Instability of the Hedonic Model and Its Effect on the Quality Adjustment of Price Index

-- The case of desktop PCs –

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1.Purposes of this research

- How is instable over time the hedonic model for desktop PC?
- How do these instabilities effect on the quality adjustment?
- Are these influences on quality adjustment different between the functional forms?

2.Framework for Measuring the Hedonic Function Depreciation

2.1 Quality Adjustment Using the Hedonic Method

$$\hat{p} = \frac{p_{new} / p_{old}}{g} * 100 - 100$$

where,

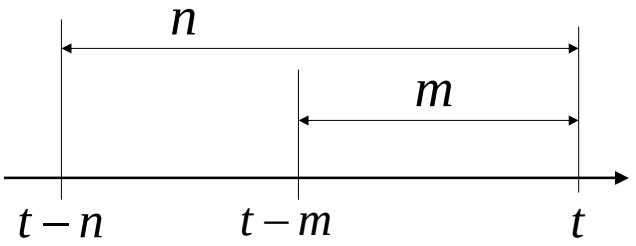
p_{new} : price of new product

p_{old} : price of old product

g : quality ratio between old and new product

$$g = \frac{\text{quality of new product}}{\text{quality of old product}} = \frac{\text{predicted price of new product}}{\text{predicted price of old product}}$$

2.2 Quality Adjustment Using the Hedonic Method



Date of price replacement.

Date of Estimating hedonic function

II

Creating hedonic function: $f(\bullet \mid t-m)$

Date of price replacement. $x_{t-n} \Rightarrow x_t$

II

Date of quality evaluation. Calculating quality ratio.

$$g(t,m,n) = \frac{f(x_t \mid t-m)}{f(x_{t-n} \mid t-m)} \times 100 - 100$$

2.3 Definition of instability measure in hedonic function

$$d(t,m,n) = \left| g(t,m,n) - g^*(t,n) \right|$$

where

$$g(t,m,n) = \frac{f(x_t \mid t-m)}{f(x_{t-n} \mid t-m)} \times 100 - 100$$

$$g^*(t,n) = g(t,0,n)$$

$d(t,m,n)$: the extent of instability in hedonic function.

$f(\bullet \mid t-m)$: hedonic price function estimated at date $t-m$.

$f(x_t \mid t-m)$: predicted price of product with characteristic x_t.

$g(t,m,n)$: hedonic quality index.

x_t : characteristic vector of a product at date t.

3. **Data and Estimation Methods**

3.1 Characteristic and price data

- **The samples are the release-month prices of products placed on sale between 1999 and 2003.**
- **The prices of each product appears in our data set only once even if it continues to be sold in the market.**
- **1,539 samples**
- **Monthly data**
- **Data source : *Nikkei PC* (published by Nikkei BP)**

  mail-order price samples.
- **Prices are at the retail level.**

3.2 Explanatory variables of Characteristic and price data

- CPU clock frequency
- RAM capacity
- Hard Disk Drive (HDD) capacity
- TFT monitor dummy × monitor size
- Time dummy (date of release)

3.3 Functional form in hedonic regression

- **Linear**
- **Log-Linear**
- **Log-Log**
- **Box-Cox Linear**
- **Double Box-Cox**

3.4 Sample period

- 12 months is adopted as the sample period in estimating each hedonic regression.
- To capture the time-series changes in the parameters, we conduct rolling regression with twelve months data.

3.5 Product set for virtual price replacement

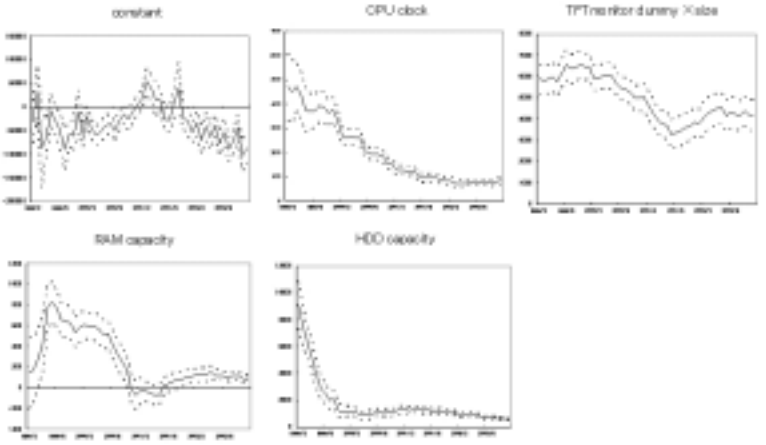
- Assume that the prices and characteristics of the representative product is backward moving averages over 6 months.
- Simulate the virtual price replacements using the representative products at each point.

