The Stability and Initial Conditions in a Forward-Looking Model

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Abstract

This paper analyzes the influence that model variables, such as parameters and initial values, have on policy changes in a forward-looking type simulation. In the first half, the policy function characterized with parameters is analyzed in terms of influence on the stability of a steady state. In the second half, an initial value is analyzed on the influence on a dynamic policy simulation. As a result, it is indicated that parameters and initial values have large influence on a simulation, and attention must be paid to them.

1 Introduction

Recently, a forward-looking type simulation has been vigorously used in policy analysis. When policy changes occur in the future, and there is a possibility that the present policy changes will be affected in the future, forward-looking type simulations can analyze the influence of the policy changes that considers future economic fluctuations. However, since the model is a two-point boundary value problem, formulization of the portions of the future and the past is accompanied by many difficulties compared with the backward type. Hence, various studies have been performed about the formulization.

Bryant and Zhang (1996a, 1996b) considered alternative specifications on the intertemporal fiscal closure rules. Those specifications are taken in MULTIMOD in the IMF and Asian LINK Model in ESRI. In FRB/GLOBAL and the National Institute's global econometric model (NiGEM) in NIESR of the United Kingdom, an error-correction model is adopted as the fundamental model structure. These formulizations have been adopted in the stage that chooses functions, to stabilize the influence of the future and the past. These studies have contributed to the stability of forward-looking type models greatly. However, it will be meaningful to examine those theoretical studies and effects, which tend to be forgotton. Therefore, we examine the influence of the existing fruits of work granted to the stability of simulations, and the influence of the initial value which tend to be overlooked.

Our analysis focuses on analysis of the simplest linear model, and, if possible, analyzes nonlinear models by linear approximation. According to Blanchard and Kahn (1980), the eigenvalue of the matrix composed of parameters plays an important role in the stability of a linear model. Therefore, we argue about the stability of models in the property of matrices. At this time, a model can be expressed as differential equations by matrices, and can be considered as a two-point boundary value problem with an initial value and a terminal value. Usually, since a terminal value uses the value of a steady state, the influence of an initial value is also considered.

Accordingly, this paper analyzes the influence that model variables, such as parameters and initial values, have on policy changes in a forward-looking type simulation. In Section 2, the policy function characterized with parameters is analyzed in terms of influence on the stability of a steady state. In Section 3, an initial value is analyzed on the influence on a dynamic policy simulation. In Section 4, the conclusion summarizes the result obtained by these analyses.

2 Alternative Fiscal Policy Reaction Functions and Stability

Altering policy reaction functions in a macroeconometric model has important impacts not only on its simulation results but also on the stability of computations to obtain the results. In this section, we investigate how introducing some policy reaction functions and altering the functions can change the extents of each parameter's value, in which we can conduct simulations stably.

Trying to simulate economic performance by using a macroeconometric model with forward-looking expectations, we sometimes encounter difficulties of conducting simulations and occasionally do not obtain any appropriate results with estimated or assumed parameter values. There are some reasons not to solve simulation paths properly in certain models. One of those reasons may be that, with given parameter values, the system of the model never satisfies theoretical conditions for stability of dynamic system. Note that these conditions are strongly related to signs of the eigenvalues of the system. In this section, we first calculate the eigenvalues of a simple linear forward-looking type model, where no policy reaction function exists, and show the ranges of each parameter value, in which the system is economically stable. After that, we introduce two types of fiscal policy reaction function, and point out that introducing these functions can remarkably enlarge the range of appropriate parameter values.

Even in the case that the system has proper eigenvalues, another problem might occur when we conduct simulations. There is a possibility that the steady state of the model would be implausible -- i.e. negative GDP and/or negative consumption etc.--, even if the eigenvalues of the system fit the theoretical stability condition. It seems to be rather unusual to confront this problem in practical macroeconometric models. However, the basic system analyzed below is somewhat fragile, which means many possibilities of the implausible steady state exist. Thus, we also pay attention to the values of the steady state.

Some studies have addressed the effects of alternating policy reaction functions on simulation results. Bryant and Zhang (1996a, 1996b) considered alternative specifications about the intertemporal fiscal closure rules. One of those specifications is called DST (Debt-Stock Targeting), which is taken in the MULTIMOD in IMF and Asian LINK Model in ESRI. The other one is called IIP (Incremental Interest Payments). In the MGS model by Sachs and McKibbin, a slightly different variant of IIP has been used. Using a simple neoclassical growth model, Bryant and Zhang illustrated how altering specifications could influence economic performance. McKibbin (1999) also explored the influence of altering specifications. Although his specifications were slightly different from those of Bryant and Zhang, he conducted simulations with a large model (MGS2 model) and showed the importance of choosing policy reaction functions.

These researches have pointed out that DST and IIP specifications bring about the different simulation results. However, in the researches, enough attentions have not been paid to stability of the system, which would change if we alter those specifications. In general, we cannot deny a possibility that, although simulations are never conducted stably by using one of those specifications, we could obtain some appropriate results with the other specification. Thus, it is important to consider which policy reaction function makes the model more stable to simulate in terms of constructing models.

2.1 Conditions to Obtain Appropriate Solutions

Blanchard and Kahn (1980) showed the following conditions about the existence of a unique solution of a linear model with forward-looking variables. If the number of eigenvalues of the system outside the unit circle is equal to the number of non-predetermined variables -- i.e. forward-looking variables --, then there exists a unique solution path on which the economy converges to a steady state. Moreover, he also proposed that, if the number of eigenvalues outside the unit circle exceeds the number of non-predetermined variables, then almost all solution paths explode, and that, if the number of eigenvalues outside the unit circle is less than the number of non-predetermined variables, there are infinite solution paths that start from the same initial condition.

These propositions mean that it is necessary, for conducting simulations appropriately by using a linear forward-looking model, that the number of eigenvalues of the system outside the unit circle be exactly equal to the number of forward-looking variables. When the system does not satisfy this condition, we tend to encounter difficulty in computing simulation paths. However we cannot exclude rare cases to obtain certain solutions even with such inappropriate systems, the computed paths can never be plausible. Therefore, we must adjust the parameter values of the model to satisfy the above conditions.

Even if a system fits the condition about eigenvalues, there is another possibility to obtain inappropriate simulation paths. The parameter values, by which the model satisfies the condition of Blanchard and Kahn, do not always give a plausible steady state. In some cases, we have an implausible steady state -- i.e. negative GDP and/or negative consumption etc.--, although the eigenvalues of the system match the condition of Blanchard and Kahn. It is obvious that we can never obtain appropriate simulation paths under an implausible steady state. Thus, in investigating the stability of any model, we must pay sufficient attention not only to the condition about eigenvalues but also to its steady state.

