Risk Premium Accounting in Macro-Dynamic Term Structure Models*

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Abstract

This paper explores the sources of variation in expected excess returns on bonds within a Gaussian dynamic term structure model that: (i) conditions on information about output growth and inflation; (ii) accommodates variation in these macro variables that is orthogonal to variation in the yield curve; and (iii) captures the significant predictive power of macro variables for excess returns (over and above standard level, slope, and curvature factors). We compute excess returns on several portfolio positions in bonds that are specifically designed to reveal information about which risks are priced and the effects of macroeconomic shocks on the market prices of these risks. The evidence suggests that two risk factors underlie variation in expected excess returns, and that the growth rate of output is a particularly important determinant of both term premia and compensation for exposure to unpredictable changes in the slope of the yield curve. Finally, we explore the behavior of risk premiums associated with inflation and output risks that are spanned by the yield curve and, through the lens of our model, reassess the relationships between risk premiums, the shape of the yield curve, and macroeconomic activity.

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1 Introduction

This paper explores the sources of variation in expected excess returns on bonds within a Gaussian dynamic term structure model ($DTSM$) that: (i) conditions on information about output growth and inflation; (ii) accommodates variation in these macro variables that is orthogonal to variation in the yield curve; and (iii) captures the significant predictive power of macro variables for excess returns (over and above standard level, slope, and curvature factors) documented by Cooper and Priestley (2008) and Ludvigson and Ng (2009) and corroborated here for our sample of yields. Within the framework of this $DTSM$, we shed new light on which risks are priced by examining which bond portfolio positions generate economically significant variation in their associated excess returns, and document the contributions of macroeconomic shocks to variation in risk premiums over the past twenty years.

That the inclusion of macroeconomic information is likely to assist in determining which risks are priced in bond markets is illustrated by Figure 1. There we display the realized excess returns, over one-month and one-year holding periods, on a portfolio of bonds that reflects *pure* slope risk. That is, its payoff tracks movement in the slope of the U.S. swap curve, while being (locally) invariant to changes in the level or curvature of the yield curve (see Section 5 for details). A striking feature of these excess returns is how closely they track the negative of the growth rate of U.S. industrial production ($-GIP$) over the past twenty years.[1]

More directly, predictive regressions that include output growth and inflation, in addition to the first three principal components (PCs) of swap yields, have notably larger R^2 than those that exclude the macro variables. Moreover, and importantly for the motivation of this study, the inclusion of output growth and inflation renders the fourth and fifth PCs statistically insignificant in predicting excess returns. That is, once the macro variables are included with $PC1 - PC3$ in our predictive regressions, the counterpart to the forward factor examined by Cochrane and Piazzesi (2005) has no incremental explanatory power.

Incorporating information about the macro economy into our affine $DTSM$ in a manner that is consistent with the joint distribution of output, inflation, and bond yields precludes simply adding macro variables to a standard set of yield-based pricing factors $\mathcal{P}_t$. Following this route would lead to the counterfactual implication

[1] GIP is measured at the inception of the investments in the slope-mimicking portfolios of bonds. So Figure 1 says that output growth has substantial contemporaneous correlation with risk premiums (expected excess returns) in swap markets. This is a distinct observation from the widely documented result that the slope of yield curve *itself* has predictive content for *future* output growth. On the latter, see for example Estrella and Mishkin (1998) and Wright (2006).

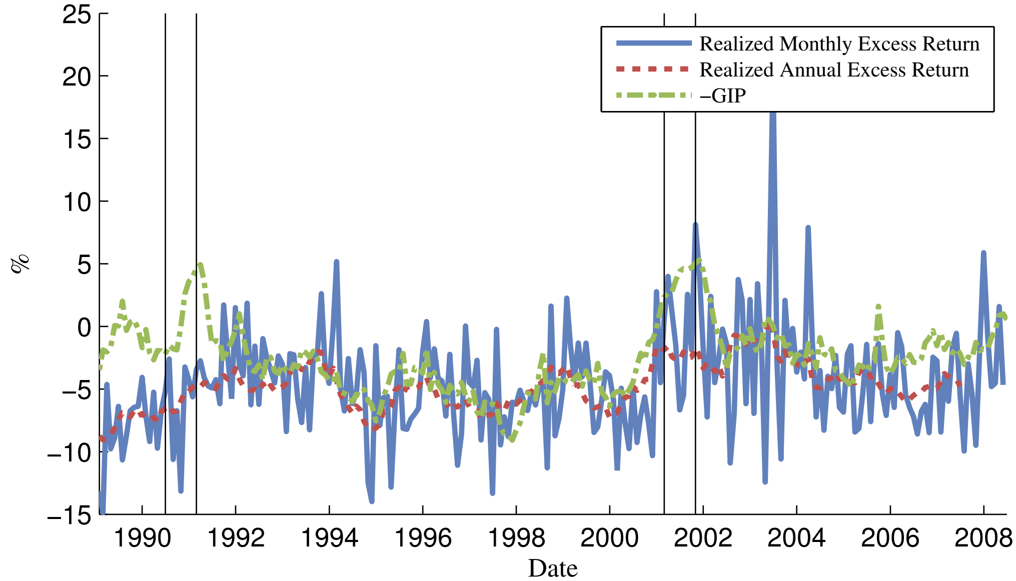


Figure 1: This figure displays the one-month and one-year realized excess returns on a portfolio of bonds with a payoff that, by construction, tracks changes in the slope of the U.S. swap curve. GIP is a smoothed version of the monthly growth rate of industrial production in the U.S.

