter the composition of output, however, introduces a direct link between the volume of trade and relative country size. The larger the output of a country, the greater will be the number of varieties of goods produced. Feenstra’s textbook (2003. p. 165) drawing on Krugman (1980) puts the “theorem” as follows:

> With two countries trading, the larger market will produce a greater number of products and be a net exporter of the differentiated good. This result is known as the ‘*home market effect,*’ because it shows that a larger home market will attract disproportionately more firms, and therefore become a net exporter. This is quite different from what we expect in a model where the number of products is constant; in that case, a larger market would have larger demand, and would therefore become a *net importer* of the good in question.

An implication of the linkage between relative country size and consumers’ desire for varieties is that a faster growing country can experience relatively rapid growth in its exports and less rapid growth in its imports than would otherwise occur. Hence, as suggested earlier, the country

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\(^{15}\) For example, Krugman (1979, 1980).

\(^{16}\) Davis (1997) and Davis and Weinstein (2001) discuss the degree to which factor endowments play an important role in trade among developed economies notwithstanding the emphasis in recent theory and empirical research on differentiated products produced under monopolistic competition. See also Schott (2003).
may be able to experience relatively faster growth without having to experience the real depreciation in the value of its currency that would be predicted by the standard elasticities approach.\textsuperscript{17}

The relatively recent emphasis in empirical research on “varieties” as an explanation for trade flows is discussed in more detail in Gagnon (2003) and Broda and Weinstein (2004b). Gagnon (2003, 2004) contributes new empirical estimates. Bayoumi (1999) includes a “varieties” dimension in his re-estimates of country-pair bilateral trade equations.\textsuperscript{18}

Because our analytical model contains no disaggregation by types of good within regions, there is obviously no direct way in the model to address the issues raised in real life by multiple varieties of goods and products. Arguably, however, a presumption exists for modifying the specifications of our import equations for the regions so that, indirectly, the model acknowledges the likely importance of varieties or home-market effects. An indirect acknowledgment entails including some proxy in a region’s import equation that is correlated with the relative diversity of goods produced abroad and at home. Given that this diversity in real life seems linked to the relative productive capacities of the home and foreign economies, a likely proxy is some measure of relative productive capacities.

The preceding line of reasoning suggests that our import equations might be amended to include a variable measuring the productive capacity of the foreign region as a proportion of “world” productive capacity, with capacities in both regions (and hence the world) measured in a common real currency unit. We therefore define a variable

\[
\text{FORGDPSHARE}_t = \frac{GDP^*_t/\mathcal{R}_t}{GDP^*_t + GDP^*_t/\mathcal{R}_t}
\]

(5)

where \(GDP^*_t\) is current-period capacity output in the home region, \(GDP^*_t\) is current-period capacity output in the foreign region, and \(\mathcal{R}_t\) is the current-period value of the real exchange rate.\textsuperscript{19} The real exchange rate is defined as the nominal exchange rate multiplied by the ratio of the price levels

\textsuperscript{17}This line of reasoning is the origin of Krugman’s hypothesis (1989) – the “45-degree rule” – that product differentiation and scale economies permit countries to grow rapidly by producing new goods that can be exported without causing an adverse movement in the terms of trade.

\textsuperscript{18}Other recent papers focusing on the “varieties” or “home-market-effect” dimensions of trade flows include Baier and Bergstrand (2001); Head and Ries (2001); Broda and Weinstein (2004a); Feenstra and Kee (2004a, 2004b); Bergoeing et al (2004); Eaton, Kortum et al (2004); and Hummels and Klenow (2004).

\textsuperscript{19}Except in the short run, capacity (“potential”) outputs in our model and actual outputs are the same; for long-run purposes, therefore, potential and actual GDPs do not differ.
(GDP deflators) in the two regions:

\[
R_t = \frac{PGDP_t}{PGDP_t^r} \frac{PGDP_t}{PGDP_t^r} \frac{PGDP_t}{PGDP_t^r} \frac{PGDP_t}{PGDP_t^r} . \tag{6}
\]

To modify the equation specification for import demand, equation (1) above is expanded to include the new \( \text{FORGDPSHARE} \) variable:

\[
IM_t = K(\left( IM_{t-1}^r \right) \left( Y_{t-1}^r \right) \left( PR_{t-1}^r \right) \left( PR_{t-1}^r \right) \left( FORGDPSHARE_{t-1}^r \right) \left( FORGDPSHARE_{t-1}^r \right) \left( U_t \right) . \tag{7}
\]

Equation (1) is expanded, again in an error-correction formulation, to read:

\[
\Delta im_t = \Phi + \beta_0 \Delta y_t + \gamma_0 \Delta pr_t + \delta_0 \Delta forGDPshare_t
+ (1 - \alpha_t) \left[ \left( \delta_0 + \delta_1 \right) \left( \frac{\gamma_0 + \gamma_1}{1 - \alpha_t} \right) pr_{t-1} + \frac{\left( \delta_0 + \delta_1 \right)}{1 - \alpha_t} \left( forGDPshare_{t-1} - im_{t-1} \right) \right] + e_t. \tag{8}
\]

As in equation (3), lower-case variables in equation (8) are the logarithms of the corresponding level variables (e.g., \( \text{forGDPshare}_t = \log(\text{FORGDPSHARE}_t) \)). The coefficient \( \delta_0 \) may be interpreted as a short-run elasticity of import demand to the foreign-GDP-share variable; the long-run elasticity to the new variable is \( \eta_{\text{forGDPshare}} = \left( \frac{\delta_0 + \delta_1}{1 - \alpha_t} \right) \). The changes in imports associated with varieties/home-market effects seem likely to occur gradually over the longer run, with very small or even zero short-term elasticity. In the simulation experiments of a varieties effect reported below, the coefficient \( \delta_0 \) is accordingly set to zero and simulations are run with alternative values of the long-run elasticity \( \eta_{\text{forGDPshare}} \) (alternative non-zero values of \( \delta_t \)).

\[\text{B. Rethinking the Significance of Relative-Price Elasticities.}\] Of all the problems and puzzles associated with cross-border trade flows and how they interact with domestic macroeconomic outcomes, probably the most consequential is the degree of relative-price substitutability across borders. Macroeconomic outcomes in a world in which relative-price elasticities for trade are low – say, with values less than -1.0 and perhaps as small as -0.5 or -0.6 – are immensely different from outcomes in a world in which such elasticities are much larger – for example, as high as -1.5, -2.0,

20 Obstfeld and Rogoff (2004) mention the issue of a varieties effect (“the possibility of change in the range of goods produced and exported by the United States”) but they suggest that such an effect “realistically is absent in the short run”; “over the longer run it might soften the terms of trade effects of various economic disturbances.”

21 Orcutt (1950) long ago pointed out difficulties in obtaining reliable estimates of price elasticities for trade flows.
or even higher. In a high-substitutability world, the occurrence of demographic and policy shocks
do not require large changes in price variables (including interest rates and exchange rates) as part
of the adjustments of quantities to the shocks. Conversely, if price substitutabilities are quite low,
the adjustments of quantities required after the occurrence of shocks must correspondingly generate
large rather than modest changes in price variables to restore equilibrium.

Econometric estimates of the relative-price sensitivity of aggregate imports, as noted earlier,
typically generate low absolute values of the elasticity, often below unity. The estimated inelastic
price responses, however, have attracted less critical attention than the high and biased-upwards
estimates of activity elasticities. Yet there is probably as much of a basis for claiming that price
elasticities are biased downwards as for the assertion that activity elasticities are biased upwards.
All things considered, the literature has overemphasized the problems with estimates of activity
elasticities and paid too little attention to the bias in price elasticities.22

As already discussed, theorists have emphasized the link between country size and numbers
of varieties as a justification for including a measure of relative productive capacities in empirical
trade equations. A complementary justification appeals to the problem that measured import prices
do not adequately capture changes in the variety of goods and the prices of the new varieties. As
suggested by Feenstra (1994) and Feenstra and Shiells (1994), a “correct” price index for imports
incorporating new varieties can be thought of as the product of the measured price index and a time-
varying bias term:

\[
\frac{p_m,_{\text{correct}}}{p_m,_{\text{measured}}} = \left( \frac{\Lambda_t}{\Lambda_{t-1}} \right)^{\sigma - 1}, \quad 0 < \Lambda \leq 1, \quad \sigma > 1
\]

where \( \Lambda \) represents the share of recorded products in imports relative to the total of imported
products (those recorded properly plus those that are not) and \( \sigma \) is some measure of the elasticity of
substitution among products. The higher the proportion of products whose prices are not recorded,
of course, the greater the degree of mis-measurement. Mis-measurement of import prices in a home
nation is likely to be greater, the greater the variety of new products being produced in the foreign
nations exporting to the home country. The variety of goods produced in the foreign nations will be

22 Riedel and Athukorala (1995, p.35) observe: “It is curious that those who argue that conventional income
elasticity estimates are biased are not equally suspicious of the price elasticity estimates that come from the same
regression equations. Krugman (1989), for example, goes to great lengths to explain why the conventional estimates of
income elasticities of export demand are biased upward, but accepts without question the empirical evidence that price
elasticities are low, perhaps because it accords with his model which emphasizes product differentiation and
monopolistic competition.”
larger, the larger the size of the foreign economies.

