This preliminary draft of the paper contains passages that three of the four authors have not yet had a chance to review.

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1. Introduction

Mortality rates for infants and children fell sharply for much of the 20th century, while life expectancy increased for adults. Somewhat later fertility rates began to decline, first in developed nations and more recently in developing countries. These major demographic shifts have already had significant effects on the age structure of the populations of most of the industrialized world, reducing the number of youths relative to adults. The transition will continue into the 21st century. The populations of most developed nations will have much higher elderly dependency ratios in the coming decades. After varying delays, further declines in fertility and a progressive aging of populations will occur in developing nations as well.

Japan is the most prominent example of an industrial nation whose fertility rates have fallen and whose population is aging. Between 1950 and 1997, Japanese fertility declined from 3.65 to 1.39 lifetime births per woman. The ratio of youths to the total population fell from over 45 percent to less than 21 percent in the second half of the 20th century. Projections for the ratio of elderly to the total population show a near doubling over the next five decades from 17.2 percent to 36.9 percent. The dramatic nature of Japan's demographic shift relative to developed countries as a whole can be seen in visual terms in Figure 1, based on revised UN projections.

Changes in youth dependency are even more important for developing nations. For several decades after 1950, youth dependency in many developing nations actually increased substantially. Projections for the 21st century tend to show marked falls in fertility, declining youth dependency, and eventual population aging -- though lagging behind by several decades the experiences of developed nations.

Substantial research in both theoretical and empirical macroeconomics has been devoted to analyzing the implications for national economies of these profound demographic changes. The focus of much of this work has been on population aging and its effects on saving, investment and growth. Unfortunately, much of the analysis of the relationship between population growth and economic growth has concentrated just on

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1 See, for example, Cutler-Poterba-Sheiner-Summers (1990), Auerbach-Cai-Kotlikoff (1991), Velculescu (2000), and Brooks (2000a).
the size and growth rate of the total population, paying little attention to shifts in the age structure of the population. From a policy perspective, research has largely focused on the increasing burdens that rising elderly dependency ratios will place on national budgets and pension systems and on the menu of possible options for reform.2

The least studied aspects of the demographic shifts are the cross-border interactions: how demographic change in individual open economies influences macroeconomic developments abroad and the global balance of saving and investment flows. In our own work we have been especially concerned with the spill-over effects of demographic changes in one country on foreign economies through changes in exchange rates and other external-sector variables and with the consequences for saving and investment flows in the world economy as a whole.3

As our research progressed, we became increasingly dissatisfied with the treatment of demographic variables in macroeconomic models. Rather than treating youth dependency and elderly dependency as exogenous inputs to macroeconomic analysis, we sought increasingly to incorporate more of the demographics as integral, endogenous parts of the modeling framework.

To integrate the effects of youth dependency, we explicitly incorporated children and child support into our theoretical framework and focused attention on the proximate effects of falling fertility as they caused declines in youth dependency ratios. Our model now nests the economic impacts of children and child support in a larger, general-equilibrium context. We likewise sought to improve our analysis of the latter stages of the aging process by endogenously incorporating increases in elderly dependency ratios and the ensuing macroeconomic effects attributable to the operation of public pension systems. By paying attention to the entire age distribution of the population and its evolution through time, we became able to analyze a broad range of demographic issues and their interaction with macroeconomic developments.


A separate background paper presents a full exposition of our analytical framework, incorporating the various changes made since the start of our research. We are also preparing a separate paper that analyzes elderly dependency and policy issues facing public pension systems. This paper summarizes our research on youth dependency.

Evidence exists in the literature suggesting that youth dependency matters for the determination of national saving, investment, and foreign capital accumulation. Mason (1987, 1988) finds that changes in the growth rate of the population and in the youth dependency ratio can have opposite effects on aggregate saving. Bloom-Williamson (1998), Higgins-Williamson (1996) and Williamson (1997) investigate the implications of youth dependency for growth in East Asia. They argue that youth dependency has a significant role in savings, investment and foreign capital dependence (an idea discussed earlier by Coale and Hoover (1958)). Numerous empirical studies, some going back to the 1980s, have identified a negative macroeconomic link between dependency ratios and saving rates. Bloom-Canning-Sevilla (2001) is a recent report surveying some of the issues.

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4 The background paper, by Bryant, Faruqee, and Velculescu, "Analyzing the Effects of Demographic Change in a Global Macroeconomic Model: Background, Theoretical Framework, and Illustrative Simulations" will be circulated in March 2003. The public pensions paper is an updated and extensively revised version of the research reported in Bryant-Faruqee-Velculescu (2002).

5 The conclusion in these papers that high growth rates in East Asia before 1990 can be explained primarily by dependency-ratio effects has been questioned. But the view in the papers that changes in youth dependency can have significant macroeconomic effects is well supported.

6 Table 2 in Bryant-McKibbin (1998), reproducing an updated version of a table by Guy Meredith in Meredith (1995), identifies many of the empirical studies of the effects of dependency ratios on saving rates and summarizes their main findings. The studies include those of Feldstein (1980), Modigliani and Sterling (1983), Masson and Tryon (1990), Horioka (1991), Masson-Bayoumi-Samiei (1995), and Meredith (1995). Recent empirical evidence suggesting that both youth and elderly dependency ratios have a significant negative effect on savings is summarized by Loayza, Schmidt-Hebbel and Serven (2000), using a large macroeconomic database released by the World Bank covering 150 countries over the period between 1965-1994. (The economically appropriate cut-off age for defining youths varies significantly between developed and developing countries, and within developing countries themselves.) As discussed in Bryant-McKibbin (1998), the macroeconomic estimates of relationships between dependency ratios and saving rates use a reduced-form approach and are contentious. Skeptics argue that the existing macroeconomic research has failed to allow properly for econometric problems and cannot be reconciled with the microeconomic evidence based on household survey data.
Our research differs from previous work in two respects. We model the age structure of populations, demographic shocks, and the consequences for macroeconomic variables in a genuinely multi-nation and general-equilibrium framework. And our framework allows us to nest the presence or absence of youth dependency in a single integrated model, which in turns permits us analytically to isolate the effects of several dimensions of youth dependency. Our analysis sheds additional light on the conclusions reached in the earlier contributions to the literature on youth dependency. Our approach is structural, however, whereas the earlier literature relies on reduced-form and linear time-series approaches. We believe that our approach points the way to a more reliable method for studying the macroeconomic effects of youth dependency.

We emphasize fertility declines as the illustrative demographic shifts in this paper. We isolate the economic effects of changes in youth dependency and study how such changes interact with other types of demographic changes. We highlight in particular the cross-border effects in open economies of changes in fertility that cause significant shifts in youth-dependency ratios and, subsequently, in the composition of the adult work force and in elderly dependency.

The timing and the sizes of the macroeconomic effects of demographic shifts can be very different when youth dependency -- more generally, the entire age structure of the population -- is taken into account. Our research strongly reinforces the presumption that changes in youth dependency have first-order consequences for the determination of exchange rates, external imbalances, and global saving and investment flows.

Section 2 summarizes key features of our analytical approach, with emphasis on youth dependency and child support. Section 3 describes benchmark simulation results for a fertility decline with and without youth dependency. The results of a global, symmetric ("closed-economy") shock are contrasted with asymmetric shocks that differ in magnitude between two countries inter-linked through trade and capital flows. Section 4 carries out sensitivity analysis with alternative specifications for how adults support children's consumption. Section 5 contains concluding remarks. A first appendix provides more details about the theoretical framework. A second appendix briefly discusses the sensitivity of our results to alternative assumptions about the intertemporal elasticity of substitution in consumption, a key behavioral parameter governing adult consumption.
2. Analytical Approach

Our analytical framework is a two-country model with cross-border flows of goods and capital. The exchange rate linking the two currencies and economies adjusts to ensure that the global current-account balance and net-foreign-asset position are always zero. Within each economy, optimizing firms produce a single composite good, determined by an aggregate production function with capital and (productivity-augmented) labor as its arguments. The composite goods from each country are imperfect substitutes; some production in each country is exported; import demands are a function of national incomes and relative prices.

Households in each country are assumed to have identical preferences over foreign and domestic goods. The treatments of household consumption, saving, and wealth accumulation build on the overlapping generations framework of Blanchard (1985), P. Weil (1989), and Yaari (1965) as extended by, among others, Faruqee-Laxton-Symansky (1997) and Faruqee (2002) to incorporate age-earnings profiles and a “bottom-up” determination of labor income. In our further extension of the framework, population growth is endogenous and households are comprised of both adults and youth dependents (children for short). Children are assumed to consume but not to earn any income. Child consumption is financed by *inter vivos* transfers from parents to their children.

In the following description of the analytical framework, we suppress most details and summarize only the modifications in the treatment of consumption, saving, and wealth accumulation that are associated with youth dependency and the economic linkages between the child and adult populations. Appendix 1 provides a fuller account.

A. Economy-Wide Saving and Wealth Accumulation

The significance of youth dependency in the model is immediately apparent from the definition of total real domestic consumption \( X(t) \) (across both the youth dependent and adult populations):

\[
X(t) = C_j(t) + C(t) ,
\]
where $C_j(t)$ is aggregate child consumption and $C(t)$ is aggregate adult consumption. These two components of consumption are interconnected. Children are assumed not to earn any labor income and not to hold any financial wealth. Child consumption is thus exclusively financed from *inter vivos* transfers to children from adults. Adult consumption, meanwhile, is formulated in a familiar life-cycle approach but modified to account for the transfers to children. The details of adult consumption are relegated to section D of Appendix 1.

Reflecting an economy-wide budget constraint for each of the countries, the aggregate stock of financial wealth (held only by adults) accordingly accumulates as:

$$FW(t) = r(t)FW(t) + Y(t)(1 - \tau(t)) - \tau_{SS}(t)Y(t) - C(t) - C_j(t),$$

where $r(t)$ is the domestic interest rate, $Y(t)$ is aggregate labor income, $\tau(t)$ represents the government’s income tax rate (applicable to both labor and capital income), and $\tau_{SS}(t)$ is the pension (payroll) tax rate. The left-hand side of equation (2) is the flow of net private saving in the economy.\(^8\)

The fiscal authorities of each country engage in real spending on goods and services, raise revenues by taxing the incomes of firms and households, and pay interest on the outstanding stock of government debt. Each country's government also operates a pay-as-you-go public pension ("social security") system that collects revenue from a pension tax and pays out pension benefits to elderly adults.\(^9\) The financial sectors of the national economies, modeled in a rudimentary way, contain monies that are only high-powered money (central-bank liabilities), the demands for which depend negatively on

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\(^{7}\) Behavioral relationships for the home country's agents and agents in the other country are identical. Accordingly this exposition omits country identifiers.

\(^{8}\) Equation (2) reflects the fact that the aggregate of support transfer payments to children is equal to the total amount of child consumption. The equation also reflects the fact that aggregate financial wealth accumulates at the rate $r(t)$ --not at the rate $r(t) + p_{\mu}(t)$-- since the amount $p_{\mu}(t)FW(t)$ is not an addition to aggregate wealth, but rather a transfer from those who die to those alive implemented through the actuarially fair Yaari-style insurance companies. The variable $p_{\mu}(t)$ is the adult mortality rate.

\(^{9}\) The modeling of transfers from working adults to elderly dependents through the public pension system is described in Appendix 1.
short-term nominal interest rates and positively on national incomes. Policy reaction functions are specified for the central bank and the government fiscal authority.\textsuperscript{10}

B. Economic Linkages between Youth Dependents and Adults

The introduction of youth dependent consumption into the model requires addressing two central issues: how the consumption of youth dependents is determined, and how and by whom that consumption is financed.