that the macro variables included in $\mathcal{P}_t$ are spanned by (i.e., perfectly predicted by) observed bond yields.[2] In fact, large components of many macroeconomic variables are orthogonal to the yield curve. For example, regressing the growth rate of industrial production on the first three principal components of swap yields gives an R^2 of only 0.14 (see Section 2 for other examples). Moreover, expanding to a high-dimensional $\mathcal{P}_t$ would surely result in an over-parameterized pricing model. For bond markets in developed economies, three pricing factors are typically sufficient to explain well over 95% of the variation in yields (e.g., Litterman and Scheinkman (1991)).

Accordingly, we posit a five-dimensional state vector X_t, one that includes inflation and output along with level, slope, and curvature variables, so that the macro information is in general not perfectly spanned by the yield information. Consistent

[2] This is an implication of the joint assumptions that the short rate r_t is an affine function of $\mathcal{P}_t$ (which includes the macro variables) and no-arbitrage pricing. Bond yields are then affine functions of $\mathcal{P}_t$ (Duffie and Kan (1996)) so, unless there is a degeneracy in the effect of the macro variables, the pricing model can be inverted to express macro variables as functions of the of yields. Complete spanning is a feature of the models in Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), Rudebusch and Wu (2008), and Ravenna and Seppala (2007a), among many others.

with a low-dimensional $\mathcal{P}_t$, we assume that bond prices are exponential affine functions of three pricing factors, each of which is an affine function of the entire state X_t. The resulting macro-$DTSM$s do not restrict *a priori* which of the factors $\mathcal{P}_t$ are priced in bond markets; nor do they restrict how yield-curve and macro information affect model-implied, expected excess returns (beyond our Markov representation of X_t). Though $\mathcal{P}_t$ does not directly include an observed inflation target or output gap, we show that our macro-$DTSM$ is fully compatible with a monetary authority following a Taylor-style policy rule.

Our primary objective is a better understanding of how macroeconomic risks are reflected in excess returns on portfolios of bonds. As a first step toward achieving this goal we construct two portfolios of bonds with the properties that their returns track changes in either the level or slope of the swap curve, and their payoffs are (locally) invariant to changes in each other.[3] The expected excess returns on these portfolios reveal that both level and slope risks are priced. Moreover, their model-implied Sharpe ratios vary substantially over the business cycle, with both evidencing strong correlation with movements in the growth of output. Inflation plays a secondary role during our sample period, though surprises in inflation are shown to have short-lived effects on risk premiums. We proceed to examine how forward term premiums respond to macroeconomic shocks and, conversely, how output growth responds to shocks to term premiums. Using these findings we reassess some of Chairman Bernanke's interpretations of the interplay between term premiums, the shape of the yield curve, and macroeconomic developments in the U.S.

As part of this analysis we explore the dimensionality of the sources of variation in expected excess returns. In the context of $DTSM$ fit to yield-curve information alone, Cochrane and Piazzesi (2005) and Duffee (2008) conclude that expected excess returns on bonds of different maturities are (to a reliable approximation) perfectly correlated. To explore this issue within our macro-$DTSM$ we develop a general strategy for imposing the constraint that expected excess returns lie in a space of dimension d, less than the number of pricing factors (3 in our case), *without constraining a priori which risks are priced* in the economy. Our analysis of macro-$DTSM$s provides compelling evidence that, for expected excess returns conditioned on swap yields, output growth and inflation, d is at least two.

Finally, a common finding in empirical studies of term structure models is that at least one of the pricing factors is highly persistent, frequently evidencing near unit-root (Dai and Singleton (2000), Jardet, Monfort, and Pegoraro (2009)) and cointegrated

[3]That is, payoffs on the level-mimicking portfolio are locally invariant to changes in the slope factor, and payoffs on the slope-mimicking portfolio are invariant to changes in the level factor. The realized excess returns displayed in Figure 1 are those of the slope-mimicking portfolio.

(Shea (1992), Giese (2008)) behaviors. Prior to estimating our macro-$DTSM$s we show that the swap data support cointegration between bond yields and inflation. Accordingly, we enforce this restriction on the historical distribution X_t and this, in turn, influences the model-implied expected excess returns.

From an economic perspective, stochastic volatility is central to the properties of risk premiums in equilibrium models embodying long-run risks (Bansal and Yaron (2004), Bansal and Shaliastovich (2007)). Similarly, Ravenna and Seppala (2006) develop a New Keynesian $DSGE$ model based on a third-order approximation to an equilibrium model that has time-varying risk premiums and stochastic volatility. While we too believe that the incorporation of stochastic volatility is an essential next step, we initially develop our risk-premium accounting framework within the Gaussian family of $DTSM$s, because the literature on arbitrage-free macro-term structure models has, to date, focused primarily on Gaussian models. Our framework can, in principle, be extended to models with stochastic volatility, with some consequences for model specification and identification. We defer these issues to future research.