One of the sensitivity tests in section 7 focuses on the relative-price elasticity $\eta_{pr}$ and shows how important the size of this parameter is for a wide range of macroeconomic outcomes.

5. Cross-Border Effects of the Elasticity of Intertemporal Substitution in Consumption

The elasticity of intertemporal substitution in consumption (EIS) is a behavioral characteristic of consumption that is not a direct reflection of cross-border goods substitutability. The EIS nonetheless has powerful effects on macroeconomic outcomes, including especially on cross-border spillovers. The cross-border effects associated with the EIS are similar in several ways to the effects of direct measures of cross-border substitutability such as varieties effects and the relative-price elasticity of trade flows. Because of these similarities and the intrinsic importance of the EIS parameter, we include variations in the EIS as part of the sensitivity tests conducted below.

The individual adult consumer in the model is assumed to solve a familiar utility maximization problem, where the utility function is the constant relative risk aversion (CRRA) form,

$$u(c(s,t)) = \frac{c(s,t)^{1-\sigma} - 1}{1-\sigma}; c(s,t) \text{ is the consumption at time } t \text{ of an individual adult born at time } s; \sigma \text{ is the coefficient of relative risk aversion, and } 1/\sigma \text{ is the elasticity of intertemporal substitution.}$$

The total wealth of the adult individual at time $t$ is the sum of $fw(s,t)$, wealth in the form of financial assets, and $hw(s,t)$, human wealth defined as the present value of expected future labor income net of taxes on labor income and in vivo transfers to children.\(^{23}\)

The solution for the utility maximization problem gives adult consumption as a linear function of total wealth:

$$c(s,t) = \frac{1}{\Psi(t)} [fw(s,t) + hw(s,t)]$$

where $1/\Psi(t)$ is the marginal propensity to consume out of wealth. The dynamics of $\Psi(t)$ are given by:

$$\dot{\Psi}(t) = -1 - \frac{1}{\sigma}[(1-\sigma)(r(t) + p_n(t))-(\theta + p_n(t))]\Psi(t)$$

where $r(t)$ is the real interest rate at time $t$, $\theta$ is the time preference rate, and $p_n(t)$ is the adult mortality rate (the inverse of life expectancy).

\(^{23}\) Transfers made by adults to support the consumption of youth dependents are discussed in detail in Bryant, Faruqee, Velculescu, and Arbatli (2004).
The marginal propensity to consume out of wealth in the general case of the CRRA utility function depends, as is well known, on the elasticity of intertemporal substitution and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equation (11). In contrast, when the EIS is assumed to be unity (the case of logarithmic utility, with \( \sigma = 1 \)) and when the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate (\( 1/\bar{\theta} = \theta + \bar{\rho} \)).

The net response of consumption to changes in the real interest rate depends on the relative strength of substitution and income effects. With a low rather than high value of EIS, consumers act less strongly to shift their consumption intertemporally; the substitution effect is smaller relative to the income effect. With a low EIS, moreover, consumers respond absolutely less to interest-rate changes. For any given shock to the economic system, the real interest rate thus must adjust by a larger amount the lower is the value of \( EIS \). Conversely, with higher values of the EIS, the substitution effect increases in importance relative to the income effect; correspondingly, shocks require smaller adjustments in real interest rates.

6. Effects of Asymmetric Sustained Fertility Declines: Benchmark Analysis

To appraise the sensitivity experiments that are the core of this paper, the reader requires a sketch of our analytical model and an overview of the qualitative conclusions that the model delivers about the effects of asymmetric demographic shocks. The account that follows provides a minimum, abridged summary.²⁴

The starting point for the empirical model is a set of equations describing the U.S. economy ("US" for short). A second artificial country, labeled for brevity as "ZZ," is a mirror image of the United States. The “world” in this stylized framework is thus composed of two economies of equal size, roughly like the United States, that are identical in domestic structure. The economies are linked together with the balance-sheet and income-flow identities that would have to hold in an actual world of two economies.²⁵

²⁴ Earlier papers provide such information in much more detail, together with a description of the underlying theoretical approach. See, for example, Bryant (2004a, 2004c).

²⁵ For example, the current-account balance and the net-foreign-asset position of the ZZ economy are exactly the negatives of the current account and the net-foreign-asset position of the US economy. The underlying larger model is the IMF staff’s MULTIMOD (see Laxton, Isard, Faruque, Prasad, and Turtelboom (1998) and Masson, Meredith, and Symansky (1990). Bryant and Zhang (1996a, 1996b) describe the original abridgement.
The two economies are connected by a single, endogenously modeled exchange rate that is proximately determined by a variant of the uncovered interest-parity relationship. Indirectly, the exchange rate is influenced by and in turn helps to determine all the macroeconomic variables in both economies. The model emphasizes the forward-looking behavior of agents and presupposes that both firms and households engage in intertemporal optimization.\textsuperscript{26} The consumption-saving sectors of the model permit an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS).

Output of the single composite good produced in each economy is a function of the economy’s capital stock and its productivity-augmented labor.\textsuperscript{27} Labor is assumed perfectly mobile within each of the two countries but completely immobile across the countries’ borders. Hence wages are equal across comparable age cohorts within each country but in general are not equal across the two countries. Over the long run labor is inelastically supplied with respect to wages and is determined by the model's demographic structural equations. Prices are sticky in the short run but flexible over a longer run. The model forces full employment of labor and capital over the long run.

When using the empirical model to perform simulations, we first develop one or more model-consistent, steady-state baseline solutions for the evolution of the two regions. For transparency, both the ZZ and US economies are assumed to follow identical paths and exhibit identical behavior along these baselines. Hence the baseline nominal and real exchange rates are constant over time at unity; the trade balances, current-account balances, and net-foreign-asset positions in the baseline are all constant at zero. Baseline solutions for the model typically assume that productivity growth occurs at a constant rate. Baseline steady-state rates of inflation are likewise assumed constant. The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables in the model. Typical baselines have these key demographic rates set at constant values.\textsuperscript{28}

To develop analytical conclusions about the consequences of demographic shifts, we run

\textsuperscript{26} A partial exception stems from an allowance for a fraction of consumers whose consumption is constrained by an inability to borrow and hence are unable to smooth their consumption intertemporally.

\textsuperscript{27} The production technology of firms is represented by constant elasticity of substitution (CES) production functions. Firms are price-taking entities that choose variable inputs and their level of investment in capital so as to maximize stock-market value. Firm investments respond to the difference between the market value and reproduction value of the capital stock (a variant of Tobin's "q" framework).

\textsuperscript{28} The empirical model is solved with a software algorithm that imposes model-consistent ("rational") expectations. Hence agents are presumed to know the structure of the model and to correctly anticipate the entire future paths of the model's exogenous variables. Imposition of model-consistent expectations is the now-standard – albeit problematic – working assumption in most empirical work in macroeconomics.
shock simulations in the model, perturbing the paths of one or more of the exogenous demographic rates and comparing the resulting shock values of endogenous variables with their baseline values. Earlier papers focused primarily on an asymmetric combination of fertility shocks. To facilitate comparisons with that earlier work, we again focus on the same asymmetric fertility declines. The child birth rate in the ZZ economy declines sharply, remains at a low level for an extended period, and then eventually recovers enough of its earlier decline to leave the ZZ economy with a stationary population. This illustrative large-cyclical shock causes a negative growth rate for the population as a whole for several interim decades. We select this fertility shock for study in part because it is roughly analogous to the recent and prospective demographic experience of Japan.\

Asymmetrically, with a contrasting smaller-gradual shock, the child birth rate in the US economy declines more slowly and monotonically until the population eventually reaches a zero growth rate. For these illustrative shocks to fertility, the adult-mortality and child-mortality rates remain unchanged at their baseline levels.\

The exogenously specified paths for the birth rate in the two asymmetric shocks are shown in the upper portion of Figure 1. The bottom part of the figure plots the corresponding endogenous growth rates for the populations as a whole, which of course reflect the patterns of the birth rates. The assumed fertility declines have major consequences for all endogenous demographic variables. The effects on youth-dependency and elderly-dependency ratios are graphed in Figure 2.