First, note that total child consumption $C_j(t)$ is defined simply as the sum over individual child consumption levels $c_j(s,t)$ scaled by the number of children $J(s,t)$ from each age cohort (indexed by $s$):\textsuperscript{11}

$$C_j(t) = \int_{t-\Delta}^{t} c_j(s,t)J(s,t)ds$$

where $\Delta$ denotes the age-range of the youth dependent population.

Second, because children do not earn any income, they are dependent on transfers from their parents to finance this consumption. In particular, total youth dependent consumption must equal the sum of individual parent-child transfers $v(s,t)$ from each adult cohort $N(s,t)$ over all the relevant cohorts (indexed by $s$):

$$C_j(t) = \int_{-\infty}^{t} v(s,t)N(s,t)ds$$

The key behavioral implications of youth dependency in the model rest on the treatment of the right-hand side variables in equations (3) and (4).

Individual child consumption $c_j(s,t)$ can be divided into two components, one that is inelastic and fixed, the other that endogenously varies through time in response to conditions in the economy. The inelastic component is unaffected by changes in the economic situation of parents—i.e., it reflects, so to speak, the “basic needs” of children

\textsuperscript{10} Each central bank follows a policy rule that ensures long-run nominal stability of the national economy, either a targeting rule for (high-powered) money, a nominal-GNP-targeting rule, or a rule combining inflation targeting with real GNP targeting. Each fiscal authority uses an “intertemporal fiscal closure rule” that is a variant of debt-stock targeting: the income tax rate is varied up or down to ensure that, over a medium run, the actual path of the government’s debt converges to an exogenously-specified target path for the debt.

\textsuperscript{11} Details regarding the population dynamics for the adult and youth dependent population are relegated to the appendix.
for (say) food, clothing, and shelter. The second component is “discretionary” and modeled as a constant fraction of adult consumption per capita. The total resources consumed by each youth dependent, \( c_j(s, t) \), is thus given by:

\[
c_j(s, t) = c_1 + c_2 C(t) / N(t)
\]

where \( c_1 \) is the fixed, basic-needs component and \( c_2 \) is the coefficient of proportionality determining the responsiveness of the time-varying, endogenous component of child consumption to adult consumption per adult.

The formulation in (5) is appealing because it nests several possible ways to model child consumption. By setting \( c_2 = 0 \), for example, one can study the implications of child consumption that is inelastic to conditions in the economy.\(^{12}\) Conversely, by setting \( c_1 = 0 \) with \( c_2 > 0 \), one can analyze the case where the basic-needs component of child consumption is absent altogether. For a given value of \( c_1 \), the parameter \( c_2 \) determines the generosity and sensitivity of discretionary child consumption relative to the consumption of an average adult. By varying the value of \( c_2 \), one can thereby investigate the effects of making the discretionary component of child consumption more or less generous than average adult consumption, and hence more or less volatile (in absolute terms). Using this general formulation yields the following expression for aggregate child consumption:

\[
C_j(t) = c_v J(t) + c_2 \delta(t) J(t)
\]

where \( J(t) \) is the size of the child population and \( \delta(t) \) is the youth-dependency ratio, defined as the ratio of children to adults (i.e., \( \delta(t) \equiv J(t) / N(t) \)).

Our treatment of the second central issue is to assume that child consumption is entirely financed through \textit{inter vivos} transfers from adults, as youth dependents do not earn labor income and have no financial wealth.\(^{13}\) In our framework, adult transfers to

\(^{12}\) The model framework allows labor productivity in the economy to grow at a constant, steady-state rate. For model simulations in which productivity growth is positive, we assume that the basic-needs component of child consumption, \( c_1 \), is not fixed absolutely but rather increases at the constant rate of productivity growth.

\(^{13}\) Thus when a youth becomes an adult and enters the workforce at the beginning of the 19\textsuperscript{th} year of life, he or she starts out with zero financial wealth.
children are thus, in economic effect, tantamount to "child tax" payments earmarked for child support. Alternatively stated, adult parents are assumed not to derive any direct utility gain from child consumption and thus not to determine, as an integral part of their own consumption decision, the level at which children consume. Our simplified formulation, embodying a rule-of-thumb standard for setting child consumption and treating adults as the only decisionmakers in the economy, allows us to avoid fundamentally altering the consumer problem facing adult agents.14

We further assume that transfers are cohort-specific across adults. Specifically, transfers are assumed to be hump-shaped, rising with the age of the parent initially before declining. The hump-shaped profile allocates transfers most heavily to middle-aged adults with larger families of dependent children and less to younger adults with growing families and older adults with grown children. For a specific approximation of a hump-shaped profile, we adopt the following:

\[ v(s,t) = v(t)[z_i e^{-\omega_1(t-s)} + (1-z_i) e^{-\omega_2(t-s)}] ; \quad z_i > 1, \quad \omega_2 \geq \omega_1 \geq 0. \quad (7) \]

The weighting function in (7), in square brackets, is normalized relative to the youngest adult cohort (where \( s = t \)). In other words, \( v(t) \) represents the transfers that the newest cohort of adult parents provide to their (newly born) children. Parents in older cohorts provide relatively more or less support than \( v(t) \) depending on their age, according to the weighting function.

The specification in (7) nests several more simplified cases used in our initial explorations of youth dependency. If both curvature parameters are set to zero (\( \omega_1 = \omega_2 = 0 \)), support payments for children are completely age-invariant (every adult age cohort makes exactly equivalent transfers). For the special case in which \( \omega_1 = \omega_2 \) and both parameters are positive, (7) produces an exponentially declining rather than hump-shaped distribution of transfers across cohorts; that is, the newest cohort of adults provides the largest amount, with the amount declining uniformly for older and older

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14 See Appendix 1 for a description of the adult consumer’s problem. For an alternative formulation of children's consumption in which child consumption directly enters the parents' utility function, see for example Becker and Barro (1988).
cohorts. The most plausible configuration of the parameters matching the pattern of age-specific fertility is the hump-shaped case where \( \omega \geq \omega_i \geq 0 \).\(^{15}\)

With the cohort distribution of adult transfers in (7), the economy-wide total for adult transfers can be written in terms of two components:

\[
V(t) = V_1(t) + V_2(t) \; ;
\]

(8)

\[
\dot{V}_1(t) = \frac{\dot{V}(t)}{V(t)} V_1(t) + z_i b_n(t) N(t) v(t) - (\omega_i + p_n(t)) V_1(t) ;
\]

(9)

\[
\dot{V}_2(t) = \frac{\dot{V}(t)}{V(t)} V_2(t) + (1 - z_i) b_n(t) N(t) v(t) - (\omega_2 + p_n(t)) V_2(t) .
\]

(10)

Using the adding-up condition that aggregate child consumption must be equal to total adult transfers, the law of motion for \( v \), individual transfers by the newest adult cohort, is given by:

\[
\dot{v}(t) \left[ c_i J(t) \left[ \frac{\dot{J}(t)}{J(t)} + c_i \delta(t) C(t) \left[ \frac{\delta(t)}{\delta(t)} + \frac{\dot{C}(t)}{C(t)} \right] \right] \right] = \left[ \frac{\delta(t)}{\delta(t)} + \frac{\dot{C}(t)}{C(t)} \right] - \frac{b_n(t) N(t) v(t)}{c_i J(t) + c_i \delta(t) C(t)} + \omega_i \frac{V_1(t)}{V(t)} + \omega_2 \frac{V_2(t)}{V(t)} + p_n(t) .
\]

(11)

C. Implementing the Framework in a Global Empirical Model

For our empirical analogue of the theoretical framework, we use a stylized and simplified two-region abridgement of a larger world model. The underlying model, containing many separate countries and regions, is the IMF staff's MULTIMOD model. Our abridgement of MULTIMOD is a substantially revised and updated version of a two-region abridgement originally created in the mid-1990s by Bryant and Zhang (1996a, 1996b).\(^{16}\) The refined two-region model is a research environment in which the global macroeconomic consequences of demographic change can begin to be systematically

\(^{15}\) The case of exponentially declining weights takes into account the likelihood that the children of middle-aged adult cohorts may have already themselves reached adulthood and are no longer dependent on parent-child transfers. But it fails to recognize that family sizes tend to grow in the early years of adulthood after youths have come of age and begin to have children.

studied. Our ultimate research agenda is to incorporate insights and specifications obtained from the stylized model back into the richer, more realistic context of larger world models with separate actual countries.

The starting point for the empirical model is a set of equations describing the U.S. economy ("US" for short). Then a second artificial country is created, labeled for brevity as "ZZ." The ZZ economy is an identical, mirror image of the United States. Thus the “world” in this stylized framework is composed of two economies, roughly like the United States, that are equal-sized, equivalently open, and identical in domestic structure. The economies are carefully linked together with the balance-sheet and income-flow identities that would have to hold in an actual world of two economies. The current-account balance and the net-foreign-asset position of the ZZ economy, for example, are exactly the negatives of the current account and the net-foreign-asset position of the US economy. The two economies are connected by a single, endogenously determined exchange rate. The exchange rate is proximately determined by a variant of the uncovered interest-parity relationship. Indirectly, the exchange rate is influenced by and in turn helps to determine all the macroeconomic variables in both economies.

The empirical model, like the theoretical framework, emphasizes the forward-looking behavior of agents and presupposes that both firms and households engage in intertemporal optimization. A partial exception stems from an allowance for a fraction of consumers whose consumption is constrained by an inability to borrow and hence are unable to smooth their consumption intertemporally. The consumption-saving sectors of the model permit an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS) -- see equations (34) and (35) in Appendix 1.

Output of the single composite good produced in each economy is a function of capital and productivity-augmented labor. The production technology of firms is represented by constant elasticity of substitution (CES) production functions. Firms are price-taking entities that choose variable inputs and their level of investment in capital so as to maximize stock-market value. Firm investments respond to the difference between
the market value and reproduction value of the capital stock (a variant of Tobin's "q"
framework).\textsuperscript{17}

The stylized model treats labor as perfectly mobile within each of the two
countries but completely immobile across the countries. Hence wages are equal across
comparable age cohorts within each country but in general are not equal across the two
countries. Over the long run labor is inelastically supplied with respect to wages and is
determined by the model's demographic structural equations. Prices are sticky in the short
run but flexible over a longer run. The model forces full employment of labor and capital
over the long run.\textsuperscript{18} Because the composite goods from each country are imperfect
substitutes, each country exports some of its production to the other. Imports in each
country are a function of national income and relative prices. Agents in a given country
are assumed to have identical and time-invariant preferences over foreign and domestic
goods.

The empirical model is solved with a software algorithm that imposes model-
consistent ("rational") expectations. Hence agents are presumed to know the structure of
the model and to correctly anticipate the entire future paths of the model's exogenous
variables. Imposition of model-consistent expectations is the now-standard working
assumption in most empirical work in macroeconomics and our use of this assumption is
familiar ground. Yet the assumption is extreme. Worse, the assumption is inherently
implausible for demographic shocks that begin gradually and then wane gradually over
many years. Bryant has shown in previous research that it is feasible to modify the
model-consistent-expectations assumption by phasing in "correct expectations" about the
paths of exogenous variables with the passage of time rather than permitting expectations
to be correct immediately and fully. More broadly, much interesting research is now

\textsuperscript{17} The model’s investment equations so far follow the treatment in the Mark II version of
MULTIMOD (Masson, Meredith, and Symansky, 1990; see also Meredith 1991). Adjustment costs for
investment in capital are modeled explicitly in the Mark III version of MULTIMOD (Laxton, Isard and
others, 1998).