2.2 Basic Model with a Forward-Looking Variable

We first explore the stability of a simple model with a consumption function based on the life-cycle hypothesis. This model is rather fragile, since we postulate a Keynesian setting in other parts of the model and because there is no concavity in it.[1]

< Basic Model >

$$Y_t = C_t + G,$$

$$C_t = \theta \cdot (HW_t + BOND_t),$$

$$HW_t = (1-\tau)Y_t + \frac{1}{1+\gamma} HW_{t+1},$$

$$BOND_t = (G - \tau Y_t) + (1+r) BOND_{t-1}.$$

・Endogenous Variables (in real terms per effective labor unit)

Y: output (income) *C*: consumption

HW: human wealth *BOND*: stock of government debt.

・Exogenous Variable and Coefficient Parameters

G: government expenditure (per effective labor unit)

θ: propensity to consumption

γ: discount rate about human wealth

r: interest rate τ: tax rate

It is assumed that the both rates of growth of population and technical progress are zero. If the probability of death is also zero -- i.e. the model is the Ramsey type --, γ must equal *r* theoretically. What's more, if the intertemporal utility function is logarithmic, θ must coincide with γ.

Note that, in this model, the forward-looking variable -- i.e. non-predetermined variable -- is *HW*, and the predetermined variable is *BOND*. These mean that the numbers of both non-predetermined and predetermined variables are just one. We can rewrite the model as follows:

$$HW_{t+1} = A_1 \cdot HW_t + A_2 \cdot BOND_t + Z_1,$$

$$BOND_{t+1} = A_3 \cdot HW_t + A_4 \cdot BOND_t + Z_2,$$

or

$$\begin{bmatrix} HW_{t+1} \\ BOND_{t+1} \end{bmatrix} = A \begin{bmatrix} HW_t \\ BOND_t \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}, \quad A \equiv \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

A_1, A_2, A_3 and A_4 are constants formed by the parameters of the model.

This system has two eigenvalues. When one of these values is outside the

[1] Bryant and Zhang (1996b) assume that a production function is of the Cobb-Douglas type. So that, their models have a concavity, even though they exclude policy reaction functions. This concavity seems to make their models more stable to compute simulation paths than our models are.

unit circle and the other one is inside the unit circle, there exists a unique path on which the economy converges to the steady state. That is, we can always conduct simulations stably, only if

$$|\lambda_1| > 1, \quad |\lambda_2| \leq 1, \qquad (1)$$

where λ_1 and λ_2 are eigenvalues about A.

As described above, it is also necessary to check the steady state values. In this model, there are many cases where we obtain an implausible steady state. Thus, in addition to (1), we must consider the following conditions:

$$\overline{Y} > 0, \quad \overline{C} > 0, \quad \overline{HW} > 0, \qquad (2)$$

where each upper bar means its steady state value.

Since this system has just two dimensions, we could find the ranges of appropriate parameter values analytically. However, this analytical approach seems to be rather cumbersome and give less intuition even in this simple model. Therefore, we directly compute the number of eigenvalues outside the unit circle about each set of parameter value, and also check the conditions described in (2) by computing the steady state values of the endogenous variables.

Figures 1-3 are the cases that r=0.03, r=0.05 and r=0.07, respectively. In all of these cases, we always set τ as 0.35. The mark in each cell of these figures means the following:

■: plausible steady state,
and the number of eigenvalues outside unit circle = 1 (appropriate case),

x: plausible steady state,
and the number of eigenvalues outside unit circle = x,

(x): implausible steady state,
and the number of eigenvalues outside unit circle = x.

We should notice, however, that the both marks of '■' and '(1)' mean stable cases. We cannot obtain appropriate solutions in the case of '(1)' because the steady state is implausible. In other cases -- i.e. '2' and '(2)'--, the system becomes unstable.

The figures show that the ranges of parameter values, where we can obtain appropriate simulation paths, are very narrow in this model. Furthermore, we can also confirm the following properties by considering details of the figures:

(i) If $\gamma=\theta=r$, then the economy has no appropriate path.

(ii) If $\theta>\gamma>r$, then we can conduct simulations stably but never obtain plausible paths.

(iii) If $\gamma>\theta>r$ and (θ/γ) is near one, then we can conduct simulations stably but never obtain plausible paths.

(iv) If $\gamma>\theta>r$ and (θ/γ) is sufficiently small, then we cannot conduct simulations stably, and even though we obtain some paths, the paths are never plausible.

The assumption of constant τ is one important reason why the ranges of appropriate parameter values are so narrow. As τ is assumed to be constant, the possibility of an explosion in *BOND* is strong because there exists no other mechanism to mitigate the explosion in this model.[2] Furthermore, by examining Figures 4-6, it can be recognized that the smaller τ is, the stronger the possibility of the explosion in *BOND* is.[3] That is, if τ become lower, '■' changes into '(2)' in some cells. The ranges of parameter values, where the model can be stable, became narrow as τ decreased.

2.3 Introducing a Fiscal Policy Reaction Function -- DST type --

This subsection and next subsection show that introducing a fiscal reaction function can remarkably enlarge the extent of appropriate parameter values of the model. First, we focus on a DST type policy reaction function, and point out that this type of reaction function can make the model more stable. The main idea of the DST type is the following: The government must keep its debt from exploding, and it has a certain target level of the debt stock. If the debt stock exceeds the target value, then the government must raise the tax rate to prevent an explosion of the debt stock. We can write this idea as

$$\Delta\tau_t = \alpha_1 \frac{BOND_t - BOND_t^T}{Y_t} + \alpha_2 \frac{\Delta(BOND_t - BOND_t^T)}{Y_t}, \qquad (3)$$

where both α_1 and α_2 are adjustment parameters, and $BOND_t^T$ denotes the target value of debt stock in period t. The second term of the right-hand side of (3) is added to mitigate the cyclical instability.[4]

Now, we postulate that the target value of the debt stock is constant through time, that is, $BOND_t^T=BOND^T$ for any t = 0, 1, 2,・・・. So then, the model in the former subsection can be modified as follows:

< Model with DST type fiscal policy reaction function >

$$Y_t = C_t + G,$$

$$C_t = \theta\cdot(HW_t + BOND_t),$$

$$HW_t = (1-\tau_t)Y_t + \frac{1}{1+\gamma}HW_{t+1},$$

2 Note that the explosion of *BOND* means that the model is no longer stable.
3 In the cases of these figures, we set r as 0.05.
4 See section 4.3 in Bryant and Zhang (1996a).

$$BOND_t = (G - \tau_t Y_t) + (1+r)BOND_{t-1},$$

$$\tau_t = \tau_{t-1} + \alpha_1 \frac{BOND_t - BOND^T}{Y_t} + \alpha_2 \frac{BOND_t - BOND_{t-1}}{Y_t}$$

・Endogenous Variables (in real terms per effective labor unit except τ)

Y: output (income) C: consumption

HW: human wealth $BOND$: stock of government debt

τ: tax rate

・Exogenous Variable and Coefficient Parameters

G: government expenditure (per effective labor unit)

$BOND^T$: target value of government debt stock

θ: propensity to consumption

γ: discount rate about human wealth

r: interest rate α_1: adjustment parameter α_2: adjustment parameter

Since this model is no longer a linear system, in general we cannot consider the global properties of the model analytically. However, by means of a linear approximation in the neighborhood of the steady state, we can obtain the necessary condition for stable computations of the model as similar to the former model's condition.