All figures in the paper contain three vertical lines. The two solid lines indicate the beginning year and the ending year of the exogenous shock to birth rates. The dashed vertical line indicates the year after which effective labor forces cease to change significantly (have reached their new steady-state values).

For the ZZ large-cyclical shock, as the cohorts of reduced size pass into and through the working-age population, the effective labor force begins a protracted, sharp decline – at a rate much steeper than that for the adult population as a whole (Figure 3). Eventually, that decline is partially

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29 The major decline in fertility in Japan took place in the second half of the 20th century. Our illustrative shock roughly combines Japan's past fertility experience and projections of its demographic future.

30 Further details about the creation of steady-state baselines and the details of the asymmetric shocks are provided in Bryant (2004a, Appendix 2).

31 The initial baseline level of the effective labor force is some 1.7 times greater than the level of the total adult population, reflecting the calibration of the labor forces with their incorporation of the age-specific relative productivities of different aged workers. The levels of the effective labor force in the model represent, in effect, the number of labor "efficiency units," not the total number of workers. The growth rate of the adult populations lag behind the growth rate of the total (adult plus youth) populations by 18 years. For the ZZ large-cyclical shock, although the total population begins to decline fairly soon after the onset, the adult population and the effective labor force thus
reversed as the proportion of youth in the ZZ economy rises again and ultimately stabilizes at its new eventual level. For the US smaller-gradual shock, the behavior of the adult population and effective labor force is dramatically different. When both economies eventually reach new steady states in which the size and composition of populations are stationary, the levels of the US population and effective labor force that result from the smaller-gradual shock are nearly twice as large as those that result from the ZZ large-cyclical shock (Figure 3) even though the ratios of the effective labor force to adult population have converged to the same value (Figure 4).32

The large differences in the demographic consequences of the two shocks cause major differences in the corresponding macroeconomic outcomes for each economy. We summarize here the main features of those outcomes for a simulation generated with “benchmark” values of the model’s parameters. The benchmark values are identified and discussed in our earlier papers. For the purposes of this paper, the most critical parameters are the elasticities in the trade equations. The long-run relative price elasticities for imports for each country are kept at their benchmark estimates ($\eta_{pr} = -1.097$). The long-run activity elasticities are kept at the imposed benchmark value of unity ($\eta_y = 1$). For the benchmark analysis, “varieties” effects are excluded ($\eta_{fshare}$ is set to zero by setting $\delta_0 = \delta_i = 0$). The speed-of-adjustment coefficient for the error-correction term in the import equations, $(1 - \alpha)$, which determines how promptly the volume of imports adjusts to its long-run determinants, is calibrated at a benchmark value of 0.75 ($\alpha = 0.25$). Another critical parameter, the elasticity of intertemporal substitution in consumption in the ZZ and US economies, is set at the benchmark value of 0.5. Alternative values for these key parameters will be examined below.33

32 Changes in the effective labor forces reflect not only the demographic shocks but also the complex effects of humped age-earning profiles over the life cycle on the determination of labor incomes and human wealths. As the demographic shocks pass through the age-earning profiles in the two economies, the dynamic effects of the demographic shifts, significant in themselves, get still further amplified. When individuals first enter the labor force, they have relatively low productivity and are relatively low savers. Then as younger workers age, gain experience, and have higher productivity, they in effect ascend the left side of the hump of the economy’s age-earning profile. Individuals reach their years of peak earnings and high savings when they are in their forties and fifties. Eventually, they start to descend the right side of the humped age-earning profile, and consequently their labor incomes and saving decline. At that point, their consumption must be increasingly financed out of their privately accumulated financial wealth as supplemented by pension transfers from the government. We first stressed the importance of these age-earning-profile effects in a 2001 paper, published as Bryant and McKibbin (2004); see in particular the comparisons in that paper between simulations with and without the age-earning profiles present.

33 More detailed analysis of the outcomes with benchmark parameters is provided in the earlier papers (Bryant, 2004a; Bryant, Faruqee, Velculescu, and Arbatli, 2004).
The following summary should be read in conjunction with examination of a 6-panel page of charts, Figures 5 through 10. Each of those figures pertains to a particular variable. The two solid curves in Figures 5, 6, 8, and 10 show the paths for the ZZ and the US variable when the fertility decline is asymmetric. The ZZ path for a variable, the focus of the discussion here, is plotted with a thicker, more prominent curve than the solid curve for the US path. The thin, dashed curves in the figures are shown for background reference; they indicate the “closed-economy” outcomes for the variable in both countries, either when the large-cyclical fertility decline, or alternatively the smaller-gradual decline, occurs in both economies identically.34

Given the decline in the ZZ effective labor force relative to the adult population, the ZZ economy-wide aggregate levels of human wealth, financial wealth, output, consumption, and the aggregate capital stock all decline to eventual levels that are significantly lower. Because the effective labor force is lower relative to the capital stock, the marginal product of capital falls and the ZZ real interest rate declines, by more than the US real interest rate but by less than it would have to decline if the ZZ economy were closed (Figure 5). The ZZ capital-output ratio rises substantially in the medium and longer runs and remains at the higher level forever. The US experiences a smaller medium-run and permanent rise in its capital-output ratio than the increase in the ZZ economy; the US increase is less than the rise that would occur if its economy were closed. These different interest-rate, capital-stock, and output evolutions in the two economies are associated with major differences in saving and external-sector behaviors, which in turn are associated with changes in the relative sizes of the economies.

Saving and financial wealth per adult in the ZZ economy rise sharply relative to baseline in the shorter and early medium runs (Figure 6). The increases in financial wealth are explained partly by the effects of the age-earning profiles on saving and partly by higher disposable incomes and savings for adults reflecting the smaller support of child consumption.35 In contrast, saving and

34 We refer in the earlier papers to shocks in which a fertility decline occurs identically in both the ZZ and US regions as symmetric (“global”). When a shock is identical in both regions, the model produces identical simulation paths for both economies. External-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity. In effect, each economy behaves as though it were completely closed, which is of course true for the world as a whole. Hence we refer to the symmetric cases (both regions experiencing the large-cyclical shock, or both experiencing the smaller-gradual shock) as “closed-economy” simulations. The closed-economy analysis in the earlier papers facilitates interpretation of the most basic, domestic consequences of fertility declines and serves as a benchmark for analytical interpretation of open-economy effects.

35 Numerous channels cause private saving, and hence financial wealth, to rise in the ZZ economy. Among them is the fact that the population aging caused by the relatively larger fertility decline requires the government authority operating an “intermediate-balanced” pension system (assumed in the benchmark simulations) to raise pension taxes and reduce pension benefits (relative to baseline), which in turn is an incentive for increases in private saving.
financial wealth per adult in the US economy fall somewhat; those US paths for saving and per-adult financial wealth are consistently below the paths that would prevail if the US economy were closed and experienced the slow-gradual fertility decline (Figure 6). The large differences in saving behavior and hence in financial wealth between the ZZ and US economies are attributable not merely to their different-sized demographic shocks but also to major effects working through the exchange rate and external-sector transactions.

As the asymmetric shocks progress in the two countries, an interest differential in favor of the US economy opens up (the interest rate falling less in the US than in the ZZ economy) that proximately puts pressure on the real and nominal exchange rates. After several initial decades, both the nominal and real values of the ZZ currency begin a strong appreciation. The real exchange value of the ZZ currency appreciates substantially further by the medium run, reaching a peak appreciation before reversing and falling back (Figure 7). In the new long-run steady state eventually reached by both economies, the nominal and the real exchange rate settle at appreciated levels significantly higher than in the baseline.