Period		the number of products	value yen	CPU clock MHz	RAM MB	HDD GB	TFT monitor the diagonal inch
start	end						
1999/2	1999/7	195	262193	410.98	96.87	9.83	6.72
1999/3	1999/8	133	268433	431.83	93.82	9.88	7.01
1999/4	1999/9	130	216804	430.12	91.97	9.30	7.14
1999/5	1999/10	179	256505	445.63	88.29	10.55	7.32
1999/6	1999/11	161	238698	453.84	88.57	10.86	7.07
1999/7	1999/12	122	248108	458.08	70.30	11.40	7.41
1999/8	2000/1	116	229450	499.46	73.78	13.60	6.57
1999/9	2000/2	149	235658	507.47	77.26	13.41	6.83
1999/10	2000/3	167	227013	513.69	78.33	13.33	8.27
1999/11	2000/4	168	197978	520.21	78.19	13.66	4.74
1999/12	2000/5	165	202133	546.92	79.39	17.46	5.16
2000/1	2000/6	222	202779	597.85	82.88	18.39	4.64
2000/2	2000/7	218	187878	591.21	87.03	18.80	3.83
2000/3	2000/8	211	184481	600.06	87.11	18.65	3.51
2000/4	2000/9	207	193435	619.79	97.95	19.23	3.63
2000/5	2000/10	248	201877	680.03	92.88	24.87	4.47
2000/6	2000/11	240	189172	700.71	96.39	24.83	4.27
2000/7	2000/12	232	189897	700.55	94.62	25.56	4.16
2000/8	2001/1	254	193548	759.42	95.78	28.44	4.57
2000/9	2001/2	261	189470	760.82	97.81	32.31	4.80
2000/10	2001/3	241	204130	808.47	99.02	34.36	5.17
2000/11	2001/4	176	195864	833.34	103.48	35.70	5.36
2000/12	2001/5	148	201825	859.12	106.42	37.82	5.40
2001/1	2001/6	161	223178	948.39	111.84	46.89	7.01
2001/2	2001/7	150	228040	982.47	116.81	52.80	8.30
2001/3	2001/8	132	219122	1008.02	122.06	53.86	8.63
2001/4	2001/9	128	216305	1036.12	129.59	53.91	9.28
2001/5	2001/10	119	213222	1037.49	130.13	53.78	8.68
2001/6	2001/11	171	210926	1136.31	138.32	58.71	8.78
2001/7	2001/12	165	206126	1206.92	163.47	59.90	9.61
2001/8	2002/1	131	209406	1216.02	217.48	64.35	9.47
2001/9	2002/2	148	213127	1281.24	228.32	63.47	8.48
2001/10	2002/3	150	209980	1423.76	233.81	64.73	8.14
2001/11	2002/4	158	207673	1443.86	233.32	65.86	7.74
2001/12	2002/5	114	193481	1491.23	247.02	64.36	7.09
2002/1	2002/6	100	211233	1587.13	216.00	70.80	8.44
2002/2	2002/7	116	209210	1630.34	214.92	71.13	8.06
2002/3	2002/8	95	205105	1657.71	256.00	73.33	7.89
2002/4	2002/9	98	203332	1683.42	237.33	76.23	7.83
2002/5	2002/10	121	206161	1781.20	216.00	79.68	8.41
2002/6	2002/11	142	220059	1871.99	265.01	85.26	9.36
2002/7	2002/12	169	203506	1819.54	284.22	85.23	9.27
2002/8	2003/1	128	214388	1898.87	287.00	92.81	11.01
2002/9	2003/2	136	223860	2011.08	270.84	100.00	11.82
2002/10	2003/3	131	232668	2039.75	271.63	102.44	12.59
2002/11	2003/4	113	222284	2113.82	282.88	98.32	11.81
2002/12	2003/5	126	191612	2203.21	211.04	84.26	8.36
2003/1	2003/6	179	202930	2277.54	263.74	93.06	8.77
2003/2	2003/7	156	195615	2233.64	282.42	89.00	9.39
2003/3	2003/8	141	188100	2221.13	280.31	83.11	7.27
2003/4	2003/9	109	183636	2380.89	300.28	91.26	7.70
2003/5	2003/10	175	207949	2499.22	329.14	111.60	10.00
2003/6	2003/11	138	219159	2498.84	333.07	123.38	11.29
2003/7	2003/12	84	210306	2118.33	314.78	121.31	11.81