The results are shown in Figures 7-9, where we assume r=0.03, r=0.05 and r=0.07, respectively. The values of basic parameters other than θ, γ and r are the following: G=10. $BOND^T$=50. α_1=0.10. α_2=0.30.[5] Although the mark in each cell of these figures has the same meaning as the former subsection, we should note that both '3' and '(3)' are also inappropriate because the number of non-predetermined is still one, even in this model. What's more, since τ becomes an endogenous variable, we must add the following condition:

$$0 < \bar{\tau} < 1. \tag{4}$$

Considering the figures, we can immediately understand that the ranges of appropriate parameter values of this model are obviously larger than those in the former model. If $\gamma > \theta > r$, there exists a possibility that even a sufficiently small (θ/γ) can be allowed to conduct simulations appropriately. Remembering the figures about the basic model, we recognize that the ranges of stable computation are enlarged. That is, some unstable marks ('(1)') in Figures 1-3 change into stable marks ('■') in Figures 7-9. In other words, introducing a DST type policy reaction function can improve the stability of the model. However, we cannot improve other results in the former section -- i.e. (i), (ii) and (iii) --, even though a DST type policy reaction function would be included into the model.

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5 The results are never changed with any other $BOND^T$ (>0). Also, if $\alpha_1+\alpha_2 > r$, with other α_1 and α_2, the results are almost the same as the figures. However, if $\alpha_1+\alpha_2 < r$, the ranges of appropriate values of the parameters become narrow.

2.4 Introducing a Fiscal Policy Reaction Function -- IIP type --

In this section, we investigate a model in which a IIP type fiscal policy reaction function, instead of a DST type, is included. A IIP type policy reaction function can be described as

$$T_t = \bar{\tau} Y_t + t_t^{adj}, \quad t_t^{adj} = r_t BOND_t - \bar{r}\,\overline{BOND}, \tag{5}$$

where $T_t = \tau_t Y_t$. The intuition of (3) is that the government must adjust tax revenues by an amount just sufficient to offset increase or decrease in its interest payments on government debt.[6] Remember that we have postulated the interest rate as an exogenous parameter. That is, the interest rate is constant through time in our models. Using (5) for the fiscal policy functions instead of (3), we can rewrite the model as follows:

< Model with IIP type fiscal reaction function >

$$Y_t = C_t + G,$$

$$C_t = \theta \cdot (HW_t + BOND_t),$$

$$HW_t = (1 - \tau_t) Y_t + \frac{1}{1+\gamma} HW_{t+1},$$

$$BOND_t = (G - \tau_t Y_t) + (1 + r) BOND_{t-1},$$

$$\tau_t = \bar{\tau} + r \frac{BOND_t - \overline{BOND}}{Y_t}$$

・Endogenous Variables (in real terms per effective labor unit except τ)

- Y: output (income) — C: consumption
- HW: human wealth — $BOND$: stock of government debt
- τ: tax rate.

・Exogenous Variable and Coefficient Parameters

- G: government expenditure (per effective labor unit)
- $\bar{\tau}$: tax rate on the steady state — θ: propensity to consumption,
- γ: discount rate about human wealth — r: interest rate

Although there is no explicit target variable, the steady state value of τ has a similar role to a target variable, because it is apparently redundant on the steady state in this model. This means that we must regard the steady state value of τ as an exogenous variable.

As same as the above section, using the linear approximate system, we can

[6] See section 4.4 in Bryant and Zhang (1996a).

compute the numbers of the eigenvalues outside the unit circle about each set of parameter values. The results are summarized in Figures 10-12. Again, these figures correspond with r=0.03, r=0.05 and r=0.07, respectively. In these cases, we set other parameters: G=10 and $\bar{\tau}$=0.35.[7]

By using a IIP type fiscal policy reaction function, we can remarkably extend the ranges of appropriate parameter values to the opposite side of the cases of a DST type. Therefore, whenever both θ and γ are larger than r, we can allow many sets of parameter values to obtain the appropriate simulation results, even if (θ/γ) would exceed one.[8] Although we cannot improve the two results of the basic model, that is, (i) and (iv) in section 2.3, introducing a IIP type policy reaction function into the model can eliminate the conditions of (ii) and (iii) for conducting simulations properly.

2.5 Brief Conclusion

In this section, we have considered alternative fiscal policy reaction functions in the light of stability of models. When we try to conduct some simulations, we might encounter problems that simulation paths cannot be obtained, or that the paths are implausible even if we can obtain them. This section showed that, even when a model is a simple linear system, there would be many cases in which those problems occur. Moreover, we investigated the effects of introducing and alternating fiscal policy reaction functions in terms of stability of the model. Alternative policy reaction functions focused on here were of DST type and IIP type. We confirmed that adding the policy reaction function of both types into the model could extend the ranges of appropriate values of the parameters. Moreover, the effects of DST type and IIP type policy reaction functions take the opposite direction regarding extension of the ranges. Thus, we can sometimes make the model more stable by altering the policy reaction functions, even when it seems to be unstable.

[7] If we raise the steady state tax rate, then the ranges of appropriate parameters' values can enlarge, just as with the basic model.

[8] However, this expansion of the ranges does not mean that stability is strengthened, because the introduction of a IIP type function only changes the implausible steady state cases into plausible ones.

3 Quantitative Evaluation of the Influence of Initial Values

In a forward-looking type simulation, exogenous values are usually needed for an endogenous variable with leads and lags in initial and terminal points. Researchers determine these values based on a certain basis. Generally, because the exogenous values in a terminal point (called "terminal values") are given with the values of a steady state, they are not arbitrary. However, the exogenous values (called "initial values") in an initial point use actual values in many cases. Actual values vary according to the start time of the simulation, and the data to adopt. Therefore, it is important to evaluate the influence of the initial value to simulation results (We here call it "initial value effect"), when we examine simulation results. Therefore, this section evaluates the initial value effect by altering initial values.