\textsuperscript{18} The current version of the model follows the treatment of prices and wages in the Mark II version
of MULTIMOD. Capacity utilization can differ in the short run from long-run full use of capacity. But the
model includes wages and employment implicitly and hence does not explicitly track unemployment.
Further refinements in the empirical model will include alternative treatments of the dynamics of prices and
wages.
being carried out that applies "learning" ideas to the evolution of expectations. For example, agents can be assumed to learn adaptively in the sense that they make forecasts using forecast functions formulated on the basis of available information but revise those forecast functions over time as new information becomes available. In further research, we ultimately hope to make modifications in our assumed treatment of expectations. For the time being, we report the results with the now-familiar, full model-consistent expectations.

When using the empirical model, we first develop one or more model-consistent, steady-state baseline solutions for the evolution of the ZZ and US economies. For transparency, both economies are assumed to follow identical paths and exhibit identical behavior along these steady-state baselines. Hence the baseline exchange rate is constant over time at unity and the trade balances, current-account balances, and net-foreign-asset positions in the baseline are all constant at zero. Baseline solutions for the model typically assume that productivity growth occurs at a constant rate. Baseline steady-state rates of inflation are likewise assumed constant.

In most baselines constructed so far, the elasticity of substitution in the CES production function has been set at unity (the Cobb-Douglas case). Our preferred value for the EIS in consumption is 0.5, but we have studied values as low as 0.3 and as high as unity (logarithmic utility). The fraction of consumers modeled as borrowing-

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19 See, for example, Evans and Honkapohja (2001) and references cited there.

20 Slightly different baseline solutions are needed depending on the assumptions made for the total generosity of child support (c1 and c2 in equation (5) and hence the baseline path of Cj(t) relative to baseline C(t) in (1)), the composition of child support between basic-needs and endogenous, time-varying components, and the distribution of child support across different ages of parent cohorts (the values of w1 and w2 in equation (7)). Baseline solutions of the model also depend on how the public pension system is implemented -- for example, whether the pension trust fund is continuously balanced or can become unbalanced -- and how baseline values are set for the pension tax rate τss or the pension benefit rate βss.

For an illustrative elderly benefit rate in baselines, we have often used a value of 0.32; that rate is roughly the order of magnitude of the average size of benefits relative to average wages in both the Japanese and U.S. public pension systems. If the pension trust fund in the baseline is assumed to be kept exogenously fixed at zero and the elderly benefit rate is 0.32, the baseline pension tax rate consistent with those assumptions typically falls into the range 0.12 to 0.14. This pension tax rate also is broadly in line with actual experiences in the Japanese and U.S. public pension systems.

21 For further discussion, see Appendix 2.
constrained is usually set somewhere between zero and one third. For expositional simplicity, we have assumed that central banks use a money-targeting reaction function in setting the short-term nominal interest rate. We use values for the coefficients in the governments' tax-rate reaction functions that cause government debts in shock simulations to return fairly promptly to their target, baseline paths.

The fertility (child birth) rate, the child mortality rate, and the adult mortality rate are the key exogenous demographic variables in the model. Typical baselines have these key demographic rates set at constant values. When studying demographic shifts, we run shock simulations in the model, perturbing the paths of one or more of these exogenous demographic rates, and then compare the resulting shock values of endogenous variables with their baseline values.

3. Simulating a Decline in Fertility With and Without Youth Dependency

A. Baseline Solutions and Specification of Illustrative Shocks

The initial baselines in this paper assume that in both economies the child birth rate has a constant value of 0.02504 (the new child cohort each year being slightly more than 2-1/2 percent of the adult population that year). The child mortality and adult mortality rates are constant at, respectively, .0075 and .015; the adult mortality rate implies that individuals at age 19 entering the workforce expect to live roughly another 67 years. These baseline assumptions about fertility and child mortality have the consequence that youths enter the baseline adult labor force at the constant rate .02 (2 percent per year), which in turn results in the total population growing slowly in the baseline at the constant rate .005 (1/2 percent per year). Labor productivity and the price level both grow in the baselines at 1/2 percent per year. Consumers' intertemporal elasticity of substitution is set at 0.5. One third of consumers are assumed to be borrowing constrained and therefore consume only out of current-period income rather than smoothing intertemporally.

Our analysis in this paper seeks to isolate the effects of demographic shifts with and without youth dependency taken into account. Thus two slightly different baseline solutions for the model are prepared. For shorthand, we label these as "CV"-- a baseline
where children and the "V" version of the pension system are both included -- and "0V"-- an otherwise similar baseline except that children are not included.

For the CV case, we use a set of benchmark assumptions about child support. The total generosity of baseline individual child support is equivalent to one half the amount of the average for baseline individual adult consumption. Of this amount, two fifths is a fixed, basic-needs component; the remaining three fifths is time-varying and dependent on variation in baseline adult consumption. The coefficients $\omega_1$ and $\omega_2$ in equation (7) are set at estimated values of, respectively, 0.084 and 0.102, resulting in an age distribution of child support across adult parents that is hump-shaped and concentrated most heavily on adults in their late twenties.

What we label as the V variant of the pension system assumes a continuous balance between pension tax revenues and pension benefit payments. Thus for every period $PTFGAP(t)$ -- see equation (28) in Appendix 1 -- is always strictly zero. The V variant keeps the pension trust fund continuously balanced by a combination of adjustments in $\tau_{SS}$ and $\beta_{SS}$ (rather than by adjustments exclusively in one or the other). Variations in $\tau_{SS}$ and $\beta_{SS}$ are required when shocks are applied to the CV and 0V cases; in the CV and OV baselines themselves, however, $\tau_{SS}$ and $\beta_{SS}$ remain constant at values of, respectively, 0.1276 and 0.32.

To illustrate the properties of the model in responding to demographic shifts, we focus on a shock simulation in which the fertility rate declines sharply, remains at a low level, and then eventually recovers part of its earlier decline. This shock results for several decades in a negative growth rate for the population as a whole. We select this

\[c_1 = \frac{C(t)}{N(t)} \quad \text{and \ varies \ over \ time \ only \ with \ growth \ of \ productivity; \ the \ coefficient} \quad c_2 \quad \text{is \ set \ at \ 0.3.}\]

\[\text{For \ this \ V \ variant,} \quad \tau_{SS} \quad \text{is \ adjusted \ upward \ by \ enough \ to \ offset \ 50 \ cents \ of \ any \ trust-fund \ deficit \ in \ the \ previous \ period \ and} \quad \beta_{SS} \quad \text{is \ adjusted \ downward \ by \ enough \ to \ offset \ the \ other \ 50 \ cents \ of \ the \ previous \ period’s \ deficit \ (and \ vice versa \ for \ surpluses).}\]
illustrative shock for study in part because it is roughly analogous to the recent and prospective demographic experience of Japan.\(^{24}\)

In a brief first analysis, we assume that this fertility decline occurs identically in both the ZZ and US regions. When a shock is identical in both regions, the model produces identical simulation paths for both economies. External-sector balances remain at zero and the exchange rate remains unchanged at its baseline value of unity. In effect, each economy behaves as though it were completely closed, which is of course true for the world as a whole (hence we refer to this case as a "closed-economy" simulation). We initially focus on this "symmetric" case because it facilitates interpretation of the most basic, domestic consequences of the shock and because it serves as a benchmark for analytical interpretation of open-economy cases. We then devote most of our analysis to an "asymmetric" (country-specific) variant of the shock. For that open-economy case, we impose a sharp decline and eventual partial recovery in fertility only on the ZZ economy. The US economy, in contrast, is assumed to experience a much milder decline in fertility.

The demographic assumptions embodied in our illustrative shocks are shown graphically in Figure 2. In the **symmetric** case (fatter curves in the figure), the child birth rates in both economies fall persistently from their baseline level of roughly 2.5 percent to about 1.0 percent over a period of some 5-1/2 decades; they remain at that low rate over the next three decades; then they gradually rise back to a new level of about 1.7 percent; they then remain stationary at that level thereafter. The adult mortality rates and the child mortality rates in both economies remain unchanged throughout at their baseline levels. The consequences of the fertility decline for the growth rates of the total population are shown in the bottom part of the figure (fatter curve). Growth rates for the total populations reflect the pattern in the birth rates. Population growth becomes negative by the third decade of the simulation and falls all the way to a negative rate of 1/2 percent per year during the period when the birth rates are at their lowest point.\(^{25}\) As

\(^{24}\) The major decline in fertility in Japan took place in the second half of the 20th century. Part of our illustrative shock in this paper is motivated by Japan's past fertility experience and part by projections of its demographic future.

\(^{25}\) This path for population growth is broadly analogous to that projected by the Japan National Institute of Population and Social Security Research, Ministry of Health and Welfare, medium variant
the birth rates then recover, the population growth rates gradually become less negative, eventually reaching a new steady-state rate of zero as the birth rates level out at their new steady-state rates.\textsuperscript{26}

The asymmetric, open-economy variant of the shock is also illustrated in Figure 2. The paths for the child birth rate and population growth rates in the ZZ economy are identical with the paths in the global, symmetric case. The asymmetry comes in the treatment of the US economy. The US birth rate and hence the US population growth rate start from the same baseline rates as in ZZ but then decline much more slowly. The US rates eventually reach the same steady-state rates as those in the ZZ economy, resulting ultimately in a stationary population in the US as well as in ZZ. Yet the US rates do not follow a U-shaped demographic pattern of sharp fall and then partial recovery; rather, they decline gradually and monotonically. For the asymmetric case, therefore, the fertility decline in the US is much milder.

The assumed fertility declines have major consequences for all endogenous demographic variables. The effects on youth dependency and elderly dependency are graphed in Figure 3. For the asymmetric shock, the ratio of children to the total population in ZZ shows a pronounced decline in the early decades. Once the ZZ child birth rate has stopped falling further and eventually begins to recover, the ZZ youth-population ratio eventually recovers; the eventual new steady-state level for the youth ratio, however, is well below its initial level before the fertility decline began. The ratio of elderly to the total population in ZZ at first rises only slowly, but subsequently a pronounced aging of the population occurs rapidly. From a baseline value of some 28-1/2 percent of the population, elderly individuals ultimately reach a peak plateau of nearly 52 percent of the population; after that peak is reached, the elderly ratio falls back substantially, eventually settling at about 39 percent of the population (significantly

\textsuperscript{26} The growth rates of the adult population lag behind the growth rates of the total population by 18 years. For the first few years of the shock, the adult populations thus continue to grow slightly even though new births of children and hence the growth rates of the total populations are falling sharply. The shock is constructed so that the economies in the long run ultimately settle into a new steady state where the total populations and their distribution by age are stationary.
above its initial baseline level, but well below the intermittent peak reached when the
demographic shock has its greatest effect on the relative numbers of elderly). The two
ratios for the US economy change gradually and monotonically from initial values to the
new ultimate steady-state values characterizing both economies. When the shock is
global and symmetric, as in Figures 2 and 4, both countries’ demographic variables
follow identical paths (fatter curves in the figures).

For the symmetric and asymmetric versions of the assumed fertility declines,
Figure 4 plots the evolutions of the levels of adult populations and effective labor forces.
During the early years of the fertility decline, the adult populations and the effective labor
forces continue to increase at the baseline positive rate of population growth.27 Even
when the total population begins to decline (in the global shock, and for ZZ alone in the
asymmetric shock), the adult population and effective labor force continue to increase.
The increases in the effective labor force reflect the fact that the number of young
workers, who are less productive, decline relative to the number of older, higher-
productivity workers. Then as the demographic shock passes into the ranks of the
highest-productivity workers, the effective labor force (both countries in the global
shock, and ZZ alone for the asymmetric shock) begins a protracted, sharp decline -- at a
rate much steeper than that of the adult population as a whole. The decline in the
effective labor force is eventually reversed as the proportion of youth in the economy
rises again and ultimately stabilizes at its baseline level. For the asymmetric case, the US
population and effective labor force behave very differently from the ZZ variables. After
the demographic shock has passed completely through the entire age distribution and the
economic effects are fully worked out, for both the global and the asymmetric shocks the
labor forces, adult populations, and total populations ultimately settle to stationary levels
(the eventual steady-state growth rates being zero).