2 Empirical Motivations: Unspanned Macro Risks and Cointegration with Bond Yields

Central to our subsequent development of macro-$DTSM$s are the empirical observations that macroeconomic risks are unspanned by bond yields and that the components of output growth and inflation that are orthogonal to the yield curve have predictive content for future excess holding period returns on bonds. In this section we document the relevance of these observations for our data and sample period.

We study monthly data on the principal components (PCs) of zero-coupon bond yields constructed from the LIBOR and swap yield curves,[4] along with the core CPI inflation rate (INF) from the Bureau of Labor Statistics and the growth rate of industrial production (GIP) from the Federal Reserve's G.17 release, over the sample period from January, 1989 through June, 2008. We use LIBOR/swap data, instead of U.S. Treasury data, because the latter embody potentially large liquidity premiums,[5]

[4]On each date, these are bootstrapped from 6m LIBOR and the available subset of the 1y-10y swap rates, under the assumption of constant forward rates between maturities. LIBOR rates are provided by the British Bankers' Association (downloaded from Datastream), and swap rates are mid-market rates from Bloomberg. Data were taken from the last trading day of the month. On 5 occasions (9/99, 10/99, 12/99, 4/02, and 5/04), the 1y swap rate was unavailable in Bloomberg and data from Datastream was used instead.

[5]See Duffee (1996) for a discussion of money-market effect on the short end of the Treasury curve,

and most large financial institutions view the LIBOR/swap curve as the "primitive" yield curve underlying their fixed-income operations.

PCi is the i^{th} principal component of the continuously compounded 6 month, one- through five-year, seven-year, and ten-year nominal zero coupon bond yields. The loadings underlying the construction of the first three PCs (in order) have the familiar "level," "slope," and "curvature" patterns. For our data set and sample period, the correlation between $PC1$ and the ten-year zero-coupon bond yield is 0.93, and the correlation between $PC2$ and the spread between the ten-year and six-month zero yields is 0.88. That the level factor is essentially the long-term bond is typical in U.S. bond markets, as the sample correlation between the ten-year yield and the spread between the ten-year and six-month yields (the slope of the yield curve) is low, only -0.11 in our sample.

Inclusion of INF and GIP is natural as they are the central ingredients in standard representations of Taylor-style policy rules. The most common way in which the existing macro term-structure literature constructs macroeconomic variables such as inflation and output growth is to take annual moving averages (e.g., Ang and Piazzesi (2003), Rudebusch et al. (2006), Bandholz et al. (2007), Hordahl et al. (2006)). The disadvantage of working directly with monthly CPI inflation is that it is a highly volatile series which seems to reflect large, transitory shocks, one source of which may be measurement error. To filter out this noise, we construct an exponentially decaying weighted average of past inflation, in the spirit of a hidden components model whereby true inflation follows an AR(1) process and observed inflation is equal to true inflation plus an *i.i.d.* measurement error. The growth rate of industrial production is filtered similarly.[6]

Prior to exploring the descriptive properties of our data it is natural to inquire which, if any of these time series exhibit near unit-root like behavior. The conclusion we draw from various tests, discussed in more depth in Appendix A, is that $PC1$, $PC2$, and INF exhibit behavior that is well approximated by unit root processes, whereas $PC3$ and GIP appear stationary.

and Feldhutter and Lando (2007) andKrishnamurthy and Vissing-Jorgensen (2008) for evidence that U.S. Treasury yields embody a substantial, indigenous convenience premium.

[6]The reduced-form representation of a hidden components model is an ARMA(1,1) process (Granger and Newbold (1986)). In turn, the predicted one period ahead value of an ARMA(1,1) model is an exponentially-weighted moving average of past monthly values, with a decay coefficient equal to the moving average parameter. We emphasize that what is critical for our analysis is that both INF and GIP, smoothed in this manner, be relatively undistorted by measurement errors or transitory shocks that obscure the links between the macro economy and bond yields. These constructions need not correspond directly to their counterparts appearing in say a structural Taylor rule or the aggregate demand or supply equations of a $DSGE$.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$	GIP	INF
GIP	−0.1298	−0.2106	−0.2749	−0.3737	0.0623	1.0000	
INF	−0.7379	0.3828	0.3324	−0.0980	−0.0652	−0.2601	1.0000

Table I: Correlations between Yield and Macro Factors

There are economically large fractions of variation in our macro variables that are not spanned by variation in swap yields. Projecting GIP onto the first three PCs of bond yields gives and R^2 of only 0.14. Adding $PC4$ and $PC5$ as regressors raises the R^2, but only to 0.28. The projection of INF onto the first three PCs gives the much larger R^2 of 0.80. However, this finding must be interpreted with caution since we just documented near unit root behavior in INF, $PC1$, and $PC2$ so this relatively high R^2 may be spurious (Granger and Newbold (1974)). Indeed, when we project changes in INF onto contemporaneous changes in $PC1$ and $PC2$, as well as the level of $PC3$, the R^2 falls to 0.01. We conclude from this evidence that, in specifying macro-$DTSM$s, it is important to allow for variation in the macro variables that is unspanned by the term structure of bond yields.