3.1 Theoretical Analysis

To avoid complication, we analyze a linear forward-looking type dynamic model. The vector of endogenous variables with lead variables, that with lag variables, and a vector without any lead and lag are defined as x_t,z_t and y_t, respectively. Generally, a linear forward-looking type dynamic model is shown as

$$\begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = A \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} + \begin{pmatrix} B \\ O \\ O \end{pmatrix} x_{t+1} + \begin{pmatrix} O \\ O \\ C \end{pmatrix} z_{t-1} + p_t. \tag{3.1}$$

Assuming that $I - A$ is a non-singular matrix, solution of these simultaneous equations at the t time is

$$\begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = (I-A)^{-1} \begin{pmatrix} B \\ O \\ O \end{pmatrix} x_{t+1} + (I-A)^{-1} \begin{pmatrix} O \\ O \\ C \end{pmatrix} z_{t-1} + (I-A)^{-1} p_t. \tag{3.2}$$

At this time, we give definitions as

$$\tilde{A} \equiv (I-A)^{-1},\ \tilde{A} = \begin{pmatrix} \tilde{A}_{xx} & \tilde{A}_{xy} & \tilde{A}_{xz} \\ \tilde{A}_{yx} & \tilde{A}_{yy} & \tilde{A}_{yz} \\ \tilde{A}_{zx} & \tilde{A}_{zy} & \tilde{A}_{zz} \end{pmatrix},\ q_t \equiv \begin{pmatrix} q_t^U \\ q_t^M \\ q_t^L \end{pmatrix}. \tag{3.3}$$

Then, (3.2) can be rewritten such as

$$\begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = \begin{pmatrix} \tilde{A}_{xx}B \\ \tilde{A}_{yx}B \\ \tilde{A}_{zx}B \end{pmatrix} x_{t+1} + \begin{pmatrix} \tilde{A}_{xz}C \\ \tilde{A}_{yz}C \\ \tilde{A}_{zz}C \end{pmatrix} z_{t-1} + \begin{pmatrix} q_t^U \\ q_t^M \\ q_t^L \end{pmatrix}. \tag{3.4}$$

At this time, it turns out that y_t is the function of x_{t+1}, z_{t-1}. Therefore, a portion important for obtaining the solution of this dynamic model is

$$\begin{aligned} x_t &= \tilde{A}_{xx}Bx_{t+1} + \tilde{A}_{xz}Cz_{t-1} + q_t^U, \\ z_t &= \tilde{A}_{zx}Bx_{t+1} + \tilde{A}_{zz}Cz_{t-1} + q_t^L, \end{aligned} \tag{3.5}$$

At this time, an initial point is set as $0(= t - N)$, and a terminal point is denoted as $T(= t + M)$. Moreover, initial and terminal values are defined as $(\bar{x}_T, \bar{z}_0)$, respectively. When (3.5) are substituted repeatedly, we get

$$\begin{aligned} x_t &= (\tilde{A}_{xx}B)^{T-t}\bar{x}_T + \sum_{m=1}^{T-t}(\tilde{A}_{xx}B)^{m-1}\tilde{A}_{xz}Cz_{t+m-2} + \sum_{m=1}^{T-t}(\tilde{A}_{xx}B)^{m-1}q_{t+m-1}^U, \\ z_t &= (\tilde{A}_{zx}B)(\tilde{A}_{xx}B)^{T-t-1}\bar{x}_T + (\tilde{A}_{zx}B)\sum_{m=1}^{T-t-1}(\tilde{A}_{xx}B)^{m-1}\tilde{A}_{xz}Cz_{t+m-1} \\ &\quad + (\tilde{A}_{zx}B)\sum_{m=1}^{T-t-1}(\tilde{A}_{xx}B)^{m-1}q_{t+m}^U + \tilde{A}_{zz}Cz_{t-1} + q_{t-n+1}^L. \end{aligned} \tag{3.6}$$

Therefore, the initial value effect is

$$\frac{\partial x_t}{\partial \bar{z}_0'} = \sum_{m=1}^{T-t}(\tilde{A}_{xx}B)^{m-1}\tilde{A}_{xz}C\left(\frac{\partial z_{t+m-2}}{\partial \bar{z}_0'}\right), \tag{3.7}$$

$$\frac{\partial z_t}{\partial \bar{z}_0'} = (\tilde{A}_{zx}B)\sum_{m=1}^{T-t-1}(\tilde{A}_{xx}B)^{m-1}\tilde{A}_{xz}C\left(\frac{\partial z_{t+m-1}}{\partial \bar{z}_0'}\right) + \tilde{A}_{zz}C\frac{\partial z_{t-1}}{\partial \bar{z}_0'}. \tag{3.8}$$

The terms of q_t on the right-hand side of (3.6) have disappeared by this partial differential. This result indicates that the influence of initial values parts from that of exogenous variables in simulation periods. Therefore, it proves that the policy simulation analyzed by the change of an exogenous variable does not need to take into consideration the initial value effect in a linear forward-looking type simulation model.

The solution of this equation is obtained by solving (3.8). But (3.8) is a difference equation about $\frac{\partial z_i}{\partial \bar{z}_0'} i = 1, \cdots T$ and cannot obtain the solution analytically. Therefore, it is analyzed by carrying out the numerical computation.

3.2 Numerical Computation

The basic model uses what is shown in Section 2. Since each variable of leads and lags is similar to the basic model, respectively, it is shown not by vectors, which were used in the previous analysis, but by scalars. In the basic model, x_t, z_t are $HW_t, BOND_t$, respectively. The parameters in the basic model are given as Table 1. Although these values are assumed, conditions of the steady state about which it argued in Section 2 are satisfied. In addition, exogenous variable G_t is fixed as $G_t = 1$ in the base case. And simulation periods are set as 200 periods.

Evaluation of the initial value effect is performed by the following procedures. A steady state is calculated first and the values of the steady state are given to endogenous variables required as terminal values, using the steady state type basic model which removes the lead and the lag. The solution at this time is defined as $HW_t^*, BOND_t^*$. After that, the basic model, which is illustrated in the previous section, is calculated by $BOND_0 = 1.1 \cdot BOND_0^*$ and $HW_T = HW_T^*$. The solution at this time is defined as $HW_t^a, BOND_t^a$. Finally, the influence of alteration of initial values is examined by the ratio before and after, $\frac{HW_t^a}{HW_t^*}, \frac{BOND_t^a}{BOND_t^*}$.

3.2.1 Influence of Alteration of an Initial Value

In Table 2, the effect of change of the initial value in Basic model, HW_t, $BOND_t$, C_t, Y_t, is shown. In the policy simulation, since the result from the simulation start period to about 10 or 20 periods is important, the computation result of 20 periods is shown. Table 2 points out that the influence of an initial value decreases with the period. However, the reduction rate of the influence falls little by little. Therefore, it turns out that the initial value has had influence in the long run. Moreover, the influence of the initial value to HW_t is half that of $BOND_t$. It is indicated that the influence of the initial value is also large to the variable with a lead. Therefore, we can say that the initial value has had a large influence on the whole simulation.