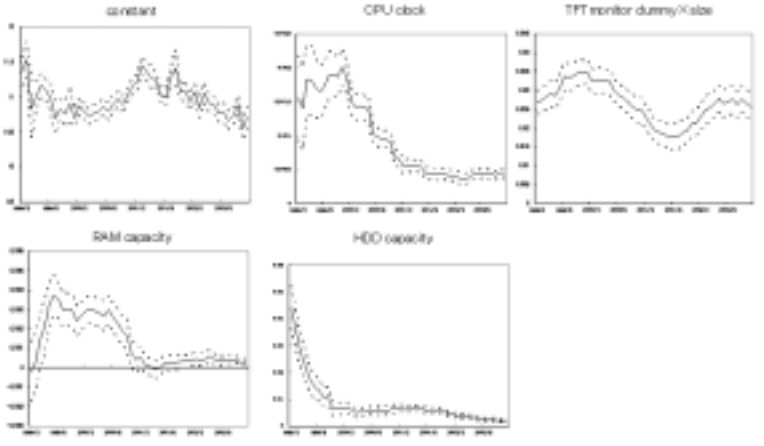
5. Characteristic of sample data

(the representative products at each point)

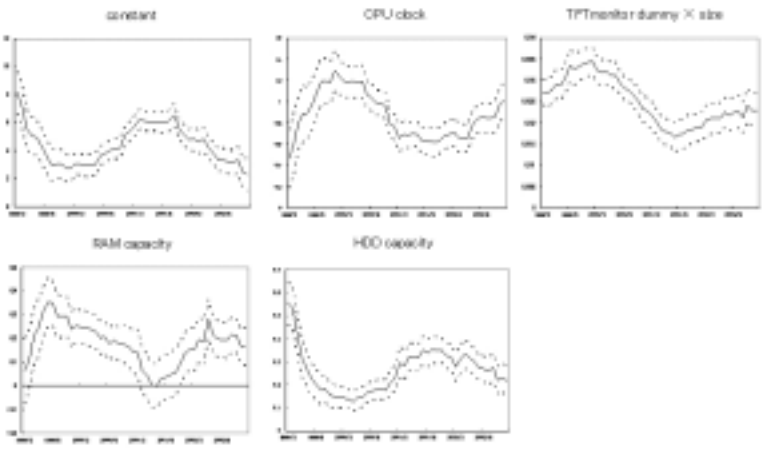
6. Hedonic Function Estimation Results : Linear form



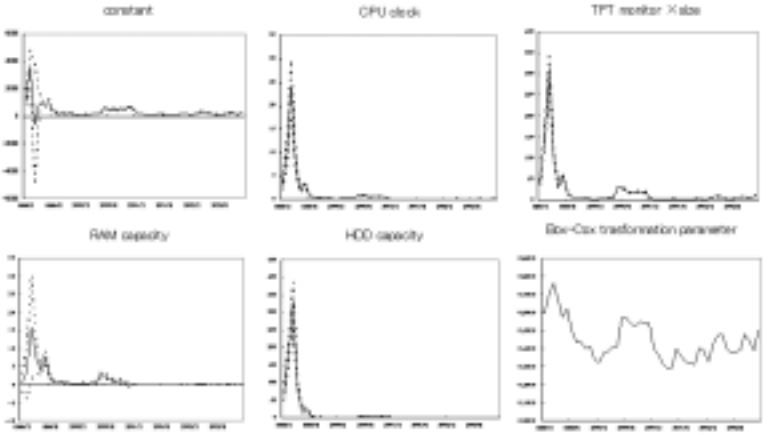
6. Hedonic Function Estimation Results : Semi-log form