Perhaps the most compelling motivation for exploring risk premium accounting in the presence of INF and GIP comes from projections of realized excess returns onto yield curve information and these macro variables. Whereas many econometric implementations of $DTSM$s focus on three-factor models (essentially on $PC1$ - $PC3$ as the risk factors), Cochrane and Piazzesi (2005) found that inclusion of the incremental information in $PC4$ and $PC5$ improved their forecasts of excess returns on bonds. Similarly, Duffee (2008) extracts an "expectations factor," constructed from bond yields alone, that also had predictive content for excess returns over and above $PC1$ - $PC3$. Both of these studies are entirely yield based in that no macroeconomic information is included in their modeling.

The contemporaneous correlations among the PCs and our macro variables are displayed in Table I. Note that, among the variables included, GIP has the largest correlation with $PC4$. This suggests that the role of $PC4$ in excess return regressions might in part be a surrogate for effects of output growth on the risk compensation in bond markets. Also the correlation between INF and $PC1$ is high, suggesting that the level of the yield curve, to a large degree, reflects low frequency movements in inflation. INF also has sizable correlations with the slope and curvature PCs, $PC2$ and $PC3$. We explore all of these links more formally in later sections within the context of our estimated $DTSM$s.

Table II displays the R^2s of the projections of realized excess returns from holding an n-month bond from time t to time $t+12$, $rx_{t+12}^{(n)}$ onto the yield-based PCs, GIP,

LHS \ RHS	$PC1$-$PC3$	$PC1$-$PC5$	$PC1$-$PC3$, GIP, INF
$rx^{(24)}_{t+12}$	0.20	0.32	0.39
$rx^{(60)}_{t+12}$	0.30	0.36	0.38
$rx^{(120)}_{t+12}$	0.34	0.36	0.39

Table II: Regression R^2s from Excess Return Regressions

and INF. The R^2s from the projections onto $PC1 - PC5$ are comparable to those reported by Cochrane and Piazzesi (2005),[7] confirming that there is incremental informational content to $PC4$ and $PC5$ for forecasting excess returns. Importantly for the motivation of this study, inclusion of GIP and INF in these projections renders $PC4$ and $PC5$ statistically insignificant. Further, with macro information, the improvement in R^2s is larger, particularly for shorter maturities of the underlying bonds. Together, these findings suggest that it is macroeconomic risk that underlies a large part of the variation in excess returns that is not captured by the first three yield PCs. Our subsequent analysis of $DTSM$s examines how these macroeconomic shocks are channeled through the market prices of risks in bond markets.

Our subsequent empirical analysis is based on a first-order vector-autoregressive (VAR) representation of $(PC1, PC2, PC3, GIP, INF)$. Ang, Piazzesi, and Wei (2003) and Jardet, Monfort, and Pegoraro (2009) posit higher-order VARs in studying $DTSM$s with (spanned) macro pricing factors. However, neither of these studies include inflation (a highly persistent process) nor $PC3$ in their state vector. Additionally, their much longer sample period includes the economically significant structural shifts in monetary policy in the late '70's and early '80's. In contrast, for our sample period of 1989 - 2008, choice of bond yields, and expanded state vector, a variety of formal model selection criteria all point to a first-order multivariate process (see Appendix B), consistent with our maintained assumption.

We conclude this section with an exploration of common trends among $PC1$, $PC2$, and INF. If these variables are cointegrated, or nearly so, then imposing this restriction within our dynamic pricing models may lead to a more reliable assessment of the their contributions to risk premiums in bond markets. Focusing on a first-order Markov representation, we proceed to test for cointegration using Johansen (1988)'s method for Gaussian VARs. Since $PC3$ and GIP are stationary, the number of cointegrating vectors r has to be $r \geq 2$. The number $r - 2$ tells us the

[7] Our PCs are not identical to those used by Cochrane and Piazzesi, because we construct PCs using bonds with maturities out to ten years, whereas their maximum maturity is five years.

number of nontrivial cointegrating relationships or, equivalent, $5 - r$ is the number of common trends underlying $PC1$, $PC2$, and INF. We find that we can reject $r = 2$ against an unrestricted VAR at the 5% significance level (LR statistic 35.52 versus critical value 34.91), but not $r = 3$ or $r = 4$. We interpret this as evidence in favor of cointegration and that there is at least one and perhaps two common trends underlying $(PC1, PC2, INF)$. Henceforth we focus on the nonstationary VAR specification that imposes the weakest set of restrictions, corresponding to $r = 4$ or a single common trend.

3 $DTSM$s with Unspanned Macro Risks

The premise of our analysis is that three risk factors adequately represent the correlation structure of bond yields for our range of maturities,[8] and variation in inflation and output growth is not spanned by these factors. The most general Gaussian affine model that captures these features of the historical data has the one-period interest rate r_t related to a set of three pricing factors $\mathcal{P}_t$ according to

$$r_t = \rho_0 + \rho_1 \cdot \mathcal{P}_t, \tag{1}$$

and has $\mathcal{P}_t$ and the macro variables $M_t' \equiv (GIP, INF)$ related to a latent, five-dimensional state vector f_t according to

$$\mathcal{P}_t = \phi_{\mathcal{P}0} + \Phi_{\mathcal{P}0} f_t \ \text{ and } \ M_t = \phi_{M0} + \Phi_{M0} f_t, \tag{2}$$

where $\Phi_{\mathcal{P}0}$ and Φ_{M0} are 3×5 and 2×5 matrices, respectively.