From this result, it was indicated that the initial value affects simulation results. Next, simulation periods are examined as a factor that affects the initial value effect.

3.2.2 Alteration of Simulation Periods

This analysis examines how much the influence of an initial value varies, when simulation periods are shortened. The case which made the simulation periods 100 periods or 50 periods is compared with the standard case.

The result is shown in Table 3. According to the 2nd column of Table 3, even if only 100 periods shorten simulation periods, the influence of an initial value hardly changes. On the other hand, the 3rd column of Table 4 indicates that the cut of the simulation periods by 150 periods makes influence of an initial value small. Since the terminal value is set as the value of the steady state in spite of simulation periods, the influence of a terminal value has been eliminated in this analysis. Therefore, it turns out that shortening simulation periods extremely weakens the influence of the initial value. However, since shortening simulation periods is arbitrary, it is desirable to perform the simulation of a sufficiently long period.

3.2.3 Influence of the Initial Value to Policy Simulations

Generally, in policy simulations, the policy effect is evaluated by the rate of deviation from a base case. Section 3.1 points out that the changes level of the endogenous variables in the policy simulation which alter exogenous variables are not influenced by the initial value. However, to calculate the rate of deviation, the simulation solution of a base case is required as the denominator, and we also indicate that initial values affect it. It has been suggested that initial values have influenced the rate of deviation. Therefore, this section examines how much the result of policy simulations changes, when initial values differ.

In this paper, the policy that increases the value of an exogenous variable 10 percent is used as a policy simulation, $G_t = 1.1$. And the ratio before and after a policy change evaluates the influence. We consider how much the ratio changes, when an initial value is altered. Before altering an initial value, the result of Y_t without the policy change is Y_t^{*b}, and that with the policy change is Y_t^{*p}. After altering an initial value, the result of Y_t without the policy change is Y_t^{ab}, and that with the policy change is Y_t^{ap}. The difference of evaluation of the policy change by the difference in the initial value is indicated in Table 4. In the 2nd and 3rd column of Table 4, the effect of the policy change without alteration for an initial value is equal to that with alteration. We can confirm that the change of the level of the endogenous variable by the policy change does not have any relation to initial values. On the other hand, the 4th column of Table 4 shows that the effects of a policy change, which are expressed with the ratio before and behind that, vary by an initial value. According to the 4th column of Table 4, if an initial value is different by 10 percent, evaluation of a policy change will change about 0.24 percent in early periods of a simulation. Although it is not a so large change for alteration of ten percent of an initial value, it turns out that the evaluation is affected. And although not shown in a table, if an initial value doubles, it will change 2 percent or mores. Therefore, when initial values differ greatly from the actual value, it turns out that evaluation of a policy change changes significantly.

3.3 Brief Conclusion

In this section, we considered the extent to which an initial value has an influence on a simulation. This analysis shows the following. First, it turns out in the linear model that the influence of the initial value can separate with other exogenous variables. Second, it turns out that the initial value may have influence on the result of simulations. Third, it turns out that the influence of an initial value varies, when simulation periods are short. Lastly, in the policy simulation that alters the values of exogenous variables, while initial values do not affect the influences of the policy change themselves, they affect the ratio used to evaluate the policy change. Although initial values are scarcely taken into consideration in model buildings, we also should be careful of them.

We have pointed out the problems that concern initial values, but the following correspondences are required for the solutions. We should use long periods in simulations. And, in the simulation, although initial values are given with actual values or the values of a steady state in many cases, we should examine carefully whether those values are really suitable ones.

No nonlinear cases have been analyzed. These will be taken up in future studies.

4 Conclusion

This paper explored problems that occur when model variables, such as parameters and initial values, are changed in a forward-looking type simulation. We first considered the influence of changes in parameter values on the stability of a steady state. Second, we analyzed the influence of a change in an initial value of exogenous variable on a dynamic policy simulation.

In the first half, we especially focused on the effects of introducing alternative fiscal policy reaction functions (DST type and IIP type). Even when a model is a simple linear system, there would be many cases in which no simulation could be performed because of the instability of the model. We confirmed that adding one of the policy reaction functions into the model could extend the ranges of appropriate values of the parameters where we can perform the simulation. Moreover, the effects of DST type and IIP type policy reaction functions have the opposite direction on extension of the ranges.

In the second half, we considered how much an initial value has an influence on a simulation, and showed the following: in a linear model, the influence of the initial value depends on the length of the simulation period. Although the change in the initial value alters the ratio used to evaluate the policy change, it is not so large, at least in the case of a 10-percent change in the initial value. In general, we should examine carefully whether or not the initial values are really suitable for the simulations.

Although all analyses in this paper are based on a simple linear model, we assume that the main results shown here would not change drastically even if the basic model adopts other endogenous variables or a certain nonlinearity. The extension of the basic model is one of our future tasks.

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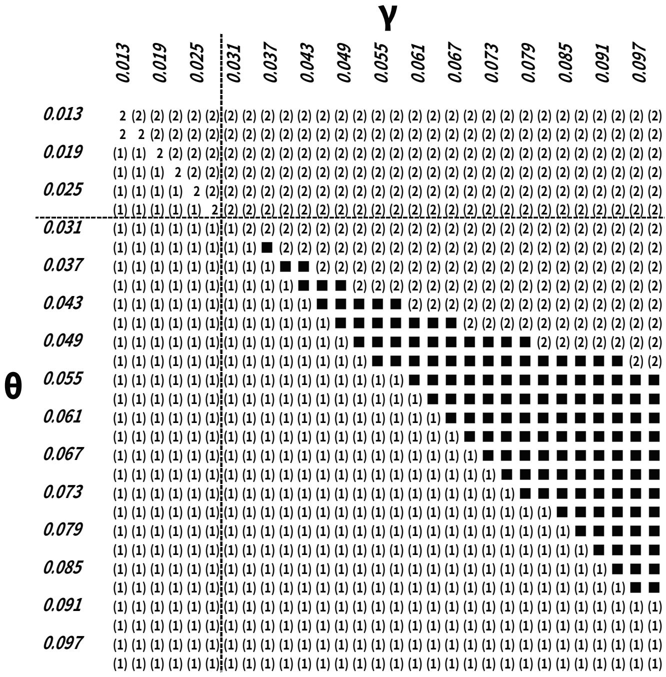

Fig. 1: The Case of Basic Model (*r*=0.03)