The time paths of variables in the simulation charts that follow are shown as
deviations from the baseline solution of the model; the units of the deviations are
specified along the vertical axes of the charts. If a variable has a value of zero in a figure,

27 The initial baseline level of the effective labor force is some 1.6 times greater than the level of the
total adult population, reflecting the calibration of the labor force with its incorporation of the age-specific
relative productivities of different aged workers. The level of the effective labor force in the model
represents, in effect, the number of labor "efficiency units," not the total number of workers.
at that point the variable is unchanged from its baseline path. Because demographic shocks have consequences over very long periods, our model simulations are carried out over long horizons, typically for at least 300 years; the charts report the results over those long periods.

B. **Symmetric Shock (Closed Economy)**

To gain intuition about how demographic forces influence macroeconomic outcomes, consider first the closed-economy case where the fertility-decline shock occurs identically in both countries.

The interaction of the effective labor force with adult transfers to support child consumption critically influences the model's macroeconomic dynamic behavior. When individuals first enter the labor force, they have relatively low productivity and are relatively low savers. Then as younger workers age, gain experience, and have higher productivity, they in effect ascend the left side of the hump of the economy's age-earning profile. Individuals reach their years of peak earnings and high savings when they are in their forties and fifties. Eventually, they start to descend the right side of the humped age-earning profile, and consequently their labor incomes and saving decline. At that point, their consumption must be increasingly financed out of their privately accumulated financial wealth (apart from pension transfers from the government). As demographic shocks pass through the age-earning profile, the dynamic effects of the demographic movements, significant in themselves, get still further amplified.

These dynamics become even more pronounced and significant when the model incorporates the consumption of children. During the period when the fertility decline in the illustrative shock is producing a marked reduction in the youth population, the adults providing children with support have progressively *smaller* transfer payments to make to children. The demographic change thus frees up resources relative to the situation where child-support transfers are absent. The way transfers to children are distributed among adult cohorts also interacts directly with the age-earning-profile effects.

As the demographic shock moves through the populations of the two economies, the levels of the adult population and the effective labor force become much smaller than they otherwise would have been ("global, symmetric" curves in Figure 4). The economy-
wide aggregate levels of human wealth, financial wealth, output, consumption, and the aggregate capital stock all are accordingly expected to decline to markedly lower levels.\textsuperscript{28}

The presence or absence in the model of youth dependency importantly determines the levels of the real interest rate and the capital-output ratio. Both in the initial baselines and throughout the shock simulations, the level of the real interest rate must be higher in a simulation with than a simulation without child consumption. More generally, the greater the generosity of child support, the higher must be the real interest rate. Other things being equal, the capital stock and economy-wide output and consumption are lower when children must be supported, as adults have to set aside more resources to cover the needs of children and hence have smaller savings, leading to less capital accumulation and hence to lower output and consumption per adult. The real interest rate (reflecting the marginal product of capital) will therefore be higher, the higher are child needs.

The dynamic pattern of real interest rate movements through time is shown in Figure 5. The real interest rate declines as the effective labor force declines; the effective labor force is lower relative to the capital stock, and hence the marginal product of capital must fall. When the effective labor force eventually recovers, the real interest rate also recovers. In the longest run, after the capital stock is again high relative to the labor force, the real interest rate then gradually moves down toward its final steady-state value.

Although the dynamic pattern of the real interest rate is broadly similar with and without youth dependency, the interest-rate movements in the medium and long runs are more pronounced when youth dependents are modeled. This greater amplitude when child support is present is due to the freeing up of resources for adults when the numbers of children are fewer. More resources for the consumption and saving of adults means that there is less need to deplete the capital stock. The capital stock relative to the effective labor force is higher in the model when child support is taken into account,

\textsuperscript{28} Economy-wide aggregates for macroeconomic variables cannot be readily used to make normative or welfare judgments about the consequences of demographic shocks such as a decline in fertility. Per capita or per adult measures of macroeconomic variables are likely to be a more useful focus for normative comparisons of pre-shock and post-shock outcomes.
which necessitates a more pronounced decline in the marginal product of capital, and hence in the real interest rate.

Human wealth, financial wealth, and adult consumption are three of the most important real macroeconomic variables in the model. The simulated movements in two of these variables, measured in per-adult terms, are shown in Figures 6 and 7. For example, Figure 7 shows percentage deviations from baseline of the ratio of economy-wide adult consumption to the number of individuals in the adult population. As before, the curves represent the benchmark cases with and without youth dependency.

In the initial decades of the shock when the youth dependency ratio is falling sharply, financial wealth per adult and human wealth per adult rise relative to baseline. The larger the assumed generosity of child support, the larger are the rises in human wealth and financial wealth relative to what would have occurred without the decline in the numbers of children. The increases in financial wealth are explained by higher disposable incomes and savings reflecting the smaller support payments to children to be made in the shorter-run future. In the medium run, as the decline in fertility is reversed and the child population again increases, financial wealth and human wealth per adult fall steadily relative to baseline until they are well below baseline levels. Then over the long run they rise back toward baseline. Simulation paths for consumption per adult likewise show a rise in the shorter run, a sustained fall in the medium run, and an eventual rise back toward baseline. The cyclical ups and downs in consumption per adult are more pronounced when adults make transfers to support child consumption. The broad pattern of cyclical movement for each of the variables human wealth, financial wealth, and consumption is qualitatively similar to the cyclical pattern of movements in the effective labor force.

C. Asymmetric Shocks (Open Economy)

The preceding brief summary of the consequences of a symmetric shock, identical for both economies, identifies several key features of how demographic events can influence macroeconomic outcomes. Those features continue to be important, of course, when demographic shifts are asymmetric. But the consequences of demographic trends are significantly modified by the openness of national economies. Our primary interest is
in situations where one part of the world economy experiences different shocks and different outcomes from those occurring elsewhere. In the remainder of the paper, we thus study the case in which the large fertility decline and subsequent partial recovery are assumed to occur in the ZZ economy while more gradual demographic changes occur in the US economy.

Our asymmetric shock assumes that the ZZ child birth rate follows the same path as for the symmetric global shock while the US child birth rate declines only slowly and gradually to a new steady state level (identical to the level to which the ZZ child birth rate eventually recovers after its sharp decline in the first fifty years -- see again Figure 2). The growth rate of the total population in the US economy therefore never turns negative, but merely gradually declines from the initial baseline rate to the eventual long-run steady-state value of zero (the populations in both economies eventually becoming stationary). In contrast to the cyclical changes in the ZZ youth and elderly dependency ratios, the US experiences only a smooth, gradual decline in its youth ratio and a steady, gradual increase in its elderly ratio (Figure 3). Analogously, while the ZZ working population and effective labor force exhibit large declines and marked cyclical movements, the US adult population rises slowly (at a decreasing rate) to its new stationary level (never declining) and the US effective labor force has only a very modest decline after a long, sustained, but modest increase (Figure 4).

As in Figures 5 through 7, Figures 8 through 15 that follow compare simulations with and without youth dependency (the benchmark CV and OV cases). For each of the two model simulations, however, we now show curves for both the ZZ and US since of course the outcomes for each economy are now quite different. The ZZ variables in the charts are plotted with thicker, more prominent curves than those for the corresponding US variables.

Deviations of the real interest rates from baseline for the ZZ and US economies are plotted in Figure 8. During the first two decades of the shock, before the adult population and the labor force have yet experienced the consequences of the fertility decline, the real interest rates change relatively little. Thereafter, the cumulating sharp fall in the ZZ effective labor force leads to a progressively larger fall in the ZZ real interest rate. The extent of the fall in the ZZ real interest rate, however, is somewhat
damped because of the ZZ economy’s openness to the rest of the world. The real interest rate in the US shows an analogous, but much smaller, decline than that in the ZZ economy. To Nominal interest rates in the two economies follow qualitatively similar paths, but with the ZZ interest rate falling well below that in the US.

As with the global symmetric shock (Figure 5), and for the same reasons, interest-rate movements in the medium and long runs are more pronounced when youth dependents are taken into account rather than assumed absent.

Movements in human wealth and financial wealth are conditioned in both economies by the fundamental demographic forces discussed above. But with the much larger demographic shock buffeting the ZZ economy, the relative positions of the two economies are very different. For example, as seen in Figure 9, ZZ financial wealth per adult in the shorter run rises relative to baseline even more strongly than in the closed-economy case and then diminishes much less strongly in the medium and longer runs. In marked contrast, US financial wealth per adult does not rise in the short run and falls sharply in the medium and long runs. The differences between the two economies are larger when the model allows for the effects of youth dependency than when it does not. The differences in saving behavior and hence in financial wealth between the ZZ and US economies are attributable not merely to their different-sized demographic shocks but also to major effects working through the exchange rate and external balances.

The model enforces a variant of the uncovered interest parity condition. Hence a sizable interest differential between the two economies -- once it opens after the initial years of the shock -- puts strong pressure on the real and nominal exchange rates (Figure 10). The ZZ currency begins a sustained appreciation, first in nominal terms, then with a lag in real terms. By the ninth or tenth decade of the shock, both the nominal and real values of the ZZ currency have appreciated by a large amount. The real exchange value of the ZZ currency appreciates substantially further over the next several decades, reaching a peak appreciation of more than 80 percent before reversing and falling back. In the new long-run steady state, both the nominal and the real exchange rates settle at levels very much higher than in the baseline solution. Allowing for youth dependency in

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29 Nominal interest rates in the two economies follow qualitatively similar paths, but with the ZZ interest rate falling well below that in the US.

30 An upward movement in the exchange rate in the model represents an appreciation of the ZZ currency (a depreciation of the US currency).
the model increases the degree to which the ZZ currency appreciates in the medium and long runs.

To understand why the asymmetric shock results in a real exchange rate permanently higher in the ultimate than in the initial steady state, recall that the fertility decline is transitory in terms of demographic rates of growth but has permanent effects on the levels of demographic and macroeconomic variables. In particular, as the ZZ fertility rate falls progressively below baseline during the shock, the ZZ population and effective labor force fall further and further below baseline. The ratio of the ZZ to the US effective labor force and the ratio of the two countries' populations fall correspondingly. Once the decline in the ZZ fertility rate is partially reversed, eventually the ZZ and US demographic rates of growth again become equal. The ratio of the ZZ to the US effective labor force, however, remains permanently smaller (see Figure 4). In the medium run and in the new long-run steady state, therefore, the quantity of ZZ-produced goods is markedly smaller than the quantity of US-produced goods. Given unchanged preferences in each economy for the two types of goods, relative prices in the world economy must change. A permanent real appreciation of the ZZ currency (an improvement in the ZZ economy's real terms of trade) is an integral part of the required change in relative prices.

Large changes in exchange rates generate powerful expenditure-switching incentives between the two economies. Thus by the fourth decade of the shock, the ZZ economy begins to import substantially more of the now relatively cheap goods produced in the US. ZZ exports to the US relative to baseline are inhibited by the appreciation of the ZZ currency. The ratios of real imports to real GDP in the two economies are shown in Figure 11. After the underlying demographic variables have begun to return to their ultimate steady-state values, the ZZ import ratio has risen to more than 15 percentage points above its baseline value (it ultimately falls back to some 9-11 percentage points above baseline). The US import ratio reaches a level some 7 percentage points below baseline (ultimately coming to rest around 5 percentage points below baseline). As would be expected from the differential effects on the real and nominal exchange rates, taking youth dependency into account increases the degree to which the ZZ economy raises imports and the US economy cuts back on imports.
For the initial decades of the shock, the ZZ real trade balance relative to real GDP changes little. As the shock progresses thereafter, however, the expenditure-switching effects permit the ZZ economy to run a larger and larger deficit on real trade account (Figure 12). This net import of real resources from abroad provides a cushion of support to the ZZ economy that permits it to sustain a significantly higher level of consumption than would otherwise be possible (discussed further below).