The basic explanation for the permanent appreciation of the ZZ currency – the goods-preferences assumptions of the traditional elasticities approach incorporated in the model – has already been emphasized above. The asymmetric fertility declines are transitory in terms of differences between the regions’ demographic rates of growth but have permanent effects on the relative levels of demographic and macroeconomic variables. The larger shock to the birth rate in the ZZ economy causes the ZZ population and effective labor force to fall further below baseline than the declines in the US economy and hence causes shrinkage in the ZZ relative to the US population and labor force. Eventually the ZZ and US population growth rates again become equal. But the ratio of the ZZ to the US effective labor force and the ratio of the two countries’ populations remain permanently smaller. Correspondingly, ZZ macroeconomic aggregates such as output and the capital stock become smaller than the corresponding macroeconomic aggregates abroad (Figure 9). By the medium and long runs, therefore, the quantity of ZZ-produced goods available for sale and consumption in the world is markedly smaller than the quantity of US-produced goods. Given

36 The underlying model enforces a variant of the uncovered interest parity condition as part of the behavior determining the exchange rate. An interest differential in favor of the US economy must be offset, other things being equal, by an expected depreciation of the US currency (an appreciation of the ZZ currency).

37 The decline in relative size of the ZZ economy is considerably smaller measured in terms of the ratio of real GDPs than when measured by the ratios of populations or labor forces. Issues about the relative sizes of the two economies are discussed further in section 7 below.
unchanged preferences in each economy for the two types of goods, relative prices in the world economy have to change to reflect the now relatively scarcer ZZ-produced good. A permanent real appreciation of the ZZ currency -- an improvement in the ZZ economy's real terms of trade -- is an integral part of the required change in relative prices.

Changes in exchange rates generate expenditure-switching incentives between the two economies. Thus by the medium run the ZZ economy begins to import substantially more of the now relatively cheaper goods produced in the US. ZZ exports to the US relative to baseline are inhibited by the appreciation of the ZZ currency. For the initial decades of the shock, the ZZ real trade balance relative to real GDP changes little. Thereafter, however, the expenditure-switching effects cause the ZZ economy to run a progressively larger deficit on real trade account. This net import of real resources from abroad provides a cushion of support to the ZZ economy that permits it to sustain a significantly higher level of consumption than would otherwise be possible. The US economy experiences the opposite effect: it must export real resources abroad and correspondingly curtail its consumption relative to what would otherwise be possible.

The medium-run trade deficit of the ZZ economy is not associated with a deficit on current account. The ZZ economy not only imports more from abroad. It also saves more relative to baseline; financial wealth rises relative to baseline (Figure 6). A fraction of the higher ZZ financial wealth is invested abroad at the higher interest rates available abroad. Hence the ZZ economy over the medium run starts to earn a higher flow of investment income from abroad. The net investment income payments received are more than enough to offset the increased deficit on trade account, with the result that the ZZ economy begins to experience a significant current-account surplus. The surplus reaches a peak during the medium run; thereafter it falls and even returns close to balance for several decades as the two economies move toward their new long-run steady states. Eventually, in the very long run the ratio of the ZZ current balance to nominal GDP settles at a moderate surplus ratio.

The net foreign asset positions of the two economies are the integral over time of the current-account imbalances. The ZZ economy – despite the relatively larger shock it experiences, causing the economy's output and aggregate consumption to fall well below the levels that would have been observed without the shock – builds up a positive net foreign asset position, on which it earns a sizable return (Figure 8).

The openness of the ZZ and US economies thus decisively influences the macroeconomic consequences of the demographic shocks. Because of the openness of the economy, ZZ domestic variables are partly cushioned from the full impacts of the large ZZ fertility decline. As a
counterpart, US domestic variables are adversely buffeted by the larger shock emanating from the ZZ economy. An important component of these cushioning and buffeting effects is associated with the changes in exchange rates. The permanent appreciation in the real value of the ZZ currency enables the ZZ economy to enjoy a large permanent improvement in its real terms of trade. The opposite effect, a deterioration in real terms of trade, contributes to the adverse effects on the US economy.\footnote{Effects on the real exchange rate, trade balances, current-account balances, and net-foreign-asset positions of the two economies are larger when analysis takes into account the macroeconomic effects of children (youth dependency) than when it does not (Bryant, Faruqee, Velculescu, and Arbatli, 2004). When the analysis takes into account public pensions, many of the macroeconomic effects and cross-border spillovers are somewhat smaller than for model variants that exclude public pensions (Bryant 2004a).}

Analysis of the macroeconomic effects of demographic changes should carefully differentiate between aggregate levels of variables and their per-capita and per-adult values. The larger fertility shocks occurring in the ZZ economy inevitably cause larger negative effects on ZZ aggregate output and consumption. The ZZ path for aggregate real consumption falls much further below baseline than does the US path for aggregate real consumption. Yet the ZZ path for aggregate real consumption is significantly \textit{above} the path that would be experienced in the hypothetical case where the ZZ economy is completely closed and therefore unable to cushion its larger shock through transactions with the rest of the world. The openness of the economy works to mitigate the size of the negative effects on the aggregates.

When the per-adult or per-capita values of consumption in the ZZ economy are considered, the cushioning effects of openness are even more consequential. Notwithstanding the fact that the demographic shock in the ZZ economy is larger than in the US economy, ZZ per-adult consumption is actually \textit{higher} than US per-adult consumption. The difference between the two economies is sizable in the initial decades and is even more marked in the long run (Figure 10). The cushioning effects are so substantial when measured in per-adult terms that individual adults in the ZZ economy are significantly better off \textit{not only relative to individual adults in the US economy but better off absolutely relative to the no-shock baseline}. US consumption per adult is markedly \textit{lower} than in the baseline despite the fact that the US population, aggregate US real GDP, and aggregate US consumption are all at \textit{higher} levels than in the baseline.
7. Sensitivity Analyses with Key Parameters Determining Cross-Border Substitutability

This section builds on the benchmark analysis by conducting sensitivity experiments that vary the values of parameters determining the degree of cross-border goods substitutability. It explores how the benchmark macroeconomic outcomes resulting from the asymmetric demographic shocks are altered when varieties (home-market) effects are introduced. And it highlights the critical importance of the values of the import relative-price elasticity $\eta_{pr}$ and the values of the elasticity of intertemporal substitution in consumption $EIS$.

A. Varying the Elasticity of Varieties Effects. The benchmark analysis ignored varieties effects: the coefficients on the $for GDP share$ variable in equation (8) were set to zero ($\delta_0 = \delta_1 = 0$) so that there was no long-run elasticity of imports to that variable ($\eta_{fYshare} = 0$). In what follows, the short-run elasticity $\delta_0$ continues to be set at zero. Now, however, alternative values of the coefficient $\delta_1$ are chosen to provide a range of values for $\eta_{fYshare}$ between 0 and unity. This range seems likely to bracket a plausible estimate of the importance of varieties effects. The specific alternative values selected for $\eta_{fYshare}$ are 0.25, 0.50, 0.75, and 1.0. A simulation with the asymmetric demographic shocks is run for each of these alternative values and all are compared with the benchmark simulation in which $\eta_{fYshare} = 0$.39

In each of the new simulations, the import equations for both countries are treated symmetrically (both the ZZ and the US economies are assumed to have $\eta_{fYshare} = 0.25$; both have $\eta_{fYshare} = 0.50$; etc.). Thus if the share of US output in world output increases (numerator and denominator both measured in a common real currency unit), a non-zero positive value for $\eta_{fYshare}$ means that the ZZ economy imports more from the US than it would otherwise import taking into account only ZZ real activity and ZZ relative import prices. Analogously, if the share of US output in world output increases (ZZ output shrinking in relation to world output), the varieties effect causes the US economy to import less from ZZ than would otherwise have been the case. For the asymmetric fertility shock that we study, a positive varieties effect thus, other things equal, increases the quantity of ZZ imports and reduces the quantity of ZZ exports. These direct quantity effects, however, are only one of several gross influences on trade flows. As will be seen below, the

39 The speed-of-adjustment coefficient for the error-correction term in the import equations, $(1 - \alpha_t)$, is kept at the benchmark value of 0.75.
direct gross effects can be more than offset in the shorter and medium runs by indirect gross effects that result from changes in the real exchange rate, in the prices of trade flows, and in domestic macroeconomic variables. The net effects of all the influences on trade flows are complex and differ across different time horizons.