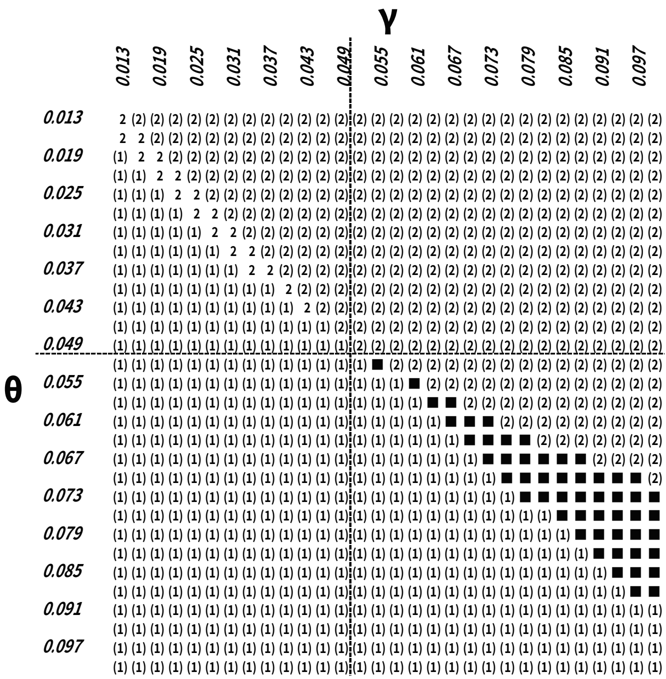

Fig. 2: The Case of Basic Model (*r*=0.05)

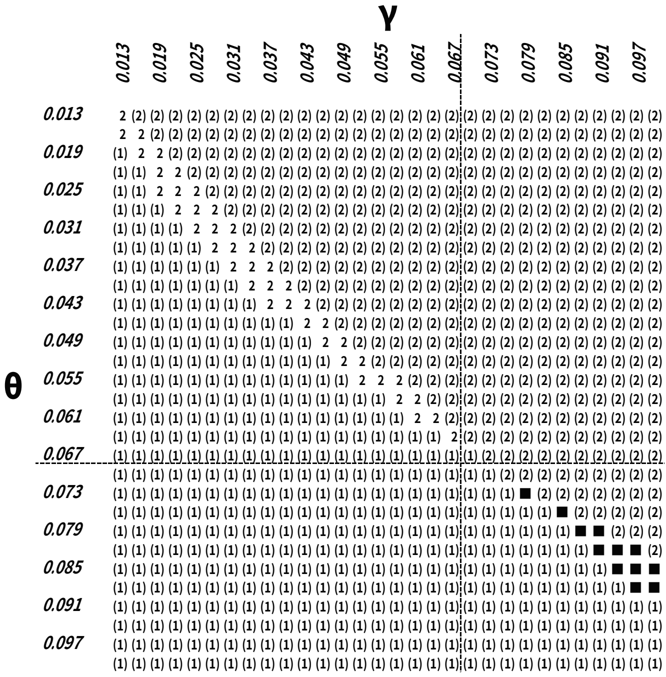

Fig. 3: The Case of Basic Model (*r*=0.07)

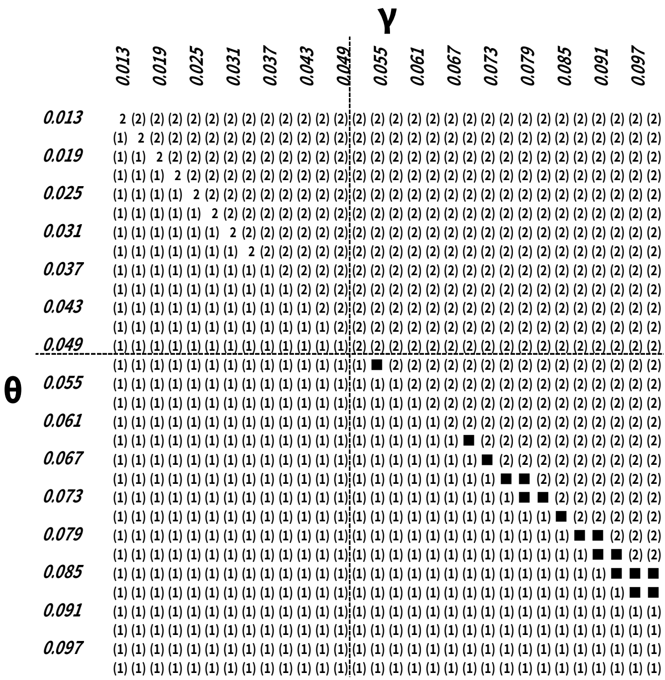

Fig. 4: The Case of Basic Model (τ=0.10, *r*=0.05)

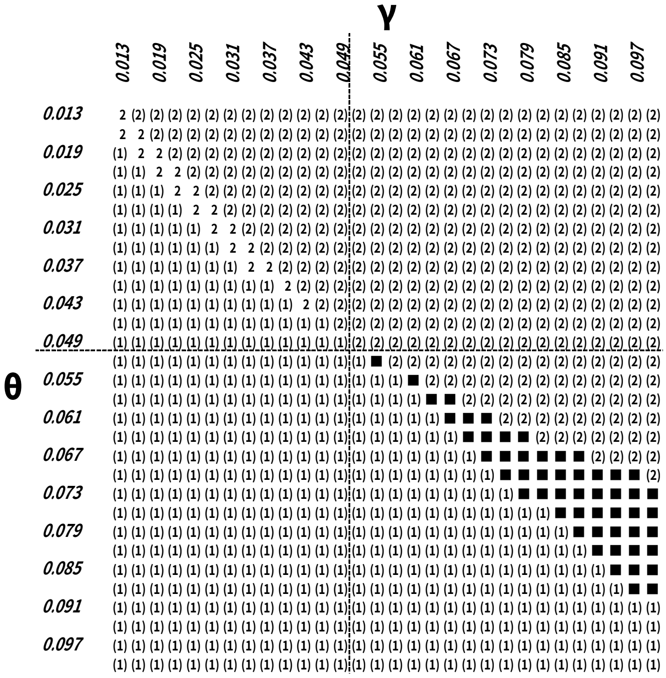

Fig. 5: The Case of Basic Model (τ=0.35, r=0.05)