Assuming that f_t follows a Gaussian $VAR(1)$ under the risk-neutral pricing measure $\mathbb{Q}$, so too does $X_t' \equiv (\mathcal{P}_t', M_t')$:

$$\Delta \begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{Q}} \\ * \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} & 0 \\ * & * \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_X} \epsilon_{Xt}^{\mathbb{Q}}, \tag{3}$$

where $\epsilon_{Xt}^{\mathbb{Q}} \sim N(0, I_5)$ and we let $\Sigma_{\mathcal{P}\mathcal{P}}$ denote the upper 3×3 diagonal block of Σ_X. The matrix $K_{\mathcal{P}M}^{\mathbb{Q}}$ (the right, upper block of the feedback matrix $K_X^{\mathbb{Q}}$) is set to zero to ensure that the macro variables M_t are not spanned by bond yields. For

[8] Dai and Singleton (2002) and Piazzesi (2005) find that the addition of a fourth factor helps in capturing variation at the very short end of the yield curve owing (in part) to institutional features of the money markets. We focus largely on maturities between one- and ten-years. The model is easily extended to accommodate additional pricing factors for applications with an expanded set of maturities.

otherwise bond yields would in general be affine functions of both $\mathcal{P}_t$ and M_t, and so M_t could be extracted from yield information alone. The last two rows of $K_X^{\mathbb{Q}}$ are not identified from bond prices alone (see below), and they have no affect on pricing or risk premiums. Together, (1) and (3) imply that zero-coupon bond yields are affine functions of $\mathcal{P}_t$ (Duffie and Kan (1996)).

There are many observationally equivalent macro-$DTSM$s encompassed by this general setup, all of which are related by invariant transformations (see, e.g., Dai and Singleton (2000)). We propose a particularly convenient canonical model that is specified in terms of observable pricing factors $\mathcal{P}_t$ with a $\mathbb{Q}$ distribution governed by a set rotation invariant, and hence economically meaningful, parameters.

3.1 Our Canonical Macro-$DTSM$

A $\mathbb{Q}$-canonical model is achieved by imposing normalizations on the parameters $(K_{0\mathcal{P}}^{\mathbb{Q}}, K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}, \Sigma_{\mathcal{P}\mathcal{P}}, \rho_0, \rho_1)$ of (1–3) that ensure econometric identification and no arbitrage. To derive our canonical model we first rotate to a model in which the pricing factors $\mathcal{P}_t$ are the first three PCs, with the $\mathbb{Q}$ distribution of $\mathcal{P}_t$ governed by (3). This is possible, because the pricing model can be inverted to express $\mathcal{P}_t$ in terms of any three linearly independent combinations of yields; we choose $\mathcal{P}_t' = (PC1_t, PC2_t, PC3_t)$. Then, under the assumption that the eigenvalues $\lambda^{\mathbb{Q}}$ of $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$ are real and distinct,[9] and for a given long-run $\mathbb{Q}$-mean $r_\infty^{\mathbb{Q}}$ of r_t, we establish a unique mapping between the parameter set $(r_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma_{\mathcal{P}\mathcal{P}})$ and $(K_{0\mathcal{P}}^{\mathbb{Q}}, K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}, \rho_0, \rho_1)$. That is, the $\mathbb{Q}$ distribution of the PCs in (3) is fully characterized by the four parameters $(r_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}})$ and the free parameters in $\Sigma_{\mathcal{P}\mathcal{P}}$. A formal derivation of of these rotations and mappings is presented in Appendix C.

Our canonical model is completed with the assumption that the data-generating process of the (now observable) state X_t is the Gaussian process

$$\Delta \begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & K_{\mathcal{P}M}^{\mathbb{P}} \\ K_{M\mathcal{P}}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{3t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_X} \epsilon_{Xt}^{\mathbb{P}}, \tag{4}$$

where $\epsilon_{Xt}^{\mathbb{P}} \sim N(0, I_5)$. This representation allows for general feedback between the pricing factors $\mathcal{P}_t$ and the macro variables (GIP, INF).[10] Moreover, (4) implies

[9] This canonical model is a variant of the formulation in Joslin, Singleton, and Zhu (2009). They also address cases where the eigenvalues of $K_X^{\mathbb{Q}}$ are not distinct or are complex.

[10] In this respect, (4) is very similar to the descriptive six-factor model studied by Diebold, Rudebusch, and Aruoba (2006). As in their analysis, we emphasize the joint determination of the macro and yield variables (potential two-way feedback). We add the structure of a no-arbitrage pricing model so that it is possible to explore the properties of risk premiums in bond markets.

that there is a component of M_t that is not perfectly predicted by current and past values of $\mathcal{P}_t$, thereby capturing a key feature of the distribution of X_t documented in Section 2. Additionally, it follows that the excess return over h periods on a bond with maturity n issued at date t,

$$rx_{t+h}^{n,h} = -y_{t+h}^{n-h}(n-h) + y_t^n\, n - y_t^h\, h, \tag{5}$$

will in general be forecastable by all components of X_t. So, potentially, there is a component of the macro information (GIP, INF) that is unspanned by the yield factors $\mathcal{P}_t$ and that forecasts excess returns. Again, the evidence in Section 2 suggests that such unspanned predictor variables are quantitatively important.