B. Varying the Relative-Price Elasticity of Trade Flows. The sensitivity of import demands to the relative price of imports has received considerable attention in the literature narrowly focused on cross-border trade. Yet the wider macroeconomic consequences of the relative-price sensitivity of traded goods – the feedbacks from trade flows to outputs and capital stocks and consumptions, and thus to the relative sizes of economies – has not received much emphasis. Our earlier papers also did not emphasize these wider effects.

To highlight the relevant issues, we conduct sensitivity tests embedding alternative values for the long-run relative-price elasticity in the model. The benchmark value for $\eta_{pr}$, -1.097, is probably somewhat above a median estimate of the relative-price elasticity for aggregate U.S. imports in the literature. If the downward bias in empirical estimates is in fact large, values for $\eta_{pr}$ well above this benchmark can be readily imagined. We thus allow for a range of higher values here. Those higher values, intrinsically worth investigating in their own right, also compensate in a loose sense for the mis-measurement of the official statistics for import prices. To widen the sensitivity analysis further, in addition we include two alternatives for $\eta_{pr}$ lower in absolute value than the benchmark estimate. The alternatives investigated are -0.8225, -1.000, -1.371, -1.6455, -2.194, and -2.7425; apart from the value that is exactly unity, the alternatives are, respectively, 0.75, 1.25, 1.5, 2.0, and 2.5 times the benchmark $\eta_{pr}$ of -1.097.40

As with the alternative values for the varieties elasticity $\eta_{Yshare}$, we treat the import equations for both countries symmetrically when introducing alternative values for $\eta_{pr}$. For every simulation, both the ZZ and the US economies have the same value for $\eta_{pr}$. In real life, of course, countries may have significantly different elasticities for the relative price of imports. Our symmetric treatment here promotes analytical transparency and is a first step to improve the analyses.

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40 For each alternative value of the long-run elasticity, $\eta_{pr}$, we retain the benchmark proportionality between the short-run elasticity $\gamma_0$ and the sum $\gamma_0 + \gamma_1$; the former is always 53 percent of the latter. And for each simulation with an alternative value for $\eta_{pr}$, the speed-of-adjustment coefficient for the error-correction term, $(1 - \alpha_t)$, is again kept at the benchmark value of 0.75.
understanding of the effects of varying degrees of cross-border substitutability.

C. Varying the Elasticity of Intertemporal Substitution in Consumption. Section 5 explained our interest in exploring the sensitivity of cross-border goods substitutability to the elasticity of intertemporal substitution in consumption. The benchmark value for the EIS is one half (0.5). In what follows, we investigate three alternative values. Two alternatives raise the parameter to higher values, increasing the response of consumers to interest-rate changes and thereby strengthening intertemporal substitutability (raising the importance of the substitution effect relative to the income effect). One of these higher alternatives puts the EIS at 0.75 and the other raises it all the way to unity, the case of logarithmic utility. The third alternative drops the EIS to the low value of one third (0.333), reducing the response of consumers to interest rates and reducing the importance of the substitution effect relative to the income effect.

Empirical evidence, in our view, is more consistent with EIS having a value such as 0.5 – or even smaller – than the value of unity imposed by logarithmic utility. That is why we choose EIS = 0.5 as the benchmark. As with the other parameters subjected to sensitivity analysis here, however, substantial uncertainty exists about the correct value (for individuals, or average values for groups or for particular countries). The range from a low of 0.333 to a high of 1.0 probably brackets plausible estimates.

D. Effects on the Exchange Rate and on External-Sector Balances. To keep the exposition of the sensitivity analyses concise, we present key aspects visually in the form of multipanel charts. Panels pertain to a particular variable in the model (either a variable such as the real exchange rate or, more often, the path for a variable in the ZZ economy). Some chart pages contain three panels for a single variable. Other pages contain six panels, a left-hand column of three panels for a first variable and a right-hand column of panels for a second variable.

Each of the panels for a particular variable shows the curve for the simulation with the benchmark value of the panel’s key parameter. The benchmark curve is always the thickest, most prominent solid line. The “a” panels compare simulations differing from the benchmark case only by having one of the alternative values for the relative-price elasticity \( \eta_{pr} \). Curves in the “a” panels with absolute values of the elasticity larger than the benchmark case are plotted with solid lines; the curves for the smaller absolute values (-0.82 and -1.00) are reported with dashed lines. The “b” panels compare simulations differing from the benchmark only by using one of the alternative values of \( \eta_{f/share} \). The “c” panels focus on simulations with differing values for the EIS. The
juxtaposition of the “a,” “b,” and “c” panels highlights similarities and differences among the three types of sensitivity analyses.

Paths for the real exchange rate are shown in Figure 11. Raising the key parameter in all three sensitivity tests has sizable effects, as expected, on the degree of medium-run real appreciation of the ZZ currency. It also significantly influences the long-run appreciated level at which the real exchange rate ultimately settles.\textsuperscript{41} If the varieties coefficient is big ($\eta_{\text{fYshare}}$ as high as unity), the medium-run peak appreciation is only some two-thirds as strong as in the benchmark case; bigger values of $\eta_{\text{fYshare}}$ also increase the size of the partial reversal of the medium-run appreciation in the longer run (Figure 11b). Raising the EIS from the benchmark case of 1/2 to unity dampens the medium-run peak appreciation by about one fifth (Figure 11c). Dampening of the appreciation is especially strong when $\eta_{pr}$ rises in absolute value; for example, setting $\eta_{pr}$ as high as -2.74 reduces the medium-run peak appreciation to only about one-fourth of the amount in the benchmark case ($\eta_{pr}$ at -1.10). When the $\eta_{pr}$ and the EIS parameters are lowered relative to the benchmark case, the influences on the real exchange rate of course work in the opposite direction; the size of the medium-run peak appreciation is increased and the value at which the real exchange rate ultimately settles in the long run is raised (Figure 11a).

A similar phenomenon can be observed for all the sensitivity tests. Raising any of the three key determinants of cross-border goods substitutability has the consequence of increasing the importance of quantity adjustments in response to shocks relative to the size of adjustments in prices and price-like variables. The tendency can be seen in Figure 11 and is evident in several of the other figures that follow.

Any exogenous shocks to an economic system, and certainly large demographic shocks, require real quantity variables to adjust so that domestic economies and the world economy as a whole can attain a new real equilibrium. The necessary adjustments in key quantity variables, although not independent of what happens to price variables, are most crucially interdependent with the evolutions of other endogenous quantity variables (with all endogenous variables ultimately driven by the exogenous shock).

If shocks are asymmetric across countries, major adjustments are typically required in both the real and the nominal values of cross-border transactions. The higher is cross-border goods

\textsuperscript{41} The medium-run and long-run appreciated levels of the nominal exchange rate are also strongly influenced by raises in the key parameter for all three sensitivity tests.
substitutability with respect to relative prices, the less will price variables have to adjust to achieve the necessary adjustments in real quantity variables. Greater sensitivity of behavior to prices means that quantities (both cross-border and domestic) adjust faster and possibly more smoothly to the required new equilibrium. Conversely, if $\eta_{pr}$ is low so that cross-border goods substitutability is weak, then price and price-like variables will be required to adjust by much larger amounts to achieve the adjustments to quantities that are ultimately necessary. Price variables in these generalizations include of course goods prices -- domestic prices, import and export prices. But the relevant price-like variables also include interest rates and exchange rates (both real and nominal).

The point about quantity adjustments versus price adjustments is most intuitively evident for the $\eta_{pr}$ parameters. Yet it also applies analogously to the varieties effect parameter $\eta_{fYshare}$ and even to the value of the EIS. Consider, for example, the sizes of macroeconomic adjustments to shocks associated with alternative values for the EIS. For any demographic or non-demographic shock, the real interest rate – a price-like variable in the sense of the preceding generalization – is forced to change by a large amount with a low value of the EIS but needs to change by much less when the EIS is higher. A reduction in EIS from 1/2 to 1/3 in our simulations, for example, virtually doubles the size of the medium-run cyclical dip in the real interest rate. Raising the EIS from 1/2 to unity, on the other hand, cuts the size of the cyclical dip in the real interest rate roughly in half. The influence of the EIS value on changes in the real exchange rate, though significant (Figure 11c), are less dramatic than the influence on changes in the real interest rate.