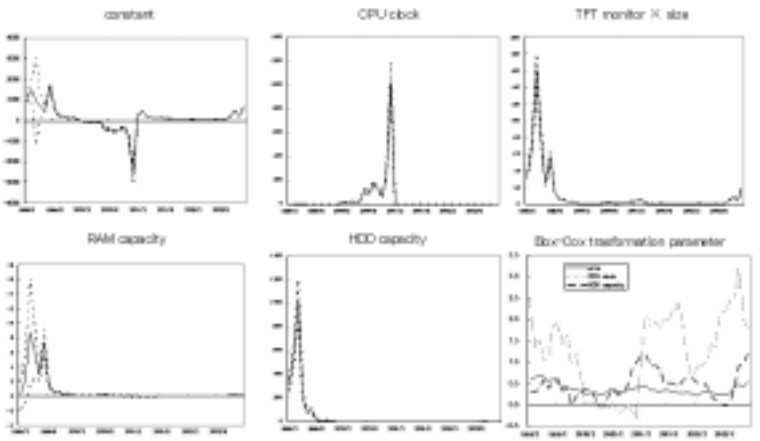
6. Hedonic Function Estimation Results : Double-log form



6. Hedonic Function Estimation Results : Semi Box-Cox form



6. Hedonic Function Estimation Results : Double Box-Cox form



6. Estimation Results of Hedonic functions

- For the linear and semi-log functional forms, the CPU clock frequency, RAM capacity and HDD capacity parameters decline overall.
- The Double Box-Cox functional form shows changes in the parameter in certain periods.
- the Double Box-Cox functional form indicates that the extend of non-linearity changes greatly depending on the estimation period.

7.Influence of the Outdated Parameters

- **d_{mn} differential approach**

$$R^{i}{}_{MN} = \Pr(dd^{i}_{mn} > 0 \mid M, N)$$
$$dd^{i}_{mn} = d^{i}_{mn} - d^{dbbox}_{mn}$$
$$m = 1, \ldots, M,$$
$$n = 1, \ldots, N$$

- **Regression approach**

$$d_{mn} = \alpha_{MN} + \beta_{MN} m + \varepsilon$$
$$m = 1, \ldots, M,$$
$$n = 1, \ldots, N$$

where

M : upper limit of m(interval of updating hedonic function)

N : upper limit of n(interval of price replacement)

Example of setting (M, N)

- *In case of* $M = 12$, $N = 6$
  - when the hedonic function is updated once per year
  - the price replacement from the old products to the new products every three months on average.

d_{mn} *Differential Approach*

$$R^{i}{}_{MN} = \Pr(dd^{i}_{mn} > 0 \mid M, N)$$

$$m = 1, \ldots, M,$$

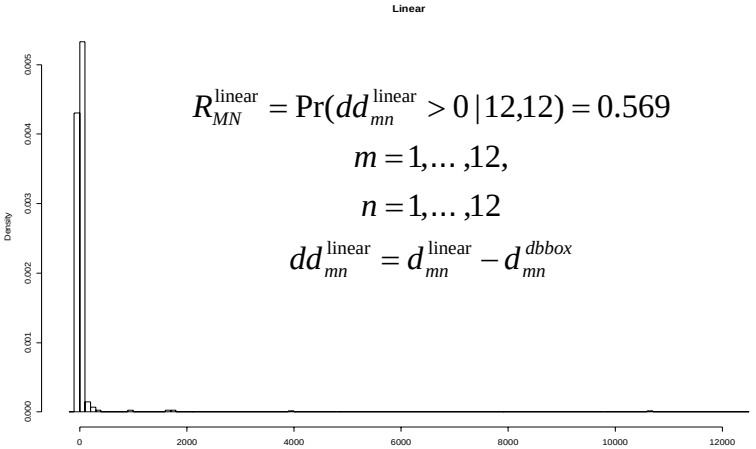
$$n = 1, \ldots, N$$

$$dd^{i}_{mn} = d^{i}_{mn} - d^{dbbox}_{mn}$$

$$i \in (\text{linear}, \text{semi-log}, \text{double-log}, \text{semi Box-Cox})$$