The medium-run trade deficit of the ZZ economy is not associated with a deficit on current account. Quite the contrary. The ZZ economy not only imports more from abroad. The ZZ economy also saves more relative to baseline so that ZZ financial wealth rises relative to baseline (Figure 9). And a substantial fraction of that higher financial wealth is invested abroad at the higher interest rates available abroad. Hence the ZZ economy over the medium run starts to earn a higher flow of investment income from abroad. The net investment income payments received are more than enough to offset the increased deficit on trade account, with the result that the ZZ economy in the medium run begins to experience a significant current-account surplus (Figure 13). The surplus reaches a peak during the eighth decade of the shock. Interestingly, the current-account surplus thereafter falls and even turns to a small deficit for several decades as the two economies move toward their new long-run steady states. Eventually, in the very long run the ratio of the ZZ current balance to nominal GDP settles at a small surplus ratio.31

The net foreign asset positions of the two economies, shown relative to nominal GDPs in Figure 14, are the integral over time of the current-account imbalances. The ZZ economy -- despite the large shock it experiences, causing the economy's output and aggregate consumption to fall far below the levels that would have been observed without the shock -- accumulates a large positive net foreign asset position, on which it earns a sizable return.

The openness of the ZZ and US economies thus decisively influences the macroeconomic consequences of the demographic shocks. Domestic variables in both economies are strongly influenced by cross-border transactions. Because of the openness

31 Note that the vertical scales for Figures 12 and 13, chosen to make it easier to identify the dynamic movements of the curves, are dissimilar; the range of the vertical distance in Figure 12 is 30 percentage points whereas it is only 6 percentage points in Figure 13.
of the economy, ZZ domestic variables are partly cushioned from the full impacts of the large ZZ fertility decline. As a counterpart, US variables are adversely buffeted by the larger demographic shock emanating from the ZZ economy. An important component of these cushioning and buffeting effects is associated with the changes in exchange rates. The permanent appreciation in the real value of the ZZ currency enables the ZZ economy to enjoy a large permanent improvement in its real terms of trade with the rest of the world. The opposite effect, a deterioration in real terms of trade, contributes to the adverse effects on the US economy.

The effects on the real exchange rate, trade balances, current-account balances, and net-foreign-asset positions of the two economies are bigger if the analysis takes into account the economic effects of children. The positive effects on ZZ saving and financial wealth resulting from the fertility decline are even larger when the analysis recognizes that resources are freed up because support payments to children are smaller. A fraction of the incrementally freed resources from lower child consumption are saved rather than consumed. The ZZ currency appreciates by a larger amount. The associated net capital flows permit the ZZ current-account surplus to be larger by the medium run than it would otherwise be without youth dependency taken into account. The resources freed up by declining numbers of children, partly invested abroad, increase the cushioning effects on the ZZ economy from its openness to the rest of the world. Similarly, the effects of youth dependency increase the degree to which the US economy is influenced by spillovers from the ZZ economy.

Figure 15, which plots the percentage deviations from baseline for adult consumption per adult in both economies, provides a suggestive indication of the relative welfare effects in the two countries. Notwithstanding the fact that the demographic shock in the ZZ economy is much larger than in the US economy, ZZ per adult consumption is actually higher than per adult consumption in the US. The difference is sizable in the initial decades of the shock; it is much higher in the new long-run steady state.

When thinking about welfare developments, it is essential not to confuse aggregate levels of variables with their per-adult (or per-capita) values. Nothing can prevent the large ZZ demographic shock from having major negative effects on ZZ aggregate output and consumption. The ZZ paths for aggregate consumption and
aggregate real GDP must accordingly fall much further below baseline than those for the US. Yet the ZZ paths for those aggregate variables are significantly above the paths that would be experienced in the hypothetical case where the ZZ economy is completely closed and therefore unable to cushion its shock through transactions with the rest of the world. The openness of the economy works to mitigate the size of the negative effects on the aggregates. Moreover, as seen in Figure 15, the cushioning effects are so substantial when measured in per-adult (or per-capita) terms that individual adults in the ZZ economy are significantly better off relative to baseline than individual adults in the US economy. The US population, aggregate US real GDP, and aggregate US consumption are all at higher levels than in the no-shock baseline. Despite that fact, US adult consumption per adult is markedly lower than in the baseline.32


The preceding discussion emphasizes that analytical conclusions about the macroeconomic effects of demographic change, not least the cross-border spillovers, can be significantly influenced by whether youth dependency is modeled or not. The particular features of the modeling of youth dependency condition the quantitative size of its effects. In this section, we briefly report the results of sensitivity analysis along three dimensions: varying the total level of generosity of child support, varying the composition of child support between basic-needs and endogenous components, and varying the age distribution among adult cohorts of the burden of child support.

We discuss results only for the asymmetric, open-economy shock where the ZZ economy experiences a much larger decline in fertility than the US economy. Except where noted explicitly, the parameters used in the analysis are the same as those underlying the simulations discussed in section 3.

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32 When the analysis ignores youth dependency and child consumption, the adverse spillover effects on US per-adult consumption are greater than when the analysis acknowledges youth dependency.
A. Generosity of Child Support

It is implicit in the analysis of section 3 that the assumed generosity of child support has an important quantitative influence on the size of effects. We now show this conclusion explicitly by reporting a bracketing range of assumptions about the magnitude of $c_j(s,t)$ in equation (5) -- and hence $C_j(t)$ in equation (6). In section 3, the level of baseline $c_j(s,t)$ for the CV case was set at 50 percent of the average amount of individual adult consumption in the baseline; of that amount, two fifths was a fixed, basic-needs component while three fifths was time-varying and dependent in simulations on variation in adult consumption. Expressed in terms of equation (5), $c_1$ was fixed as 40 percent of initial baseline per-adult consumption and varied over time only with growth of productivity; the coefficient $c_2$ was set at 0.3. The lower bound for child consumption, self-evidently, is zero (both $c_1$ and $c_2$ set to zero) and the simulation with those assumptions was the other set of curves in Figures 5 through 15.

An intermediate assumption about the generosity of child support between the two cases already considered fixes baseline child consumption at 25 percent of baseline adult consumption. A high-end assumption sets the total of baseline child consumption at 75 percent of baseline adult consumption. To contrast these additional cases with the earlier simulations, it is convenient to continue with the benchmark assumption that, for the given total of child consumption, two fifths takes the form of basic needs while three fifths endogenously varies with adult consumption.

As the level of child support is raised from zero (no economic effects of children in the model) to larger and larger fractions of adult consumption, the effects of the fertility-decline shock on domestic macroeconomic variables become larger. The intuition behind this generalization is straightforward: the higher is the ratio of child to adult consumption, the larger is the amount of real resources freed up by a decline in the number of children (those resources no longer being devoted to child consumption). Hence real interest rates must fall further, capital-output ratios must move more, and the

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33 Cutler, Poterba, Sheiner and Summers (1990) used an estimate of child support relative to adult consumption generated by Lazear and Michael (1980), then added expenditures on public education to the Lazear-Michael estimate, and came up with a calculation that child consumption is approximately 72% of adult consumption.
effects on per-adult human wealth, financial wealth, and consumption are all bigger. The
dependence of the size of real interest rate movements on the generosity of child support,
for example, is shown in Figure 16.

An analogous generalization applies to external-sector variables. The higher is
child consumption relative to adult consumption, the more the ZZ currency appreciates in
real and nominal terms and the more the ZZ economy can afford to run a larger real trade
deficit because its net investment income from abroad is larger. With higher rather than
low or zero child consumption, the ZZ current-balance surplus is bigger and the ZZ net
foreign asset position is more positive. Figure 17, plotting the ZZ ratio of the current-
account balance to nominal GDP, illustrates of the size of these differential external-
sector effects.

As expected, the level at which adults are assumed to support child consumption
powerfully influences their own consumption paths. This conclusion is highlighted in
Figure 18. When a fertility decline occurs, a high level of adult support for children's
consumption frees up large amounts of resources for the adults, some part of which raises
adult consumption immediately but another part of which is saved, building up adults' income and consumption in the future. For ZZ adults, the quantitative differences are
considerable among simulations with alternative levels of child support. The differences
become large by the medium run, determining whether per-adult consumption will fall
substantially below baseline (child support assumed completely absent) or can hover
above baseline (child support assumed at some one third or more of adult consumption).
The differences remain very large over the longest run.

B. Invariant Basic-Needs versus Endogenous Time-Varying Child Support

Our earliest efforts to incorporate youth dependency into macroeconomic models
made the simplifying assumption that all child consumption was "basic needs," fixed at
some proportion of initial baseline adult consumption. In subsequent work, we
developed the specification in equation (5) that, for any given total level of child support,
divides the total between a time-invariant basic-needs component and a discretionary
component that is time varying and endogenously dependent on adult consumption. As
explained earlier, this richer specification nests several alternative ways to model child consumption.

We illustrate the alternatives in what follows by contrasting the benchmark-case simulation used in the earlier discussion with two additional simulations, one in which the entire amount of child consumption is basic needs and the other in which the entire amount varies endogenously with adult consumption. These additional simulations are extremes, bracketing the range of possibilities. For the purposes of this comparison, the total generosity of child support for all three simulations is set at 50 percent of baseline adult consumption.\(^{34}\)

To illustrate the sensitivity of results to alternative compositions of child support between the basic-needs and the endogenous-to-adult-consumption components, we report results for the simulations for just three variables: ZZ financial wealth per adult (Figure 19), the ZZ ratio of net foreign assets to nominal GDP (Figure 20), and ZZ real adult consumption per adult (Figure 21). The differential consequences for these variables are similar to those for most other variables in the model.

As the time-varying endogenous component of child consumption increases relative to the basic-needs component, the sizes of effects on most variables relative to baseline are diminished, with the differences tending to be small in the shorter run but becoming more pronounced in the medium and then longer runs. The largest within-country differences are for variables such as per-adult financial wealth (Figure 19) and the private savings ratio. The differences in effects are especially noticeable in the exchange rate (real and nominal) and in the external-sector variables (for example, Figure 20), again primarily for the medium and long runs.

If one interprets per-adult consumption (Figure 21) as a rough measure of welfare effects, the largest favorable effects for the ZZ economy are associated with the assumption that all child consumption is basic needs and the smallest favorable effects

\(^{34}\) In the benchmark-case simulation, as described earlier, \(c_1\) in equation (5) is set as 0.2 times initial baseline adult consumption and \(c_2\) is set at 0.3. In the simulation where child consumption is all basic needs, \(c_1\) is set at 0.5 times initial baseline adult consumption and \(c_2 = 0\). For the simulation in which child consumption has no basic-needs component, \(c_1 = 0\) and \(c_2\) is set at 0.5.
for the opposite extreme assumption. The benchmark case between the extremes, for which the assumptions seem a priori more plausible, yields intermediate results.

The intuition for these results is again straightforward. A reduction in the relative importance of the basic-needs component of child consumption should moderate the effects on macroeconomic variables because a fertility decline "releases" the largest amount of resources to adults for other uses in the case where all child consumption is assumed to take the form of basic needs.