The interplay between the sizes of quantity and price adjustments manifests itself clearly in cross-border trade transactions. The panels of Figures 12 and 13 report effects on, respectively, the volume of ZZ imports per adult and the price of ZZ imports relative to the price of home-produced goods. As discussed in the benchmark analysis, the real appreciation of the ZZ currency generates expenditure-switching incentives between the two economies; thus the ZZ economy imports substantially more of the now relatively cheaper goods produced abroad and exports less to the rest of the world. But the size of the effects depends on the parameters $\eta_{pr}$, $\eta_{fYshare}$, and EIS. When any of these three is increased, ZZ aggregate imports and imports per adult are reduced relative to what they would be with lower substitutability (Figures 12a, 12b, and 12c). The size of the fall in local-currency import prices, reflecting the smaller medium-run and long-run real appreciation of the ZZ currency, is similarly reduced (Figures 13a, 13b, and 13c).

The currency appreciation and associated price changes, resulting in expenditure switching between the two economies, enable the ZZ economy to import more and export less than would
otherwise be possible, which in turn cushions the effects of the demographic shock on ZZ consumption. This qualitative observation, evident in the benchmark case, continues to be true for all the sensitivity analyses. Notably, however, the ZZ deficit on real trade account (shown in Figures 14a, 14b, and 14c as a ratio to ZZ real GDP) is smaller the higher is the price elasticity of trade flows \( \eta_{pr} \), the larger is the varieties parameter \( \eta_{fYshare} \), and the higher is EIS.

Figure 15 reports the movements of the ZZ nominal trade balance as a ratio to nominal GDP. Percentage-point declines in the nominal ratio are substantially less than the declines in the real ratio. Notice, however, that for two of the three sensitivity analyses, the net effects of parameter variation on the ZZ nominal trade balance work in an opposite direction from those on the real trade balance. Raising the price elasticity of trade flows \( \eta_{pr} \) and increasing the varieties parameter \( \eta_{fYshare} \) somewhat enlarges the medium-run ZZ nominal trade deficit (contrary to the diminution of the real trade deficit). Interestingly, this opposite direction of movement in the real and nominal trade balances is not observed when the EIS is varied; increasing the EIS above its benchmark value of 1/2 reduces both the real and the nominal trade deficit (Figure 15c).

Effects of demographic shocks on private saving and financial wealth, and also on human wealth, are sensitive to all three parameters. Differences in the behavior of the ZZ private saving ratio are highlighted in the panels of Figure 16. The greater is the absolute value of \( \eta_{pr} \) the larger are the short and medium-run boosts to ZZ private saving. Over the longest run, on the other hand, the larger values of \( \eta_{pr} \) are associated with lower rather than higher ZZ savings ratios. Bigger values of the varieties parameter \( \eta_{fYshare} \) likewise increase the medium-run rise in the private saving ratio and reduce it over the longest run (though the effects from variation in \( \eta_{fYshare} \) are less strong than for variation in \( \eta_{pr} \)). Effects of variation in the EIS, however, are again somewhat opposite. Raising the value of EIS has the result of reducing the ZZ private saving ratio in both the medium and long runs; lowering the EIS from 1/2 to 1/3 causes a much larger increase in the saving ratio than in the benchmark case (Figure 16c). The effects of variation in EIS on saving, consumption, and financial wealth stem in large part from the extent of intertemporal smoothing by consumers. For values of EIS as high as unity, consumers choose to spread out smoothly over future periods any reductions in consumption that must be made. In contrast, smaller values of the EIS reduce the salience of the substitution effect relative to the income effect; consumers accordingly do less intertemporal smoothing and make sharper short-run changes in their consumption and saving.

The medium-run real and nominal trade deficits of the ZZ economy are not, as emphasized
in the benchmark analysis, typically associated with a ZZ deficit on current account. For most values of the key parameters, saving and financial wealth rise relative to baseline (Figure 16) and some portion of the incrementally higher financial wealth is invested abroad at the higher interest rates available abroad. The ZZ economy thus earns a positive net flow of investment income from abroad (Figure 17, which shows the ratio of net foreign investment income to ZZ nominal GDP). The medium-run buildup in net investment income is greater the higher are the values of the substitutability parameters $\eta_{pr}$ and $\eta_{fyshare}$.

Over the medium run the net receipt of investment income is large enough to more than offset the nominal trade deficit, causing the ZZ economy by the medium run to experience a current-account surplus and hence to build up a positive net foreign asset position (Figures 18 and 19, in both cases expressed as a ratio to ZZ nominal GDP). Variation in the sizes of these external-sector positions associated with variation in the key substitutability parameters may at first glance seem surprising. Despite the fact that the medium-run currency appreciation is least when the $\eta_{pr}$ elasticity and the $\eta_{fyshare}$ elasticity are at their highest values (Figure 11), the medium-run ZZ current balance shows the largest surplus for high values of $\eta_{pr}$ and $\eta_{fyshare}$ (Figures 18a and 18b). Correspondingly, the medium-run ZZ net foreign asset position cumulates to a higher peak for those values (Figures 19a and 19b). Smaller relative-price substitutability and lower values for the varieties elasticity weaken rather than improve the ZZ external-sector position.

It bears stressing again that medium-run consequences during the time when the asymmetric fertility decline is occurring are partially reversed over the longest run after all the endogenous demographic changes have ceased. This long-run reversal is especially large for high values of $\eta_{pr}$ and $\eta_{fyshare}$.

Outcomes for the ZZ current-account balance and net foreign asset position when the EIS is varied are again different from the outcomes for the other two sensitivity experiments. When the EIS is set higher than in the benchmark case, the ZZ currency appreciates less, the ZZ current-account balance shows a smaller rather than larger surplus (Figure 18c), and hence the ZZ net foreign asset ratio rises to less positive values (Figure 19c). Significantly, if the EIS is as high as unity, the current-account balance in the first decades of the shock moves into deficit rather than surplus, which in turns leads for a while to a negative net foreign asset position (and hence also to little or no net inflow of investment income from abroad). This result is a cautionary reminder that even the direction – and most definitely the magnitude – of net capital flows associated with asymmetric fertility declines can be sensitive to a model's assumption about the value of the EIS.
E. Effects on the Relative Sizes of Economies. Macroeconomic interactions across borders following asymmetric demographic shocks can have major long-run influences on the relative sizes of countries. This insight, implicit in what has gone before, can be seen more clearly by examining the ratios of comparable variables for the two regions of the general-equilibrium model economy. For example, Figure 20 plots curves for the ratio of ZZ real GDP to US GDP; Figure 21 shows comparable curves for the ratio of the two countries’ real capital stocks.42

The relative-size consequences stemming from demographic shocks are most consequential over the longest run because macroeconomic feedbacks eventually lead to significant long-run changes in the relative sizes of the countries’ outputs, capital stocks, and consumptions. The larger the varieties elasticity $\eta_{\text{Yshare}}$, the greater are the long-run declines in ZZ real GDP and the ZZ capital stock in relation to the counterpart variables abroad (Figures 20b and 21b). Similarly, as the relative-price elasticity $\eta_{pr}$ is raised from the benchmark value to higher values, the ZZ economy shrinks relative to the US economy by a greater amount (Figures 20a and 21a). The relative-size effects are more modest for increases in the EIS above the benchmark value. Even in that sensitivity experiment, however, raising the EIS has the result of diminishing the ZZ relative to the US economy (Figures 20c and 21c).

What occurs if the degree of cross-border substitutability is reduced? Strikingly, the relative-size effects can be reversed. The simulations that drop the absolute value of $\eta_{pr}$ to -0.82 significantly increase the size of the ZZ relative to the US economy. For example, whereas ZZ output eventually falls to 88 percent of US output in the benchmark case and falls to about 60 percent for the high value $\eta_{pr} = -2.74$, the relative size of ZZ output increases to 123 percent of US output for $\eta_{pr} = -0.82$. There is an analogous effect of increasing the size of ZZ relative to the US, albeit more modest, when the EIS value is dropped to 1/3. The long-run expansion of the ZZ economy for low values of cross-border substitutability following a larger ZZ fertility decline is a dramatic example of the potential significance of the $\eta_{pr}$ and EIS parameter values.