$$dbbox : \text{double Box-Cox}$$

7-1 the distribution of dd_{mn} Linear form, M=12, N=12



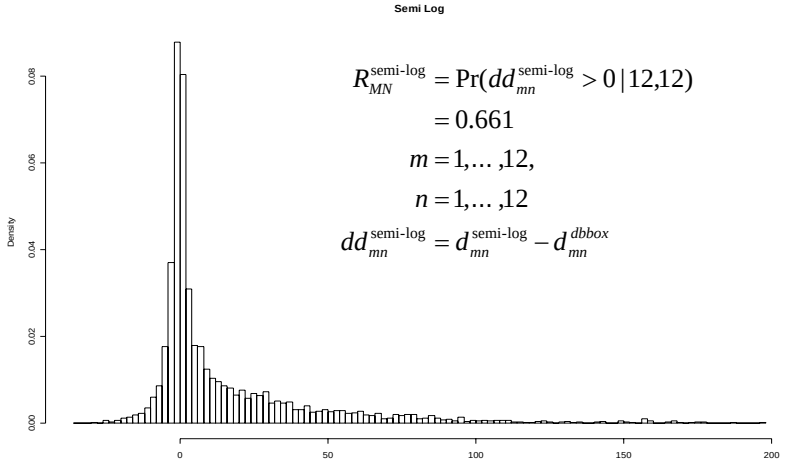
$$R^{\text{linear}}_{MN} = \Pr(dd^{\text{linear}}_{mn} > 0 \mid 12,12) = 0.569$$

$$m = 1, \ldots, 12,$$

$$n = 1, \ldots, 12$$

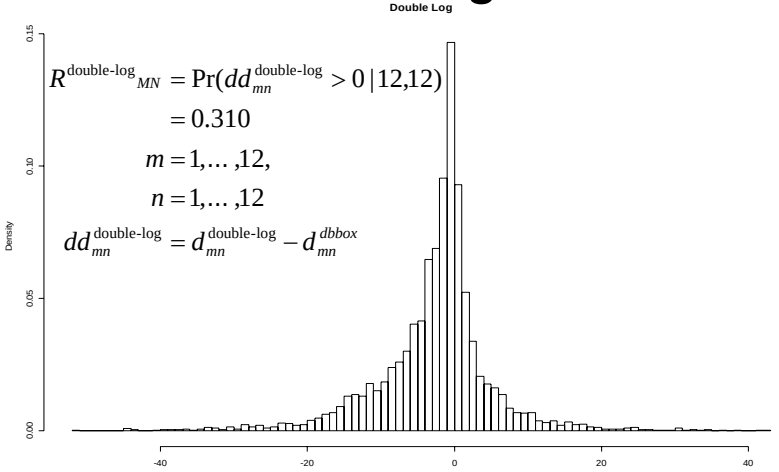
$$dd^{\text{linear}}_{mn} = d^{\text{linear}}_{mn} - d^{dbbox}_{mn}$$