Because our initial results for fertility declines and youth dependency (reported in the 2001 paper) employed the specification that child consumption was entirely the basic-needs component, those early results tended slightly to overestimate the consequences of youth dependency. As Figures 19 through 21 reveal, the extent of overestimation was relatively minor, especially for shorter-run effects. Nonetheless, we feel more comfortable with estimates that assume a significant fraction of child consumption varies endogenously with adult consumption. Pending the availability of better underlying empirical evidence about the composition of child consumption and parents' child-support decisions, our future research will continue to postulate that the basic-needs component of child consumption is only half or less of the total.

C. Alternative Distributions Among Adults of Child Support

Our initial efforts to study the effects of youth dependency did not empirically implement the richer specification shown in equation (7) for the age distribution of child support among adult cohorts. The initial analysis used either the extreme assumption that the age distribution is completely flat (every adult cohort makes the same contribution regardless of age) or some variant of the artificial assumption that the youngest adult cohort makes the largest contribution with contributions by older adult cohorts declining exponentially with their age. Equation (7), as explained earlier, nests those earlier simplified cases. With a suitable choice of the \( \omega_1 \) and \( \omega_2 \) parameters, however, equation (7) lends itself to a more plausible hump-shaped pattern for the age distribution among adults of child support.

We have conducted a variety of sensitivity tests about this age distribution. These tests suggest that a hump-shaped profile for the age distribution, in addition to being
more plausible in its own right, also yields effects on macroeconomic variables that appear more reasonable. The effects on key variables from hump-shaped profiles tend to fall in between those from the extreme of a flat contribution from every adult and the extreme of weights that decline exponentially with age with the largest contribution made by the youngest adult cohort. Typically, the effects on macroeconomic variables follow a similar pattern qualitatively, but with the smallest effects associated with the flat-distribution assumption, the largest effects with the exponentially-declining weights assumption, and the effects with the hump-shaped profile falling in between. To save space, we do not report such simulations here. In future work we plan to restrict attention to one variant or another of the hump-shaped profile (cases in which $\omega_2 \geq \omega_1 \geq 0$).

In a further set of sensitivity tests, we have experimented with choices for the $\omega_1$ and $\omega_2$ parameters that leave unchanged the benchmark identification of which adult cohorts bear the largest relative size of the burden of child support (in our estimates, adults in the 28-30 year age range) but raise or lower their relative share of the total burden. In effect, these sensitivity tests raise or lower the peak of the hump-shaped distribution without shifting the peak to younger or older adults. So long as there is already a modest-sized hump in the distribution (which is the case for our empirical estimates), it seems to make relatively little difference to the simulation results when the peak is somewhat raised or somewhat lowered. Because the results of these tests show such small differences, we omit an account of these tests, too, from this paper.

Differences among assumptions matter more, however, if we choose parameters that shift the peak of the hump profile to the left or to the right rather than up or down (for any given total generosity of the child-support burden). Shifting the peak of the hump to the left pushes more of the relative burden on to adults younger than ages 28-30. Shifting the peak to the right increases the relative contributions of older adults. Speaking loosely, as the peak of the hump shifts rightwards, grandparents and older middle-aged parents provide relatively more of the child support.

Figures 22, 23, and 24 give the flavor of the differential consequences of shifting the age distribution to the right and the left. Each of these figures reports the curves for a variable for both countries; the curves toward the top of each figure pertain to the ZZ variable; US curves are at the bottom. When younger adults are assumed to take on more
of the burden of child support relative to the benchmark case, the effects on private saving and financial wealth are increased, the exchange rate and external-sector variables are more strongly affected, the ZZ economy builds up a larger net foreign asset position over time, and accordingly ZZ per-adult consumption rises further above baseline. If the burden of child support falls more on middle-aged parents and grandparents, the effects are dampened relative to those in the benchmark case. Differences due to varying assumptions about the age distribution of child support tend to be smaller in the short run, but are more sizable over the medium and long runs as the differential effects on saving and external asset positions cumulate to larger amounts.35

5. Concluding Remarks

The analytical approach developed in the research reported here and in our other papers yields insights into the cross-border dimensions of the macroeconomic consequences of demographic change that are not yet attainable in other approaches. Our primary focus is on exchange rates, external-sector imbalances, international capital flows, and hence spillover effects to other countries.

For an open economy asymmetrically experiencing fertility declines and population aging, negative consequences accompanying the demographic shifts are typically cushioned because the negative effects are shared with the rest of the world. That cushioning and sharing may not be desirable as seen from the perspective of foreigners, but it can produce sizable welfare gains for home residents.

The open-economy dimensions are first-order in importance. Ignoring the powerful macroeconomic effects working through exchange rates and cross-border

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35 The benchmark assumptions used for the hump-shaped profile for the age distribution, based on empirical estimates, are $\omega_1 = 0.084$; $\omega_2 = 0.102$. The parameters in the simulation where younger adults assume more child support are $\omega_1 = 0.1318$; $\omega_2 = 0.160$. For the simulation shifting more of the burden on to older adults, the parameter values are $\omega_1 = 0.070$; $\omega_2 = 0.085$. We initially performed these last sensitivity results using the assumption that all child consumption takes the form of basic needs (to contrast the results with results obtained in our initial research); the versions of Figures 22, 23, and 24 in this draft are for this case; a later draft of the paper will show analogous results for our the intermediate case where basic-needs child consumption is half or less of total child consumption.
transactions is likely to provide a seriously inaccurate assessment of the net impacts of demographic change.

Our research demonstrates that analysis of the macroeconomic consequences of demographic change needs to focus on both ends of the age distribution: *youth dependency* (transfers from adults to children) and *elderly dependency* (transfers from working adults to older adults, especially through public pensions). Introduction of youth dependency into macroeconomic models generates significantly different inferences about the economic behavior of countries linked to the global economy. Models focusing only on the behavior of adult populations, ignoring children and child support, are likely to miss important aspects of how economies respond to demographic changes. Similarly, transfers to elderly dependents through public pension systems, and alternative ways in which the pension systems are managed, have large effects on the evolution of open economies. The effects of public pensions are particularly important if an imbalance in pension tax revenues and pension benefit payments causes large changes in the stock of outstanding government debt.

This paper has concentrated on youth dependency effects. Why does youth dependency have crucial implications for the steady state and the transitional dynamics of the economy? The key point is that the consumption-saving behavior of individual adults who provide in-vivo transfers to children is dramatically different, in theory and in practice, from the behavior of otherwise identical individuals without financial responsibilities for child support. As lower fertility rates reduce the financial burden on existing adults, resources are freed for additional consumption and saving. That reallocation of resources radically changes the transitional dynamics and the ultimate steady state of the economy compared to what it would otherwise be in an analysis that ignores children. Ignoring youth dependency suppresses this major source of macroeconomic effects.

The open-economy repercussions of the key point are, again, first-order important. Youth dependency's implications for consumption and saving generate significant differential effects on exchange rates and external-sector variables. A large fertility decline induces relatively higher saving, part of which goes into increased assets held abroad. Exchange rates, interest rates, the trade balance and the current account in a
well specified macroeconomic model are all powerfully influenced by changes in youth dependency. In turn, large changes in the real terms of trade of a country tend to cause sizable differential feedbacks on its output and consumption. Thus inferences about the evolution of key macroeconomic variables in an open economy -- and judgments about policy implications and economic welfare -- can be critically influenced by whether or not children are explicitly introduced into the analysis.

We further show in this paper that particular choices made about the modeling of child support and its age distribution across adult cohorts can have significant quantitative influences on analytical and policy conclusions. Higher levels of child support and skewness of that support toward invariant basic needs (away from time-varying dependence on adults' own consumption) both tend to magnify the size of macroeconomic effects. Assuming that younger adult cohorts carry a higher (lower) burden of child support also tends to heighten (diminish) the size of macroeconomic effects. Further empirical research should help analysts in forming better judgments about the details of how youth dependency is incorporated in macroeconomic models.

Many challenges remain for research into the interactions between demographic change and macroeconomic outcomes. The effects need to be carefully studied and refined in more disaggregated, empirically realistic multi-country models. Nonetheless, we like to believe that our research is a significant step forward toward a better understanding of the complexities of the demographic transition the world is now facing. Our general approach should be especially useful to other researchers who are analyzing demographic change in economies experiencing large changes in fertility rates and dependency ratios. Researchers aspiring to build macroeconomic models to study demographic shifts in developing nations, for example, have no choice but to confront the analytical challenges of introducing youth dependency into their models.
Appendix 1: Further Background on Analytical Approach

This appendix provides further details about our analytical framework, indicating how we treat population dynamics (for both the adult and youth dependent populations), age-earnings profiles for the labor force, economic linkages between the elderly and working adults through public pension systems, and the consumption decision made by adults.36

A. Population Dynamics and Youth Dependency

Our augmented framework introduces both youth dependency and elderly dependency. Unlike the standard Blanchard-Weil-Yaari framework, individuals enter the economy not as adults but as dependent children. After a period of life as dependents, youths enter the adult population and begin to supply labor in the workforce. After eventually reaching a threshold age, adults become elderly dependents and receive a public pension to supplement their other income flows. Adults have a different, typically higher mortality rate than children. We also allow for the child birth rate and the mortality rates of adults and children to vary through time. To preserve the advantages of the Blanchard-Weil-Yaari framework for aggregation across individuals, however, we maintain that framework's assumptions that mortality rates are age-invariant and that elderly adults gradually instead of discontinuously withdraw their labor from the workforce.37

We denote the size of a child cohort, indexed by $s$ ($s$ is the time of birth) at time $t$, as $J(s,t)$. Correspondingly, $N(s,t)$ is the size of an adult cohort at time $t$. The populations of children and adults are interrelated as both evolve through time.

For children, the initial size of a cohort at the time of birth $s$ is:

$$J(s,s) = b_j(s)N(s), \quad (12)$$

36 A still more detailed and systematic exposition of the theoretical framework is contained in a forthcoming separate background paper.

37 The assumption that mortality rates are age-invariant rather than age-specific departs seriously from reality. See Faruqee (2003) for further discussion of age-specific mortality issues.
where \( b_j(s) \) is the birth (or fertility) rate at time \( s \), expressed as a fraction of the contemporaneous adult population \( N(s) \). Youth dependents face the infant/child mortality rate, \( p_j(t) \), which can vary through time but is the same for all youths regardless of age. The number of survivors from the initial \( J(s,s) \) cohort at some later date \( t \) is given by:

\[
J(s,t) = b_j(s)N(s)e^{\int_s^t p_j(v)dv}.
\]

Aggregation of all child cohorts over the finite range \( \Delta \) of childhood ages and then differentiating, using the survivor formula given by equation (13), yields the following expression for the evolution of the total child population, \( J(t) \):

\[
\dot{J}(t) = b_j(t)N(t) - N(t,t) - p_j(t)J(t),
\]

where \( N(t,t) \) denotes the outflow of youths into the adult population. More specifically, through continuity, the oldest child cohort passes into adulthood:

\[
N(t,t) = J(t - \Delta, t).
\]

This cohort thus also represents the inflow of new adult workers into the economy.

Given a distinct adult mortality rate \( p_n(t) \), the survivor formula and dynamics for adult cohorts are given by:

\[
N(s,t) = N(s,s)e^{\int_s^t p_n(v)dv}.
\]

\[
\dot{N}(t) = N(t,t) - p_n(t)N(t).
\]

Equation (17) shows that the net change in the adult population in each period depends on the inflow of new entrants to the adult population less the numbers of adults that die. The rate at which new youth entrants pass into the adult population can be viewed as an adult “birth” rate \( b_n(t) \equiv N(t,t)/N(t) \). The population growth rate of adults is given by \( n(t) = b_n(t) - p_n(t) \), where the adult "birth" rate \( b_n(t) \) depends on the sequence of child birth rates in earlier periods.