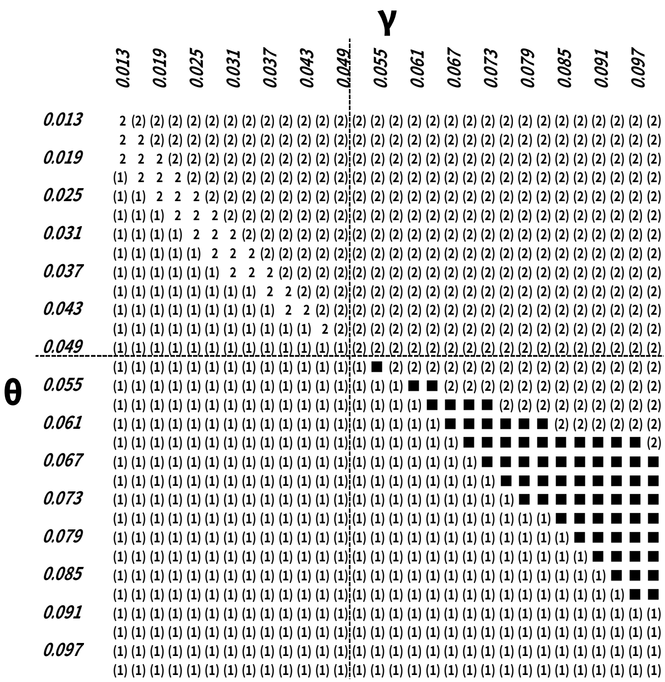

Fig. 6: The Case of Basic Model (τ=0.50, r=0.05)

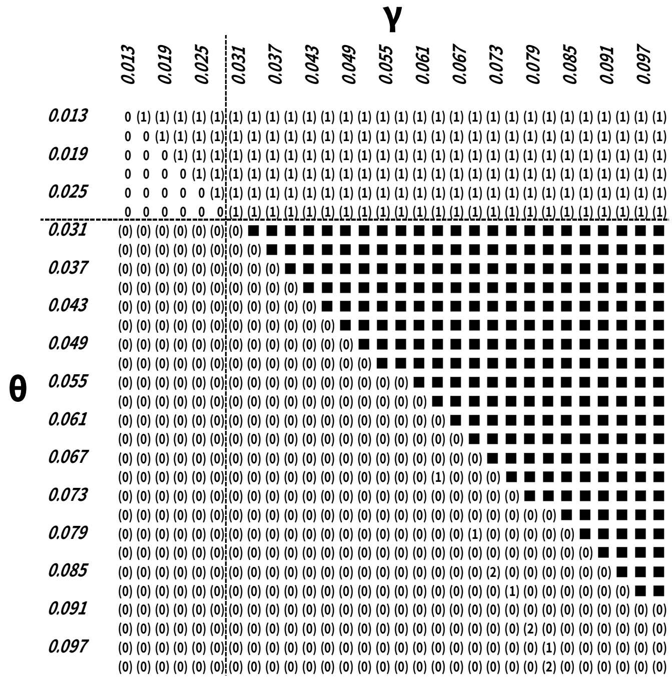

Fig. 7: The Case of DST type (*r*=0.03)

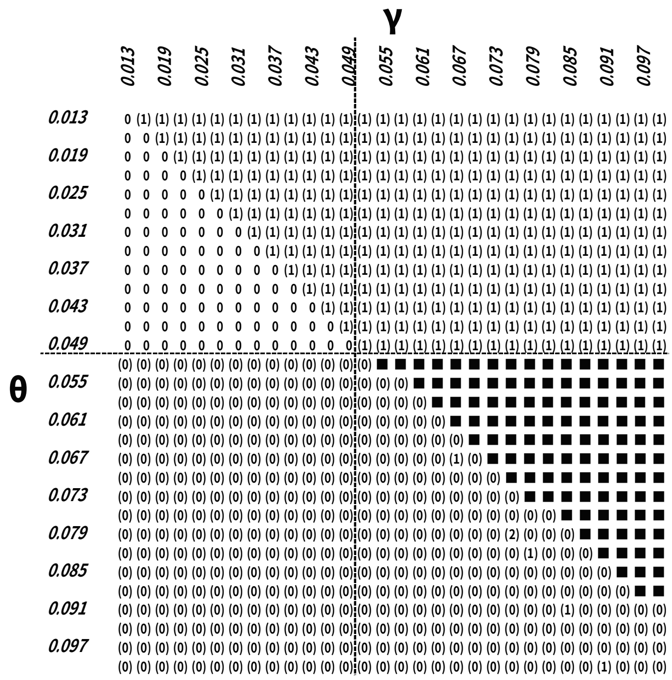

Fig. 8: The Case of DST type (*r*=0.05)

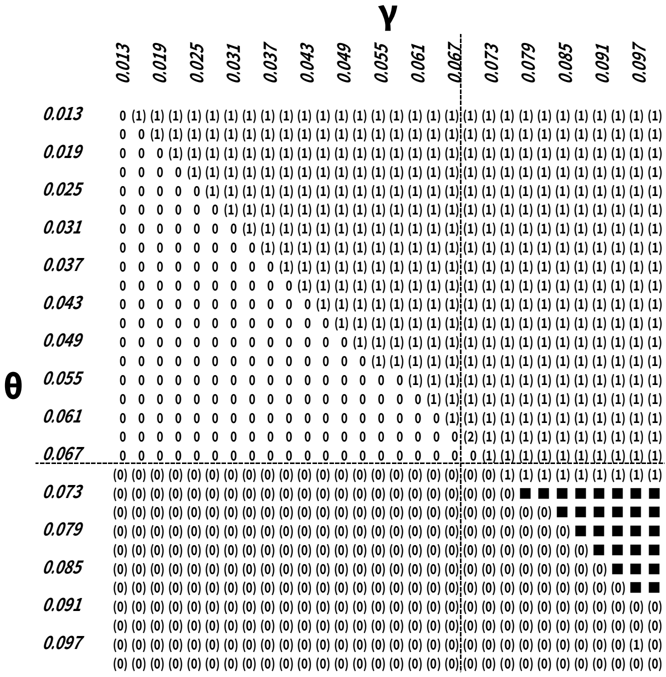

Fig. 9: The Case of DST type (*r*=0.07)

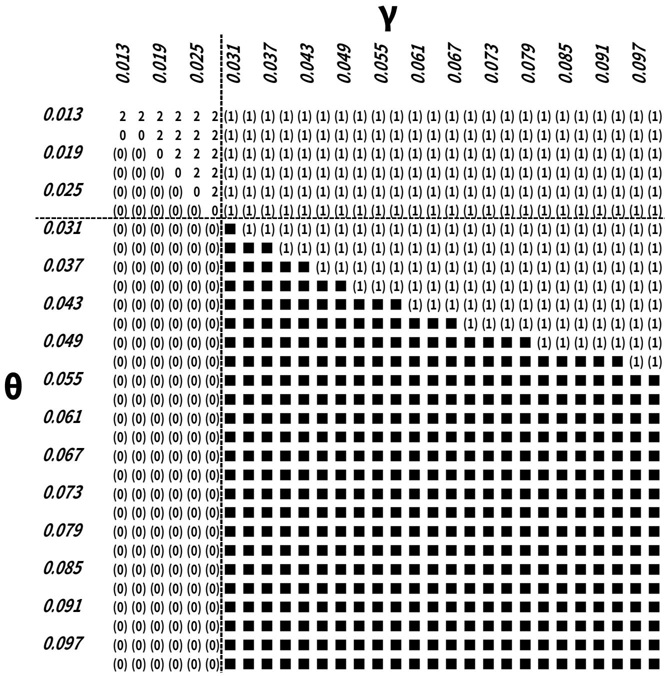

Fig. 10: The Case of IIP type (*r*=0.03)