7-2. the distribution of dd_{mn} semi-log form



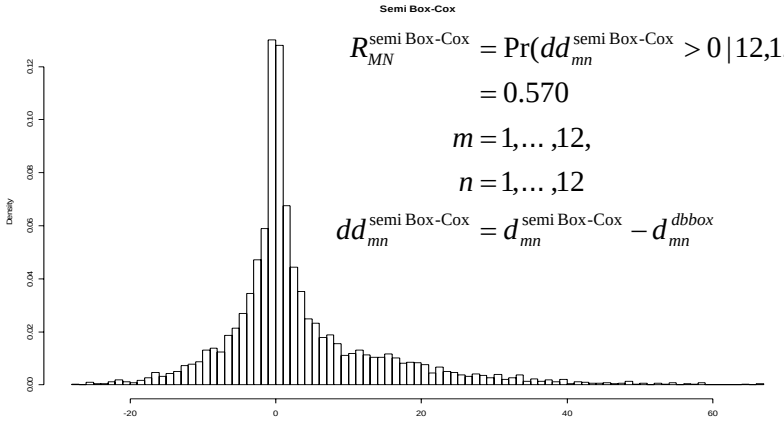
$$R_{MN}^{\text{semi-log}} = \Pr(dd_{mn}^{\text{semi-log}} > 0 \mid 12,12)$$
$$= 0.661$$
$$m = 1,\ldots,12,$$
$$n = 1,\ldots,12$$
$$dd_{mn}^{\text{semi-log}} = d_{mn}^{\text{semi-log}} - d_{mn}^{dbbox}$$

7-3. the distribution of dd_{mn} double log form



$$R^{\text{double-log}}{}_{MN} = \Pr(dd_{mn}^{\text{double-log}} > 0 \mid 12,12)$$
$$= 0.310$$
$$m = 1,\ldots,12,$$
$$n = 1,\ldots,12$$
$$dd_{mn}^{\text{double-log}} = d_{mn}^{\text{double-log}} - d_{mn}^{dbbox}$$

7-4. the distribution of dd_{mn} Semi Box-Cox form



$$R_{MN}^{\text{semi Box-Cox}} = \Pr(dd_{mn}^{\text{semi Box-Cox}} > 0 \mid 12,12)$$
$$= 0.570$$
$$m = 1,\ldots,12,$$
$$n = 1,\ldots,12$$
$$dd_{mn}^{\text{semi Box-Cox}} = d_{mn}^{\text{semi Box-Cox}} - d_{mn}^{dbbox}$$

Calculation result of R_{MN}

			M						
			6	7	8	9	10	11	12
N	6	linear	0.649	0.629	0.600	0.575	0.557	0.520	0.492
		semi-log	0.572	0.604	0.626	0.656	0.663	0.668	0.677
		double-log	0.311	0.319	0.318	0.306	0.306	0.294	0.282
		semi Box-Cox	0.574	0.575	0.570	0.569	0.551	0.541	0.529
	7	linear	0.675	0.640	0.610	0.585	0.567	0.530	0.501
		semi-log	0.583	0.603	0.622	0.653	0.660	0.664	0.672
		double-log	0.315	0.321	0.321	0.311	0.311	0.299	0.287
		semi Box-Cox	0.593	0.581	0.577	0.577	0.560	0.550	0.538
	8	linear	0.684	0.649	0.622	0.598	0.580	0.543	0.512
		semi-log	0.584	0.606	0.620	0.651	0.658	0.661	0.669
		double-log	0.312	0.320	0.325	0.315	0.315	0.304	0.293
		semi Box-Cox	0.606	0.596	0.585	0.586	0.569	0.560	0.547
	9	linear	0.694	0.660	0.635	0.612	0.594	0.556	0.525
		semi-log	0.592	0.613	0.629	0.648	0.656	0.660	0.666
		double-log	0.310	0.318	0.324	0.319	0.318	0.308	0.298
		semi Box-Cox	0.618	0.608	0.598	0.590	0.577	0.567	0.554
	10	linear	0.714	0.681	0.656	0.633	0.608	0.570	0.539
		semi-log	0.581	0.604	0.619	0.639	0.654	0.658	0.664
		double-log	0.319	0.328	0.334	0.329	0.321	0.312	0.302
		semi Box-Cox	0.613	0.605	0.596	0.590	0.581	0.573	0.561
	11	linear	0.720	0.685	0.660	0.636	0.612	0.585	0.553
		semi-log	0.572	0.596	0.611	0.631	0.646	0.656	0.661
		double-log	0.319	0.328	0.336	0.331	0.323	0.317	0.307
		semi Box-Cox	0.610	0.604	0.596	0.590	0.584	0.577	0.567
	12	linear	0.726	0.691	0.665	0.641	0.618	0.593	0.569
		semi-log	0.567	0.590	0.605	0.626	0.640	0.651	0.661
		double-log	0.321	0.330	0.338	0.334	0.326	0.320	0.310
		semi Box-Cox	0.612	0.604	0.596	0.588	0.582	0.577	0.570