One can think of long-run shifts in relative country size as the result of a long-run

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42 For both the real-GDP and the real-capital-stock ratios, the numerator and denominator are expressed in the same real currency unit. Thus the ratio can change because the macroeconomic aggregate expressed in each country’s local currency changes or because the real exchange rate changes. The FORGDPSHARE variable used in the import equations, the ratio of one country’s real GDP to “world” GDP (see equation (5)), is a transformation of the ratio of ZZ real GDP to US real GDP and we therefore do not discuss separately simulations paths for FORGDPSHARE.
“redistribution” across borders of outputs, capital stocks, and consumptions. A country whose output expands relative to total world output following the occurrence of asymmetric demographic shocks finds that high relative-price elasticities and high positive varieties effects, in essence, eventually reinforce its faster growth. Asymmetric demographic transitions in conditions of higher cross-border substitutability generate, as it were, a feedback cycle of endogeneity. Faster relative output growth (slower relative decline) for a country engenders higher relative consumption of its output, an effect which strengthens as the medium run turns into the long run. The country with higher relative growth may undergo a sizable real depreciation of its currency in the shorter and early medium runs -- indeed, will definitely do so if it is experiencing a smaller decline in fertility than the rest of the world. Once past the period when its currency depreciates, however, the earlier depreciation begins to be reversed. The eventual reversal is only partial when varieties effects are absent ($\eta_{\text{Yshare}}$ is zero) and for low values of relative-price elasticities. But for larger non-zero values of $\eta_{\text{Yshare}}$ and for larger $\eta_{pr}$, the eventual reversal becomes more sizable. If $\eta_{\text{Yshare}}$ is as large as unity, the short- and medium-run depreciation of the currency of the faster growing country becomes entirely reversed by the longest run. After its currency starts to appreciate in real terms, progressively reversing the earlier depreciation, the country’s relatively faster output growth is reinforced by the real appreciation in its currency, which leads to gradual further real appreciation, which in turn further reinforces the relatively faster output growth.

The process of “redistributing” the world’s output, capital, and consumption in relative terms in response to asymmetric demographic transitions eventually comes to a stop when both economies and the world as a whole ultimately reach a new steady-state balanced-growth

43 In our illustrative demographic shocks, the relative output effects occur because the US economy with its smaller fertility decline experiences a fall in aggregate output measured in local currency smaller than the decline in local-currency-measured aggregate output in the ZZ economy, the economy buffeted by the larger fertility decline. The rise in the US share of world real output is, in effect, somewhat masked in the short and medium runs by the real appreciation of the ZZ currency. The lesser absolute fall in US output measured in local currency does serve, other things equal, to raise the share of US output in world output. But the real appreciation of the ZZ currency, while it is occurring over the shorter and medium runs, works in the opposite direction (i.e., other things equal, it lowers the US output share).

44 After the real appreciation of the ZZ currency reaches its peak and then begins to be reversed (after the later medium run), the increases in the faster-growing (US) country’s share of world output then become more dramatic. After that point, both the changes in the real exchange rate and the relatively larger change in US output measured in local currency (relative to ZZ output) work in the same direction. So the increases in the US share of world output become more visible, and are especially so when the varieties effects are strongest ($\eta_{\text{Yshare}}$ at the high extreme of unity.). Over the longest run, moreover, the varieties effects become cumulatively stronger the higher is the $\eta_{\text{Yshare}}$ elasticity.
equilibrium. But that new steady-state outcome will not be reached until long after the triggering asymmetric demographic shocks are concluded. The larger are the assumed varieties effects on trade flows, the longer this adjustment process will be protracted and the greater will be the ensuing redistribution of outputs, capital stocks, and consumptions across countries.

**F. Welfare Consequences.** Per capita or per adult measures of macroeconomic aggregates are a more useful focus for normative comparisons of pre-shock and post-shock outcomes than economy-wide aggregates themselves. The panels of Figure 22 accordingly report the effects of the sensitivity tests on adult consumption per adult.

For each of the three determinants of cross-border substitutability, a raising of the absolute value of the parameter adversely influences ZZ per-adult consumption. High values of the relative-price elasticity $\eta_{pr}$ act to augment the long-run reversal of the medium-run upsurge in that consumption. The greater the strength of varieties effects, the less favorable the long-run evolution of per-adult consumption. The higher the value of the EIS, the deeper the cyclical dip in ZZ per-adult consumption after the medium run and the lower the value of that consumption in the long-run steady state.

The striking result that low cross-border substitutability tends to have favorable influences on the ZZ economy is evident again in the panels of Figure 22. If the $\eta_{pr}$ elasticity is set as low as -0.82, the long-run path for ZZ per-adult consumption is an order of magnitude improved relative to the benchmark case. And if the EIS is lower than the benchmark assumption of 1/2, ZZ per-adult consumption rises more in the medium run, exhibits a smaller cyclical downswing after the medium-run peak, and ultimately rises to a substantially more favorable level in the long-run steady state.

**G. A Benchmark Case with Advanced Substitutability.** To pull together the key inferences from the sensitivity tests, it is revealing to examine one further case. Whereas the preceding analysis varied the key substitutability parameters one at a time, the new simulation sets high values for all three parameters simultaneously. The new case thus presupposes a greater degree of cross-border substitutability in all three dimensions. For shorthand, this case will be labeled an “advanced substitutability” (AS) benchmark.

For the AS case, the relative-price elasticity $\eta_{pr}$ is set at -2.1940; the $\eta_{fy\text{share}}$ elasticity is set at 0.75; and the EIS is set at 0.75. These values are toward the high end of the ranges investigated
in the preceding section, but not literally at the highest values. (The benchmark-case values, again, are $\eta_{pr} = -1.097$, $\eta_{fYshare} = 0$, and EIS = 0.5.)

Outcomes with the advanced substitutability benchmark are compared with those from the original benchmark in Figures 23 through 26. The two benchmark curves are plotted as solid lines. In addition, in the background the figures contain three dashed curves indicating the effects of introducing each of the AS-case parameters separately rather than in combination.

If each of the three higher AS values is introduced separately (Figure 11), the medium-run real value of the ZZ currency is significantly reduced from the very strong appreciation in the benchmark case. When the AS higher values of the three parameters are introduced simultaneously, the currency appreciation still follows the same qualitative pattern but the medium-run peak appreciation is a mere one-fifth as large (Figure 23). By the longest run, the appreciation in the AS case has been almost entirely reversed.

The net effect of raising all three parameters to the AS values simultaneously is to greatly increase the medium-run peak surplus in the ZZ current account and to greatly enhance the buildup in the ZZ net foreign asset position (Figure 24). As discussed earlier, the separate effect of raising the EIS reduces rather than increases the medium-run current-account surplus and net foreign asset position. The strengthening effects on the external balances of raising $\eta_{pr}$ and $\eta_{fYshare}$, however, much more than offset the weakening effects from raising the EIS.

Although the net effects in the AS benchmark greatly strengthen the ZZ medium-run external balances, the longer-run reversals are much larger too. By the longest run, the ZZ net foreign asset position with AS parameters is less positive than in the original benchmark. In absolute terms, the AS net foreign asset position is moving back toward the pre-shock value of zero.

Figure 25 again shows effects on per-adult consumption in the ZZ economy, the variable in the model most revealing of welfare consequences for ZZ residents. After the short and early medium runs, the AS path falls further and further below the benchmark path. By the long run, the size of the shortfall is dramatic. The advanced-substitutability outcome is significantly less advantageous than the benchmark case (indeed, absolutely worse than baseline by the end of the medium run). Because each of the higher AS parameters individually reduces ZZ per-adult consumption below the benchmark path, the combined effects of varieties effects on trade flows, higher relative-price substitutability, and a higher elasticity of intertemporal substitution in consumption cause the cushioning effects of the economy’s openness to disappear altogether after the medium run.
Whereas ZZ per-adult consumption is undermined by advanced cross-border substitutability, the opposite is true in the rest of the world (Figure 26). The AS path of per-adult and per-capita consumption in the US economy is consistently above the benchmark path, by an increasing amount as time passes. The adverse effects on US per-adult consumption that stem from the asymmetric fertility decline are mitigated. US residents are benefited (harmed less) if varieties effects are strong, if trade flows have higher elasticity to relative prices, and if the elasticity of intertemporal substitution in consumption is high.