Since $\mathcal{P}_t$ is observable, all of the parameters in (4) are directly identified from the first and second conditional moments of X_t. Thus, our complete pricing model is parameterized by $\Theta = (\lambda^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, L_X, K_0^{\mathbb{P}}, K_X^{\mathbb{P}})$, where L_X is the Cholesky factorization of Σ_X. For estimation of these parameters we assume that $\mathcal{P}_t$ is priced perfectly by the model and that there is an additional vector of yields y_t^e that are priced with Gaussian *i.i.d.* errors. Then the joint likelihood function can be written as:

$$\begin{aligned}\ell(X_t, y_t^e | X_{t-1}; \Theta) &= \ell(y_t^e | X_t, X_{t-1}; \Theta) \times \ell(X_t | X_{t-1}; \Theta) \\ &= \ell(y_t^e | X_t, X_{t-1}; \Theta) \times \ell(X_t | X_{t-1}; K_1^{\mathbb{P}}, K_0^{\mathbb{P}}, \Sigma_X).\end{aligned} \tag{6}$$

The conditional density $\ell(X_t|X_{t-1};\Theta)$ depends on $(K_0^{\mathbb{P}}, K_X^{\mathbb{P}}, \Sigma_X)$, but not on $(\lambda^{\mathbb{Q}}, r_\infty^{\mathbb{Q}})$. Furthermore, for any Σ_X, the $(K_0^{\mathbb{P}}, K_X^{\mathbb{P}})$ that maximize the likelihood are simply the standard OLS estimates. Therefore, in estimation, we need only maximize the likelihood over L_X, $r_\infty^{\mathbb{Q}}$, and $\lambda^{\mathbb{Q}}$.

This approach offers two potential advantages over working with a latent factor model directly as, for example, in Dai and Singleton (2000). First, the pricing factors are the observable constructs of interest, namely the PCs that have interpretations as level, slope, and curvature. Second, by having the primitive parameters be the readily interpretable long-run $\mathbb{Q}$ mean of r and the eigenvalues of $K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}}$, one can often guess reasonably good starting values for estimation. The starting value for L_X comes directly from OLS estimation of (4). We have found that, using this normalization, standard search algorithms converge extremely quickly to the global optimum of the likelihood function.

Another convenient feature of our canonical model is that cointegration under $\mathbb{P}$ can be imposed directly in equation (4). The restriction that X_t follows a cointegrated process amounts to a restriction on the rank of $[K_0^{\mathbb{P}}, K_X^{\mathbb{P}}]$ ($K_0^{\mathbb{P}}$ is included as we impose no trend). We can again concentrate the likelihood function built up from $\ell(X_t|X_{t-1}; K_X^{\mathbb{P}}, K_0^{\mathbb{P}}, \Sigma_X)$ and, though the maximum likelihood estimates of $K_0^{\mathbb{P}}$ and

$K_X^{\mathbb{P}}$ now depend on Σ_X, they continue to be computable in closed form. We focus on the case of one common trend underlying X_t (four cointegrating vectors). As with the unconstrained model, we end up maximizing the likelihood over $(L_X, r_\infty^{\mathbb{Q}}, \lambda^{\mathbb{Q}})$.

3.2 An Alternative $DTSM$ with Unspanned Macro Risks

To assist in interpreting our macro-$DTSM$, it is instructive to consider an alternative $DTSM$ with unspanned macro risks M_t that are linked directly to the pricing factors. Specifically, consider the $A_0(3)$ latent factor model in which (1) holds and, under $\mathbb{Q}$, the latent $\mathcal{P}_t$ follow the autonomous Gaussian process

$$\Delta \mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{Q}} + K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} \mathcal{P}_{t-1} + \sqrt{\Sigma_{\mathcal{P}\mathcal{P}}}\, \epsilon_{\mathcal{P}t}^{\mathbb{Q}}. \tag{7}$$

Further, suppose that $\mathcal{P}_t$ is linked to a J-dimensional vector of macro variables M_t ($J \leq 3$) according to

$$M_t = \Gamma_0 + \Gamma_1 \mathcal{P}_t + \nu_t, \tag{8}$$

where ν_t is a Gaussian measurement error satisfying $E^{\mathbb{P}}[\nu_t | M_{t-s}, \mathcal{P}_{t-s}, s \geq 1] = 0$. Thus, M_t is a noisy version of linear combinations of $\mathcal{P}_t$, $\Gamma_1 \mathcal{P}_t$. Owing to the presence of ν_t, M_t is not spanned by the latent pricing factors $\mathcal{P}_t$ or by bond yields. The model in Kim and Wright (2005), discussed subsequently, is the special case of this setup with $J = 1$ and M_t equal to the inflation rate.

In terms of pricing bonds, this is a standard $A_0(3)$ latent factor model. Accordingly, by the same reasoning as in Appendix C, we can rotate this model so that the pricing factors $\mathcal{P}_t$ are the first three PCs of zero-coupon bond yields. It follows that the model-implied yields are identical to those in our canonical macro-$DTSM$.