7.5 Result of d_{mn} differential approach

- For the linear and Semi Box-Cox functional forms, R_{MN} generally exceeds 0.5 with some exceptions. Moreover, under these functional forms, R_{MN} approaches 0.5 as the value of M increases.
- For the semi-log functional form, all the calculated values of R_{MN} exceed 0.5. For this functional form, the value of R_{MN} rises along with the value of M.
- For the double-log functional form, all the calculated values of R_{MN} don't exceed 0.5. And the value of R_{MN} declines as the value of M rises.

8.1 Regression approach

$$d_{mn}^{i} = \alpha_{MN}^{i} + \beta_{MN}^{i} m + \varepsilon$$

$$m = 1, \ldots, M,$$

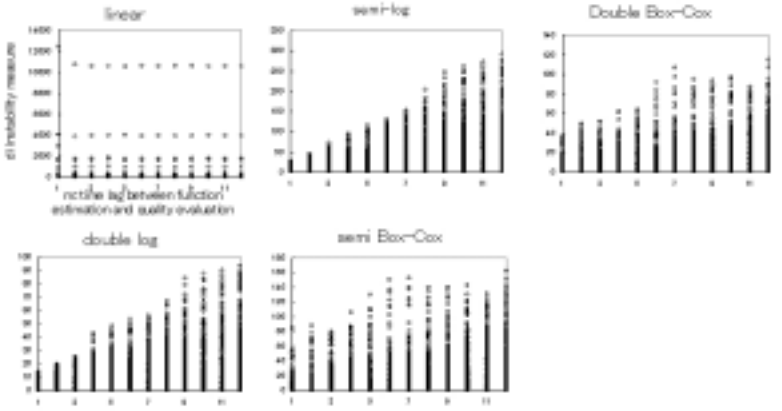
$$n = 1, \ldots, N$$

d : the extent of hedonic function instability.

m : the time - lag between the time of hedonic model estimation and that of application to actual price replacements.

$$i \in (\text{linear, semi - log, double - log, semi Box - Cox, double Box - Cox})$$