---

\(^{38}\) We parameterize \( \Delta \) to be 18. Children are assumed to be wholly dependent for the first 18 years of their life. At the beginning of their 19th year, they enter adulthood, begin supplying labor input, and no longer receive any support payments from older adults.
Given these population measures, the youth dependency ratio $\delta(t)$ is simply the ratio of the child population to the adult population:

$$\delta(t) = \frac{J(t)}{N(t)}.$$  \hspace{1cm} (18)

Differentiating this expression yields the law of motion for the youth dependency ratio:

$$\dot{\delta}(t) = b_j(t) - [1 + \delta(t)]b_n(t) + \delta(t)[p_n(t) - p_j(t)].$$  \hspace{1cm} (19)

Intuitively, the evolution of youth dependency depends on the comparative rates of growth in the child and adult populations. This differential depends, in turn, on differences in birth and death rates for children and adults (appropriately scaled). Higher fertility rates (i.e., arrival rates of children) tend to raise the youth dependency ratio, while higher child mortality rates tend to lower this ratio, other things equal.

We define an elderly dependency ratio as the proportion of the adult population above a certain threshold age level – indexed by $i(t)$. For example, $i$ might be, say, 47 years of adult life (measured from the moment of entering adulthood at the beginning of the 19th year of life). Given an index value for $i$ that is unchanged through time, the elderly dependency ratio $\phi(t)$ is defined as:

$$\phi(t) = \int_{-\infty}^{i(t)} \frac{N(s,t)}{N(t)} ds. \quad 0 \leq \phi \leq 1.$$  \hspace{1cm} (20)

In an economy with a constant adult “birth” rate, the elderly dependency ratio would also be constant. For the case where the adult birth rate and death rate are time varying, the elderly dependency ratio evolves according to:

\footnote{The growth rate of the child population can be written as:

$$J(t) = [b_j(t) - b_n(t)]/\delta(t) - p_j(t)$$

which is determined not only by the birth and death rates of children but also by their entrance into the labor force—i.e, the adult “birth” rate, $b_n$.}

\footnote{In steady-state, the following population relationships obtain:

$$\overline{b}_n = \overline{p}_n; \overline{b}_j = \overline{p}_n e^{\tau \lambda}; \overline{\delta} = \frac{\overline{b}_j - \overline{p}_n}{\overline{p}_j}.$$}

\footnote{If measured from the year of actual birth, the threshold age would be 65 years. The threshold age does not imply “retirement” per se; as noted above, there is no discontinuity of labor input at the threshold age. Rather, adults older than $i$ still continue to receive some (but gradually declining) labor income even after they reach the age at which they are defined as “elderly.”}
\[ \phi(t) = \frac{N(i(t),t)}{N(t)} - [p_n(t) + n(t)]\phi(t). \] (21)

Intuitively, the change in the elderly dependency ratio is determined by the relative size of new dependents reaching the threshold age -- the first term in (21) -- less the proportion of the elderly who die during the period -- \( p_n(t)\phi(t) \) -- and less a scaling term accounting for growth in the adult population base -- \( n(t)\phi(t) \). 42

**B. Labor Income Profiles**

The adult consumption problem follows the familiar formulation widely used since Blanchard (1985) but with two major modifications: (1) the inclusion of age-earnings profiles to introduce a life-cycle path to labor income, and (2) the explicit inclusion of parent-child transfers in the adult budget constraint. We elaborate on these two issues and other aspects of the consumer’s problem in what follows.

In real-life economies, labor earnings display a hump-shaped pattern across age groups, rising initially as younger workers accumulate experience and seniority, peaking at later middle age, and declining after the peak years of productivity have been passed and workers supply less labor and eventually retire. We account for this life-cycle age-earnings profile by varying effective individual labor input according to age. Moreover, our analytical framework incorporates such a hump-shaped profile in a way that permits a “bottom-up” rather than “top-down” determination of aggregate labor income.

The age-earnings profile is mathematically approximated by specifying the labor input of an individual cohort \( s \) at time \( t \) with three exponential terms:

\[ l(s,t) = \left[ a_1 e^{-\alpha_1(t-s)} + a_2 e^{-\alpha_2(t-s)} + (1-a_1-a_2)e^{-\alpha_3(t-s)} \right] \] (22)

---

42 Because death rates are modeled as age-invariant rather than age-specific, only the adult birth rate matters for the share of elderly dependents in the population. As discussed in Faruqee (2000a, 2000b), the model's assumption that the adult mortality rate is age-invariant has an undesirable consequence: the numbers of elderly in the total population are overestimated relative to the real-life situation in which adult mortality increases as adults become older and older.
Loosely speaking, the first two terms in (22) may be thought of as representing the
decline in an individual cohort's labor supply over time as it ages and (gradually) retires.
The third term can be interpreted as reflecting gains in earnings that accrue with age and
experience. The restriction on the $a_i$ coefficients on the exponential terms (the third of the
coefficients must be equal to $1 - a_i - a_2$) embodies a normalization that the youngest
cohort (for whom $s = t$) has effective labor input equal to unity.

Assuming that differences in earnings are attributable to these age-specific
differences in labor productivity and labor supply, the relative labor earnings of a
particular cohort can be written as:

$$y(s,t) = \left[ \text{wage}(t) \right] l(s,t),$$

(23)

where \text{wage}(t) is the economy's average wage rate which grows over time at the rate of
labor-augmenting technical change. Consequently, individual labor earnings can change
over time because of general growth in labor productivity, assumed to apply uniformly to
all cohorts (i.e., after adjustment for their age-specific relative productivities). The time-
path for individual relative earnings, meanwhile, is assumed to match the cross-sectional
age-earnings pattern observed in the data. Given (22) and (23), the parameters
$a_1, a_2, \alpha_1, \alpha_2, \text{ and } \alpha_3$ are calibrated to match non-linear least squares estimates on cross-
sectional data for age and earnings for Japan and the United States obtained by Faruqee
(2000b). The relative productivity parameters are further specified to be exogenous (i.e.,
fixed over time) in the modeling code.

If (22) and (23) are aggregated over all individual cohorts, aggregate labor income
can be written as:

$$Y(t) = \int_{-\infty}^{t} \text{wage}(t) l(s,t) N(s,t) ds = \text{wage}(t) L(t),$$

(24)

where $L$ is aggregate labor input adjusted for cohort-specific relative productivities. The
definition of labor input for the individual cohort in (22) also permits one to write
aggregate $L$ as the sum of three components $L_1, L_2, \text{ and } L_3$ where each component
reflects an exponential term in (22). Details are in the background paper. Intuitively,
changes in aggregate labor input depend on the effective labor supply of new entrants to
the labor force, the relative productivity experiences and deaths of existing workers, and
the general pace of labor-augmenting technical progress. The specific values of the five
coefficients $a_1, a_2, \alpha_1, \alpha_2$ and $\alpha_3$ in (22) obtained from estimating the age-earnings
profile play a critical role in determining the movements of effective labor supply and the
evolution of human wealth and consumption over time.

C. Public Sector and Pension Scheme

The simplified governments in our analytical framework purchase real goods and
services (typically, an exogenously set fraction of the national output), raise revenue by
taxing the incomes of firms and households, and pay interest on the outstanding stock of
their debts. The government also operates a public pension ("social security") system that
collects pension-tax revenue from workers and makes pension transfer payments to the
elderly.

We implement the public pension systems in a simplified pay-as-you-go (PAYG)
form, identical for both countries. Seen from the perspective of an individual cohort $s$,
the transfer scheme embodied in the public pension system can be summarized as:

$$ptr(s,t) = \begin{cases} 
-\tau_{ss}(t)y(s,t); & s > i(t) \\
\beta_{ss}(t) \frac{Y(t)}{N(t)} - \tau_{ss}(t)y(s,t); & s \leq i(t),
\end{cases}$$

(25)

where $ptr(s,t)$ is the transfer amount at time $t$, $\tau_{ss}(t)$ is the pension tax rate, $\beta_{ss}(t)$ is
the elderly benefit rate, $y(s,t)$ is the individual cohort's real labor income, and $Y(t)/N(t)$ is
average real labor income in the economy. Equation (25) states that younger generations
($s > i(t)$) pay pension taxes into the system and receive no benefits; older agents having
reached the threshold elderly age ($s \leq i(t)$) are "pensioners" and receive a benefit.
Pensioners still receive modest amounts of labor income after they reach age 65, and thus
pay modest pension taxes on that income.43

The aggregate amounts of pension taxes collected and benefits paid by the
government in period $t$ are:

43 Since the effective labor input and labor income of the elderly declines sharply as they age beyond
the threshold of 65 years, the pension taxes paid by the elderly become increasingly negligible as the
individuals grow older and older.
where $PT_{ss}(t)$ and $PB_{ss}(t)$ are the real values of aggregate taxes and aggregate benefits, $Y(t)$ is aggregate real labor income in the economy, $Eld(t)$ is the number of elderly, $N(t)$ is the number of adults, and $\phi(t)$ is the elderly dependency ratio.

If pension contributions and benefits are not identical, there will exist a positive or negative "financing gap" in the pension system reflecting the degree of over- or underfunding of current-period benefit payments. Accordingly, we define a variable for the financing gap (changes in the real value of the "trust fund" for the pension system):

$$PTFGAP(t) = PT_{ss}(t) - PB_{ss}(t).$$

When the pension trust-fund gap is negative, the deficit in the pension system must be financed by changes elsewhere in the government's budget: increases in revenues from income taxes, cuts in government spending on goods and services, or increased government borrowing through additional issuance of government debt.

The budget identity of each fiscal authority in our framework, expressed in real terms, may be written as:

$$gdef(t) = r(t)\frac{B(t)}{P(t)} + [g(t) - taxy(t) - PTF GAP(t)].$$

Here $gdef(t)$ is the overall real deficit in the government's budget (with deficits expressed as positive numbers, surpluses as negative). The value of interest payments on the government debt is the product of $r(t)$, the real interest rate at time $t$, and the outstanding real stock of debt, where $B(t)$ is the nominal debt stock and $P(t)$ is a general price deflator. Real spending on goods and services is $g(t)$; $taxy(t)$ represents the real value of revenues from income taxes.