Where this model differs from ours is in the implied $\mathbb{P}$-distribution of $(\mathcal{P}_t, M_t)$. The typical $A_0(3)$ model assumes that $\mathcal{P}_t$ follows an autonomous Gaussian process under $\mathbb{P}$. Combining this assumption with (8), the historical distribution of X_t is

$$\Delta \begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ \Gamma_0 \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & 0 \\ \Gamma_1 K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & 0 \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_X} \begin{bmatrix} \epsilon_{\mathcal{P}t}^{\mathbb{P}} \\ \eta_t \end{bmatrix}, \tag{9}$$

where $\eta_t = (\nu_t + \Gamma_1 \sqrt{\Sigma_{\mathcal{P}\mathcal{P}}}\, \epsilon_{\mathcal{P}t}^{\mathbb{P}})$. It follows that this "noisy M_t" model is the constrained special case of our macro-$DTSM$ (see (4)) in which lagged values of M_t are of no value for forecasting future values of either itself or of the pricing factors (PCs), once one conditions on $\mathcal{P}_{t-1}$. Therefore, the significant role for the unspanned components of INF and GIP for forecasting excess returns, documented in Section 2, is absent from this model.

We can evaluate these constraints within our model with $M_t' = (GIP_t, INF_t)$ by testing whether $K^{\mathbb{P}}_{\mathcal{P}M} = 0$ and $K^{\mathbb{P}}_{MM} = 0$ in (4) using a likelihood ratio (LR) statistic. Within the unconstrained VAR, the LR statistic for the null hypothesis that $K^{\mathbb{P}}_{\mathcal{P}M} = 0$ is 27.4 with a p-value of 0.0001, and the LR statistic for the joint null hypothesis $K^{\mathbb{P}}_{\mathcal{P}M} = 0$ and $K^{\mathbb{P}}_{MM} = 0$ is 915 with a p-value of essentially zero. Thus, allowing for the feedback from lagged M to both $\mathcal{P}_t$ and M_t is central to capturing the joint dynamics of the macro variables and the PCs.

3.3 Compatibility with Taylor-Style Policy Rules

Though fitting a Taylor rule is not our objective, nor do we view the LIBOR rate as the Federal Reserve's policy rate, it is nevertheless instructive to briefly digress and show that, conceptually, our model is fully compatible with linear, Taylor-style monetary policy rules. Suppose one hypothesizes that

$$r_t = r^* + \alpha(\pi_t^* - INF_t) + \beta(GIP_t - g_t^*) + \nu_t, \tag{10}$$

where π_t^* is the policy authority's long-run inflation target, g_t^* is potential output, and ν_t captures "residual" policy actions. Also, for the purpose of this illustration, suppose that π_t^* and g_t^* are constants.[11] Then, given $r_t = \rho_0 + \rho_1 \cdot \mathcal{P}_t$ and any (α, β), we can always rewrite r_t within our macro-$DTSM$ as

$$r_t = r^* - \alpha INF_t + \beta GIP_t + \left[\rho_0 - r^* + \rho_1 \cdot \mathcal{P}_t + \alpha INF_t - \beta GIP_t\right]. \tag{11}$$

Defining the term in brackets in (11) to be the monetary policy shock ν_t, we have reconstructed the Taylor rule.

If the policy shock ν_t is orthogonal to INF_t and GIP_t, then we can recover consistent estimates of (α, β). For this, one does not need an arbitrage-free pricing model: one can simply regress r_t on INF_t and GIP_t. Our framework provides a convenient means of computing econometrically efficient estimates of the parameters governing a Taylor rule in setting where the macro ingredients are not spanned by bond yields.

The orthogonality condition justifying estimation by linear projection is strong, however. Indeed Cochrane (2007) argues that it is an inherent feature of new-Keynesian models that the policy shock ν_t "jumps" in response to changes in inflation or output gaps. If so, then the parameters of a Taylor rule are not identified in

[11] Measures of the output gap $GIP_t - g_t^*$ are provided by several sources, including the Federal Reserve. We have chosen to use industrial production as our measure of output, rather than a measure of the output gap, since the former is available at a monthly sampling frequency.

our macro-$DTSM$. Lack of identification is a generic feature of all macro-$DTSM$s specified with: a VAR representation of the state, an affine relationship between r and the pricing factors, and a term-structure of bond yields derived from the assumption of no arbitrage. The reduced-form, VAR representation of X_t is fully compatible with the existence of a Taylor-rule based policy, but it is not possible to reverse engineer the policy authority's rule from the reduced-form.[12]

4 Excess Returns in Macro-$DTSM$s

The only risks in our macro-$DTSM$ that are potentially priced are the shocks to the pricing factors $\mathcal{P}_t$. The inclusion of M_t in X_t when fitting our macro-$DTSM$s affects the time variation in the risk premiums on $\mathcal{P}_t$, but it does not introduce new priced risks over and above those that appear in standard three-factor Gaussian $DTSM$s (e.g., Dai and Singleton (2000), Duffee (2002)).