Which benchmark case, the original or the advanced-substitutability, is likely to be a closer approximation to the behavioral parameters operative in the real world economy? We do not try to answer that important question in this paper. We conjecture that the AS parameters overstate the degree of cross-border substitutability. But we also believe that the original benchmark parameters may understate it. At the least, we tend to think that the original benchmark is wrong in assuming that varieties effects are absent entirely.

8. Concluding Summary

Our earlier papers on the cross-border dimensions of demographic change emphasized the finding that countries with faster and larger demographic transitions due to more rapid declines in fertility experience a substantial appreciation of their currencies and strengthening of their current-account balances. Such external-sector changes cushion macroeconomic aggregates in the more rapidly aging economies and thereby mitigate the negative consequences on domestic output and consumption that would otherwise have to occur if the economy were closed to the rest of the world.

This paper has explored the possibility that those findings might not be robust. Indeed, a major motive for conducting the sensitivity tests in this paper was a concern that the earlier analysis might have overstated the extent of currency appreciation for the country with the larger fertility decline and thus distorted our inferences about cross-border transmission and welfare consequences.

In the event, the basic qualitative conclusions in the earlier papers are not overturned. Especially for the shorter and medium runs, the earlier qualitative findings are reliable guides for thinking about cross-border transmission and welfare consequences. In particular, the openness of its economy does foster “cushioning” effects for the country with a larger demographic shock. The cushioning effects are present even when substantial weight is given to varieties (home-market) effects. And they are present for virtually all plausible values of other key parameters determining cross-border goods substitutability. The qualitative conclusion likewise remains robust that the cushioning effects, though favorable for the economy with the larger demographic shock, are
unfavorable for the rest of the world.

However, the findings from our earlier papers do merit significant revision in quantitative terms. The incorporation in the analysis of varieties effects, allowing for higher values of relative-price elasticities for imports, and raising the value of the elasticity of intertemporal substitution in consumption (EIS) each work in the direction of (i) reducing the extent of the medium-run currency appreciation for the country experiencing the larger fertility decline, (ii) significantly modifying its external-sector imbalances resulting from the demographic shocks, (iii) diminishing the favorable cushioning effects for its economy, and (iv) diminishing the adverse effects abroad where the fertility decline is smaller.

One can view the introduction of varieties effects and the allowance for higher values of relative-price elasticities for imports as unambiguously increasing the degree of cross-border goods substitutability in the world economy. A raising of the value of the EIS in consumption also indirectly has the effect of increasing cross-border substitutability. The increased substitutability when the EIS is varied upwards is not, to be sure, directly a cross-border phenomenon. Nonetheless, upward variation in the EIS for each country’s consumers, and hence an increase in intertemporal substitutability, also has large effects on the exchange rate and other external-sector variables, and hence on cross-border spillovers.

Quantitative variations in our findings seem especially noteworthy when attention is focused on welfare consequences (with welfare approximated for each country by deviations of real consumption per adult from a shock-free baseline). Judgments about a country’s post-shock welfare depend crucially on whether the degree of cross-border goods substitutability is believed to be low or high! For a country experiencing a larger fertility decline, per-adult and per-capita consumption are cushioned most strongly when the degree of cross-border substitutability is weak! But the opposite is true elsewhere: the rest of the world prefers an outcome where the degree of cross-border goods substitutability is high!

As far as we are aware, this line of thinking has not been highlighted in the literature. Yet it merits emphasis and further explication. If the world in fact is characterized by weak cross-border substitutability and hence tends to require price and price-like variables to adjust by large amounts to generate changes in quantities required by shocks, the welfare experiences of countries may diverge considerably in response to shocks that occur in one part of the world but not in others. Conversely, if cross-border substitutability is high, welfare divergences in the face of asymmetric shocks, though still present, may be less large and less problematic. This paper illustrates these points with examination of fertility-decline shocks. But we strongly suspect that the degree of
cross-border substitutability significantly influences geographical welfare inferences about all types of demographic shocks – indeed about all types of shocks, non-demographic as well as demographic.

Further research on welfare consequences for geographical parts of the world economy may need to refine a distinction between welfare cushioning and cross-border transmission of shocks.\textsuperscript{45} Increases in the degree of cross-border substitutability such as increases in the relative-price elasticities of trade flows may unambiguously augment the cross-border transmission of shocks, at least for many types of variables. The transmission is heightened both from the perspective of a country where a shock originates and from the perspective of other countries that, so to speak, receive the shock via cross-border transmission. Phrases such as increased integration of, or greater interdependence in, the world economy presumably imply augmented cross-border transmission, again at least for many types of variables.

Yet heightened cross-border transmission does not necessarily imply greater welfare cushioning. What is cushioning for a country where a shock originates is often the opposite of cushioning, call it buffeting, for the rest of the world. For an economy where a shock originates, increased cross-border substitutability – heightened cross-border transmission – typically may not be a favorable development. The increased substitutability across borders may reduce welfare cushioning effects even while it heightens the transmission of macroeconomic effects to the rest of the world. In sharp contrast, seen from the perspective of the rest of the world, an increase in cross-border substitutability may entail buffeting effects and welfare effects that are positively correlated: increased cross-border substitutability can heighten cross-border transmission and yet also improve welfare consequences (diminish unfavorable buffeting effects).

We emphasize a conclusion in this paper emerging from our general-equilibrium simulations that was not discussed earlier, namely, that macroeconomic interactions across borders following demographic shocks have major long-run influences on the relative sizes of countries. In effect, asymmetric demographic shocks result in a “redistribution” across borders of outputs, capital stocks, and consumptions. Such redistributions can have powerful long-run welfare consequences for individual countries, in economic but probably also in political and security terms.

\textsuperscript{45} Statements about the degree of cross-border transmission in the world economy are typically intended to be descriptive rather than normative. Statements about welfare cushioning for particular geographical parts of the world economy, on the other hand, inevitably have a normative content and must cope with the fact that what is beneficial from the perspective of one part of the world may well be adverse for other parts.
Along any of the three dimensions emphasized in our sensitivity experiments, increases in cross-border substitutability augment the quantitative importance of the long-run redistributions of macroeconomic aggregates in the world economy. For a country experiencing a larger fertility decline, for example, the redistribution effects changing the relative size of economies are a significant channel through which the medium-run favorable welfare consequences of “cushioning” are reduced in the longer run.

Long-run cross-border redistribution effects have typically not been identified in earlier analyses of demographic transitions. These effects are important. But they also need to be seen in context. The modeling framework underlying our analysis does not yet permit people, in particular workers, to be mobile across borders. Within the borders of a national economy, on the other hand, people and labor are assumed to be perfectly mobile. Thus in the model, the effects of a country-specific demographic shock fall only on the population, workers, and effective labor forces within the borders of the nation where the demographic shock occurs. There are no secondary or feedback effects, via emigration or immigration, on the sizes of the population, workers, and effective labor forces outside that country. By construction, therefore, asymmetric demographic shocks cannot have cross-border demographic effects.

In sharp contrast with the inability of people to move across borders, financial capital and goods in the model do freely move across borders. The model permits – and our analysis emphasizes – the cross-border implications for macroeconomic aggregates such as outputs and capital stocks. With the passage of time, outputs, capital stocks, and consumptions in the model can be redistributed in the world. Hence the sizes of economies measured by people and labor forces behave very differently from the sizes of the economies measured by macroeconomic aggregates.

Allowing for varieties effects in the model causes significantly larger long-run reallocations of outputs and capital stocks and consumptions across the countries’ economies than would otherwise occur. But those larger reallocations need to be seen in the context of the model’s exclusion of the mobility of people across borders. For the large country-specific demographic shocks examined in our research, it strains credulity to maintain the assumption of complete immobility across borders of people and workers. No doubt a similar point applies to large non-demographic shocks.

We conclude with a last point that is probably already in the reader’s mind but nonetheless needs stating explicitly. Analysts and policymakers require more reliable empirical estimates of the determinants of the degree of cross-border goods substitutability than the estimates currently available. The paper shows that the empirical estimates of relative-price elasticities for trade flows,
of varieties effects on trade flows, and even of the elasticity of intertemporal substitution in consumption play critical roles in determining the global consequences of demographic shocks—and probably of all types of shocks. This observation alone should be sufficient to make it a high priority for future research to try to obtain more robust estimates of key parameters determining cross-border substitutability.

REFERENCES


32 (1950), 117-32.