8.2 Correlation between d_{mn} and m



estimation result of β_{MIN}

			M						
			6	7	8	9	10	11	12
N	6	linear	0.3554	0.1840	0.1660	0.1275	0.1159	0.0970	0.0819
		semi-log	1.6977	1.7717	1.8642	1.9640	2.0147	2.0764	2.1514
		double-log	0.5255	0.5124	0.4952	0.4795	0.4767	0.4698	0.4628
		semi Box-Cox	0.7123	0.6671	0.6313	0.6449	0.6362	0.6383	0.6444
		double Box-Cox	0.5616	0.5676	0.5809	0.6003	0.6479	0.6698	0.6873
	7	linear	0.2238	0.1430	0.1281	0.1042	0.0946	0.0794	0.0669
		semi-log	1.9903	2.0689	2.1724	2.2044	2.2757	2.4557	2.5089
		double-log	0.6213	0.6123	0.5940	0.5787	0.5729	0.5603	0.5575
		semi Box-Cox	0.8451	0.7889	0.7509	0.7685	0.7630	0.7687	0.7780
		double Box-Cox	0.6357	0.6502	0.6828	0.7346	0.7564	0.7828	0.8081
	8	linear	0.2311	0.1500	0.1037	0.0750	0.0723	0.0499	0.0395
		semi-log	2.3108	2.4079	2.5021	2.6606	2.7569	2.8612	2.9895
		double-log	0.7320	0.7166	0.6999	0.6820	0.6773	0.6663	0.6597
		semi Box-Cox	1.0038	0.9327	0.8779	0.9019	0.9014	0.9109	0.9241
		double Box-Cox	0.7419	0.7526	0.7793	0.8418	0.8678	0.8009	0.9326
	9	linear	0.1829	0.0543	0.0468	0.0103	0.0149	−0.0031	−0.0230
		semi-log	2.6492	2.7760	2.8809	3.0312	3.1567	3.2855	3.4459
		double-log	0.8519	0.8341	0.8101	0.7932	0.7886	0.7797	0.7700
		semi Box-Cox	1.1630	1.0832	1.0163	1.0308	1.0462	1.0696	1.0791
		double Box-Cox	0.8348	0.8465	0.8599	0.9320	0.9792	1.0190	1.0585
	10	linear	0.1772	−0.0089	−0.0190	−0.0604	−0.0834	−0.0634	−0.0787
		semi-log	2.9773	3.1089	3.2511	3.4026	3.5675	3.7317	3.9225
		double-log	0.9880	0.9794	0.9529	0.9281	0.9260	0.8997	0.8998
		semi Box-Cox	1.3228	1.2287	1.1449	1.1519	1.1847	1.2079	1.2358
		double Box-Cox	0.9286	0.9353	0.9434	1.0083	1.0762	1.1319	1.1797
	11	linear	0.2936	0.0741	0.0122	−0.0451	−0.0371	−0.0484	−0.0656
		semi-log	3.3404	3.4805	3.6352	3.8188	4.0000	4.1898	4.4118
		double-log	1.1397	1.1225	1.1011	1.0727	1.0434	1.0230	1.0141
		semi Box-Cox	1.5313	1.4166	1.3116	1.3133	1.3401	1.3641	1.3857
		double Box-Cox	1.0546	1.0479	1.0444	1.1197	1.1767	1.2346	1.2994
	12	linear	−1.9582	−1.7789	−1.4801	−1.2403	−1.0211	−0.8695	−0.7021
		semi-log	3.7255	3.8882	4.0594	4.2638	4.4812	4.6878	4.9110
		double-log	1.3089	1.2854	1.2628	1.2354	1.2011	1.1705	1.1432
		semi Box-Cox	1.7473	1.6082	1.4800	1.4786	1.5114	1.5201	1.5577
		double Box-Cox	1.1966	1.1725	1.1538	1.2336	1.3004	1.3449	1.4128

8.3 Example:CGPI case

The "personal computers" item in CGPI

- the average monthly rate of change in the "personal computers" item in the Bank of Japan's Corporate Goods Price Index (CGPI) from January 2000 through December 2003 was approximately -3.1%.

Price index compiling Conditions

- $M = 6, N = 6 : \beta_{6,6} = 0.5616$ (double Box-Cox)
  - The hedonic function is updated once per 6 month
  - double Box-Cox form is adopted for hedonic function
  - the price replacement from the old products to the new products every 3-4 months on average.
- the prices of one-third of the surveyed prices used are marked down (an approach used when the quality of the new products exceeds the quality of the old products) in every month.

Estimated error from instability

- The error from the instability of the function is approximately 0.2% per month.

8.4 The results of regression approach

- For all of the functional forms other than linear, the value of β_{mn} increases as N grows larger.
- For the semi-log hedonic function the error in the evaluation of the quality ratio increases conspicuously along with the lag in the estimation period.
- For the double-log functional form, the error caused by the lag in the estimation period is small. This is because the elasticity of characteristics does not depend on the level of the characteristic values in this functional form.

9. Summary in Influence of the Outdated Parameters

- the extent of instability under the Double Box-Cox functional form is comparatively smaller than that under the linear, semi-log and Semi Box-Cox functional forms
- the extent of the instability under the Double Box-Cox functional form is but greater than that under the double-log functional form.

Concluding remarks(1)

- As the time-lag between the time of hedonic model estimation and that of application to actual price replacements becomes longer, the evaluation of quality by the hedonic regression method tends to diverge from the ideal value.
- In particular, when characteristics between the old and new products change greatly, the estimation error increases.

Concluding remarks(2)

- The selection of functional form should depend not only on the criterion of the sample fit when the hedonic function is estimated but also on the increase in error caused by the time lag from the original estimation to the update.
- When the improvement in product quality is rapid and the interval of function updating is extremely long, it is desirable to select double-log and other simple functional forms.