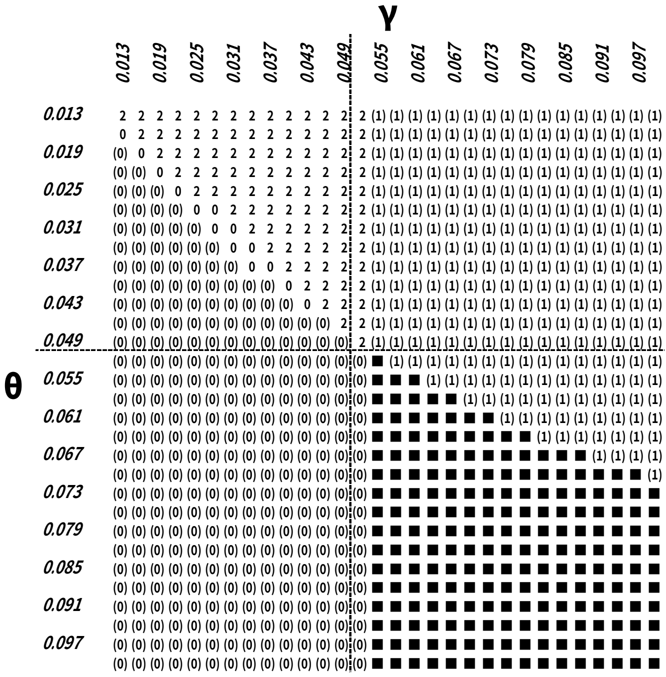

Fig. 11: The Case of IIP type (*r*=0.05)

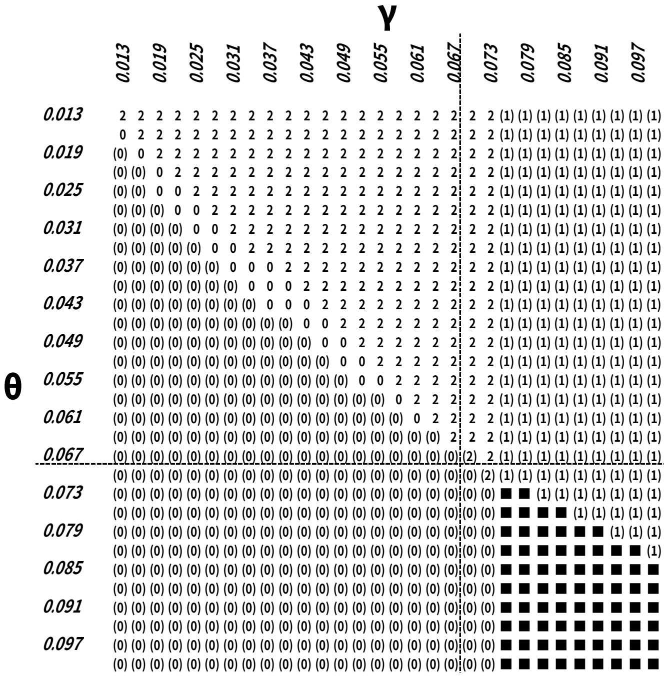

Fig. 12: The Case of IIP type (*r*=0.07)

Table 1: Parameters of Base Model

	T	θ	τ	γ	r
Value	200	0.065	0.35	0.09	0.03

Table 2: Influence of the Initial Value on Endogenous Variables

Period	HW_t^a/HW_t^*	$BOND_t^a/BOND_t^*$	C_t^a/C_t^*	Y_t^a/Y_t^*
1	1.0416	1.0988	1.0611	1.0470
2	1.0411	1.0977	1.0604	1.0464
3	1.0406	1.0966	1.0597	1.0459
4	1.0402	1.0954	1.0590	1.0454
5	1.0397	1.0943	1.0583	1.0448
6	1.0393	1.0932	1.0577	1.0443
7	1.0388	1.0922	1.0570	1.0438
8	1.0383	1.0911	1.0563	1.0433
9	1.0379	1.0900	1.0557	1.0428
10	1.0375	1.0890	1.0550	1.0423
11	1.0370	1.0879	1.0544	1.0418
12	1.0366	1.0869	1.0538	1.0413
13	1.0362	1.0859	1.0531	1.0408
14	1.0357	1.0849	1.0525	1.0404
15	1.0353	1.0839	1.0519	1.0399
16	1.0349	1.0830	1.0513	1.0394
17	1.0345	1.0820	1.0507	1.0390
18	1.0341	1.0810	1.0501	1.0385
19	1.0337	1.0801	1.0495	1.0381
20	1.0333	1.0792	1.0490	1.0376

Table 3: Simulation Periods and the Change of the Initial Value Effect

Period	$\frac{HW_t^a}{HW_t^*}, T=100$	$\frac{HW_t^a}{HW_t^*}, T=50$	$\frac{BOND_t^a}{BOND_t^*}, T=100$	$\frac{BOND_t^a}{BOND_t^*}, T=50$
1	0.00%	-0.15%	0.00%	0.01%
2	0.00%	-0.15%	0.00%	0.01%
3	0.00%	-0.16%	0.00%	0.02%
4	0.00%	-0.17%	0.00%	0.03%
5	0.00%	-0.18%	0.00%	0.04%
6	0.00%	-0.18%	0.00%	0.05%
7	-0.01%	-0.19%	0.00%	0.05%
8	-0.01%	-0.20%	0.00%	0.06%
9	-0.01%	-0.21%	0.00%	0.07%
10	-0.01%	-0.22%	0.00%	0.08%

Table 4: Alteration of an Initial Value and the Policy Simulation

Period	$Y_t^{*p} - Y_t^{*b}$	$Y_t^{ap} - Y_t^{ab}$	$\frac{Y_t^{ap}}{Y_t^{ab}} - \frac{Y_t^{*p}}{Y_t^{*b}}$
1	0.2291	0.2291	-0.2379%
2	0.2314	0.2314	-0.2377%
3	0.2338	0.2338	-0.2374%
4	0.2361	0.2361	-0.2371%
5	0.2384	0.2384	-0.2367%
6	0.2406	0.2406	-0.2363%
7	0.2428	0.2428	-0.2358%
8	0.2450	0.2450	-0.2353%
9	0.2472	0.2472	-0.2348%
10	0.2493	0.2493	-0.2342%