**D. Adult Consumption**

The individual adult consumer in our framework can be described as solving the following maximization problem:
\[
\max \int_{t}^{\infty} e^{-\left(\theta + p_{e}^{(v)}(t-v)\right)} u(c(s, v)) dv
\]  
(30)

subject to:

\[
w(s, t) = f_{w}(s, t) + h_{w}(s, t)
\]  
(31)

\[
f_{w}(t) = \left[ (r(t) + p_{a}(t)) f_{w}(s, t) + y(s, t) \right] (1 - \tau(s, t))
\]  
- \theta_{s s}(s, t) y(s, t) - c(s, t) - v(s, t)
\]  
(32)

\[
h_{w}(s, t) \equiv \int_{s}^{\infty} [y(s, v)(1 - \tau(s, v) - \tau_{s s}(s, v)) - v(s, v)] e^{\int_{v}^{(r(t) + p_{e}(t)) d\mu} dv}.
\]  
(33)

Here \(u(.)\) is the utility function of the individual adult, assumed to be of the constant relative risk aversion (CRRA) form, \(u(c(s, t)) = \frac{c(s, t)^{1-\sigma} - 1}{1-\sigma}\), where \(\sigma\) is the coefficient of relative risk aversion, and \(1/\sigma\) is the intertemporal elasticity of substitution. Adults are assumed not to derive explicit utility gain from child consumption; hence only the consumption at time \(t\) of an individual adult born at time \(s\), \(c(s, t)\), directly enters the utility function in (30). The total wealth of the adult individual at time \(t\), \(w(s, t)\) is the sum of \(f_{w}(s, t)\), wealth in the form of financial assets, and \(h_{w}(s, t)\), human wealth defined as the present value of expected future labor income net of taxes on labor income and in vivo transfers to children. Transfers made by adults to support the consumption of youth dependents, \(v(s, t)\), are discussed in section 2B of the text above. The other variables in the preceding equations are: \(\theta\), the time preference rate; \(y(s, t)\), the individual’s labor income (equation (23)); \(\tau(s, t)\), the tax rate on that individual’s income from all sources; and \(\tau_{s s}(s, t)\), the pension tax rate on labor income. Disposable income for the individual is \(\left[ (r(t) + p_{a}(t)) f_{w}(s, t) + y(s, t) \right] (1 - \tau(s, t)) - \theta_{s s}(s, t) y(s, t)\). The individual’s saving in the period -- the net change in financial wealth given by (32) -- is disposable income less taxes less own consumption less transfer payments for child support.\(^{44}\)

\(^{44}\) With a positive probability of death, adult individuals discount the future at a rate higher than their pure time preference rate. The effective discount rate for an individual is therefore \(\theta + p_{e}(t) > 0\). The presence of perfect annuity markets as first introduced by Yaari ensures that individual wealth grows at the rate \(r(t) + p_{a}(t)\), where the premium paid by the perfectly competitive insurance company to an individual during his lifetime is equal to \(p_{a}(t)\) (equation (32)).
Solving the utility maximization problem gives adult consumption as a linear function of total wealth:
\[ c(s,t) = \frac{1}{\Psi(t)} [fw(s,t) + hw(s,t)]. \tag{34} \]
where \( 1/\Psi(t) \) is the marginal propensity to consume out of wealth. The dynamics of \( \Psi(t) \) are given by:
\[ \Psi(t) = -1 - \frac{1}{\sigma}[(1-\sigma)(r(t) + p_n(t)) - (\theta + p_n(t))]\Psi(t). \tag{35} \]

The marginal propensity to consume out of wealth in the general case of the CRRA utility function depends, as is well known, on the intertemporal elasticity of substitution (EIS) and on the entire sequences of future interest rates and future adult mortality rates. This dependence is readily evident in equation (35). In contrast, when the EIS is assumed to be unity (the case of logarithmic utility, with \( \sigma = 1 \)) and when the adult mortality rate is assumed to be constant rather than time varying, the marginal propensity to consume out of wealth reduces to the simple form of a constant, the sum of the time preference rate and the mortality rate \( (1/\Psi = \theta + \rho_n) \).\(^{45}\)

When one aggregates over the consumption functions of individual decision-making units (equation (34)), total adult consumption \( C \) is given by:
\[ C(t) = \int c(s,t)N(s,t)ds = \frac{1}{\Psi(t)} [FW(t) + HW(t)], \tag{36} \]
where \( C(t) \) is aggregate adult consumption and \( FW(t) \) and \( HW(t) \) are aggregate financial and human wealth. The evolution of \( FW(t) \) is described in equation (2) in the text.

Aggregate human wealth is a stock variable representing the present value of economy-wide labor income (adjusted for the varying ages and relative productivities of different cohorts). Its change through time is given by:

\(^{45}\) The net response of consumption to changes in the real interest rate depends on the relative strength of substitution and income effects. With a low rather than high value of EIS, consumers act less strongly to shift their consumption intertemporally; the substitution effect is thus smaller relative to the income effect. For any given shock to the economic system, the real interest rate thus must adjust by a larger amount the lower is the value of EIS. Empirical evidence, in our view, is more consistent with \( \sigma \) having a value smaller than unity (for example, 0.5 or even as small as 0.3) than with the assumption that \( \sigma = 1 \) (logarithmic utility).
\[
\begin{align*}
\dot{HW}_t &= \frac{d}{dt} \int_{-\infty}^{t} hw(s,t)N(s,t)ds \\
&= hw(t,t)b_{\tau}(t)N(t) + r(t)HW(t) - Y(t)(1 - \tau(t)) - PTFGAP(t) - C_j(t) .
\end{align*}
\] (37)

Equation (37) shows that the incremental change in aggregate human wealth at time \( t \) is influenced by the additional human wealth of the newest adult cohort coming of age at time \( t \), \( hw(t,t) \). The shape of the labor-earnings profile -- embodied in the five parameters \( a_1, a_2, \alpha_1, \alpha_2, \text{and} \alpha_3 \) in equation (22) -- has a critical influence through time on the behavior of \( hw(t,t) \) and hence of aggregate human wealth, \( HW(t) \).

Income taxes and child-support transfers influence aggregate human wealth. Any imbalance in the pension trust fund between pension tax revenues and benefit payments, the variable \( PTFGAP(t) \) in (37), also affects the dynamics of aggregate human wealth. When the pension system is continuously balanced so that current-period pension taxes exactly cover current-period pension benefits paid to the elderly \( (PTFGAP(t) = 0) \), the direct effect of the pension system on aggregate human wealth nets out to zero. However, even for the continuously balanced cases where \( PTFGAP \) never differs from zero, the inclusion of a public pension system in the framework has important indirect effects on human wealth (and hence on consumption and other key macroeconomic variables) through the evolution of \( hw(t,t) \), the human wealth of the newest cohort just entering adulthood and the workforce (see below).

Given the age-earnings profile, adult support of child consumption, and the public pension system, the dynamics of human wealth for the new cohort of individuals entering adulthood and working life are:

\[
hw(t,t) = \sum_{k=1}^{3} hw_k(t,t) - hw_{11}(t,t) - hw_{12}(t,t) + hw_{22}(t,t) .
\] (38)

Equation (38) defines the human wealth of the newest individual cohort entering the adult population at time \( t \) as the sum of six components. The first three components derive from the concave time profile of labor income as described in equations (22) through (24) and are given by:

\[
\begin{align*}
\dot{hw}_k(t,t) &= [r(t) + p_{\alpha}(t) + \alpha_k]hw_k(t,t) - a_kwage(t)[1 - \tau(t) - \tau_{\alpha}(t)]; \ k \in \{1,2,3\} .
\end{align*}
\] (39)
The $hw_k(t,t)$ equations allow for both the income and pension taxes that are paid on labor income. The fourth and fifth components of (38) reflect the impact of parent-child transfers on human wealth, with $v(t)$ as determined in equation (7) in the text above:

$$
\dot{hw}_{v_1}(t,t) = (r(t) + p_n(t) + \omega_1)hw_{v_1} - z_1 v(t) ;
$$

(40)

$$
\dot{hw}_{v_2}(t,t) = (r(t) + p_n(t) + \omega_2)hw_{v_2} - (1 - z_1) v(t).
$$

(41)

The final component, $hw_{ss}(t,t)$, is included to reflect the real value of the stream of pension benefits that the new adult cohort expects to receive eventually after reaching the threshold elderly age (47 years in the future, $t + \Lambda$), discounted back to the present:

$$
hw_{ss}(t,t) = [r(t) + p_n(t)]hw_{ss}(t,t) - \left[ \beta_{ss}(t + \Lambda) \frac{Y(t+\Lambda)}{N(t+\Lambda)} \right] e^{-\int_t^{t+\Lambda} (r(s) + p_n(s)) ds}.
$$

(42)

The second term in square brackets in (42) is the per-elderly gross pension benefit expected in the future at the date today’s new adults expect to reach the threshold elderly age.

Note that even for a “pure” PAYG case in which aggregate pension taxes continuously equal aggregate pension benefits, the discounted stream of an individual new worker's future pension taxes will not necessarily be equal to the discounted stream of pension benefits that that worker expects to receive from age 65 until death. Consequently, even a continuously balanced PAYG pension system affects individual consumption and saving behavior through its effect on individual human wealth. The effects for particular individuals on human wealth, consumption, and saving are influenced even more strongly when the trust-fund gap in the pension system is not continuously zero.
Appendix 2: Alternative Assumptions about Adult Consumer Behavior

Our analytical framework permits an explicit assumption about the value of the consumers' elasticity of intertemporal substitution (EIS) -- see equations (34) and (35) in Appendix 1. This parameter plays an important role in determining several aspects of the model's behavior in simulation experiments. We briefly highlight this sensitivity in this appendix.

The situation in which the EIS is unity is the often-used assumption of logarithmic utility. If a researcher postulates values of the EIS progressively less than unity, the model's simulations of the effects on many macroeconomic variables often grow progressively larger.

The benchmark case reported in section 3 of the paper embodied an assumed value of the EIS of 0.5. To highlight the effects of varying the EIS, we report here several charts each containing a curve for that benchmark simulation plus two additional curves for simulations identical with the benchmark case except for the assumed value of the EIS. One of the additional simulations uses a high value for the EIS of unity; the other drops the EIS all the way to the value of 1/3.46

Figure 25 again plots the deviations from baseline of the ZZ real short-term interest rate in response to our standardized asymmetric fertility decline. The curve for the benchmark simulation (EIS value of 1/2) is the same curve shown in Figures 8 and 16. Assuming a lower (higher) value of the EIS means that the strength of the intertemporal substitution effect in response to interest-rate changes diminishes (increases) relative to the income effect. With a lower (higher) EIS, moreover, consumers respond absolutely less (more) to interest-rate changes. For a given size shock in the model, therefore, interest rates are forced to change by larger amounts for lower values of the EIS. In the example here, cutting the value of the EIS to 1/3 nearly doubles

46 We believe cases where the EIS takes on a value markedly less than unity are more consistent with the available empirical evidence (studies in which researchers have tried to estimate the value). For example, plausible estimate of the EIS appear to be as low as 0.5 or even 0.3.
the size of the medium-run cyclical dip in the real interest rate. Raising the EIS to unity cuts the size of the cyclical dip roughly in half.

In the external sector of the ZZ economy, when the EIS is set lower than in the benchmark case the ZZ currency must then appreciate more, the ZZ current-account balance must move into a larger surplus (shown in Figure 26), and hence the ZZ net foreign asset ratio must rise to still more positive values. Opposite effects result when the EIS is raised above the benchmark value. Interestingly, the sign of the nominal trade balances and current-account balances in the short run can be reversed for a value of the EIS as high as unity (see again Figure 26). In other words, even the direction -- and most definitely the magnitude -- of net capital flows associated with asymmetric fertility declines can be sensitive to the model's assumption about the value of the EIS.

Effects on, for example, private saving and financial wealth are also highly sensitive to the value of the EIS. Perhaps most important, for values of the EIS lower than the benchmark assumption, ZZ per-adult consumption rises more initially, has a smaller cyclical downswing during the medium-run playing out of the shock, and is ultimately significantly higher in the long-run steady state (Figure 27). In contrast, an EIS value as high as unity exacerbates the medium-run cyclical dip in per-adult consumption and results in an outcome for the long-run steady state that is much less favorable.

Note that the effects of progressively lowering the EIS tend to be non-linear. For example, changes in the effects resulting from a drop in the assumed elasticity from 1/2 to 1/3 are, for some variables, virtually as large as the changes in effects from dropping it all the way from unity to 1/2.

These differential results for alternative values of the EIS make it clear that analytical and policy judgments can sensitively depend on how researchers calibrate their models for this key parameter. In our future research we expect to study this issue further.47

47 From our research so far, model results are relatively insensitive to alternative assumptions about the fraction of consumers that are borrowing constrained. Faruqee and Laxton (2000) explain why an assumption about the fraction of consumption that is borrowing-constrained is less consequential than the assumption about the value of the EIS.
REFERENCES