Elaborating, from (4) we obtain the drift $\mu_{\mathcal{P}}^{\mathbb{P}}(X_t)$ of $\mathcal{P}_t$ under $\mathbb{P}$. The drift of $\mathcal{P}_t$ under $\mathbb{Q}$, $\mu_{\mathcal{P}}^{\mathbb{Q}}(\mathcal{P}_t)$, comes from (3) and it depends only on $\mathcal{P}_t$. Combining these components gives the market price of $\mathcal{P}_t$ risk:

$$\Lambda_{\mathcal{P}}(X_t) = \Sigma_{\mathcal{P}\mathcal{P}}^{-1/2} \left(\mu_{\mathcal{P}}^{\mathbb{P}}(X_t) - \mu_{\mathcal{P}}^{\mathbb{Q}}(\mathcal{P}_t) \right). \tag{12}$$

4.1 On the Market Prices of Inflation and Output Risks

What we cannot identify are the market prices of inflation and output growth risks. The $\mathbb{P}$ drift X_t, $\mu_{Xt}^{\mathbb{P}}$, is again given directly by (4). However, the component of $\mu_{Xt}^{\mathbb{Q}}$ associated with M_t plays no role in pricing. Hence the subvector of $\Lambda_{Xt} = \Sigma_X^{-1/2}(\mu_{Xt}^{\mathbb{P}} - \mu_{Xt}^{\mathbb{Q}})$ associated with M_t, Λ_{Mt}, is not identified from historical information on X_t and bond prices. The inability to identify Λ_{Mt} is an inherent feature of any reduced-form $DTSM$ that includes macro variables in M_t that are not spanned by bond yields and that captures this lack of spanning in pricing.

Importantly, our model is not entirely silent about priced inflation and output risks. *Spanned* INF and GIP risks– that is, the components of M_t linearly spanned by $\mathcal{P}_t$– are potentially priced in our model. We can recover the market prices of these spanned risks by constructing the relevant linear combinations of $\Lambda_{\mathcal{P}t}$ in (12).[13]

[12] Cochrane (2007) makes the much more subtle point that, even in the presence of a fully specified equilibrium model, it might not be possible to identify the parameters of the Taylor using the moment conditions that underlie typical GMM estimators of (α, β).

[13] If M_t is spanned by $\mathcal{P}_t$, then necessarily we can fully identify Λ_{Mt}. Also, the prices of *unspanned* output or inflation risks are potentially identified from security prices with payoffs that are sensitive

4.2 Excess Returns on Bond Portfolio Positions

Since the drifts of X under both $\mathbb{P}$ and $\mathbb{Q}$ are affine functions of X and the volatilities of the state are constant, the expected excess returns on portfolios of zero-coupon bonds over instantaneous holding periods are proportional to $\mu^{\mathbb{P}}_{\mathcal{P}}(X_t) - \mu^{\mathbb{Q}}_{\mathcal{P}}(\mathcal{P}_t)$. Letting $P(X_t, \tau)$ denote the price of a bond issued at date t and with maturity τ, by Ito's lemma,

$$\text{drift}\Big(P(X_t,\tau)\Big) = \big[B(\tau)\cdot(\mu^{\mathbb{P}}_{\mathcal{P}}(X_t) - \mu^{\mathbb{Q}}_{\mathcal{P}}(\mathcal{P}_t)) + r_t\big]P(X_t,\tau), \tag{13}$$

where $B(\tau)$ are the loadings on $\mathcal{P}_t$ in the exponential-affine representation of $P(X_t, \tau)$. Therefore, the instantaneous expected excess-return from investing in this bond is $B(\tau)\cdot(\mu^{\mathbb{P}}_{\mathcal{P}}(X_t) - \mu^{\mathbb{Q}}_{\mathcal{P}}(\mathcal{P}_t))$.

An immediate implication of these observations is that variation in the (instantaneous) expected excess return on any bond portfolio is induced by variation in $(K^{\mathbb{P}}_{\mathcal{P}X} - K^{\mathbb{Q}}_{\mathcal{P}X})X_t$, where $K_{\mathcal{P}X}$ denotes the first three rows of K_X. This, in turn, implies that the assumption that expected excess returns lie in a factor space of dimension d that is less than three (the dimension of the pricing factors $\mathcal{P}_t$) amounts to the restriction that the rank of $\mathcal{K} \equiv (K^{\mathbb{P}}_{\mathcal{P}X} - K^{\mathbb{Q}}_{\mathcal{P}X})$ is d. We compute the singular value decomposition $\mathcal{K} \equiv U \times D \times V'$, where U is 3×3, D is a diagonal 3×3 matrix, and V is 5×3. The rank of $\mathcal{K}$ can then be controlled by restricting the number of nonzero entries of D. For a given value of d we normalize U to be a unitary matrix and V' to be a column-orthogonal matrix.[14]

We stress that this approach to restricting d places no restrictions on which risks are priced and it is rotation invariant. Given one's assumption about the distribution of the pricing errors (which bonds if any are priced with errors and the distributions of these pricing errors), the rank of D is completely invariant to how one rotates the risk factors. This is in contrast, for example, to the analysis in Cochrane and Piazzesi (2008). They first construct their excess return (forward) factor; then they form residuals of their PCs relative to the forward factor; then they estimate their model assuming only level risk is priced. Fixing the distributional assumption of the

to these specific risks. For instance, incorporating the prices of inflation-protected securities such as TIPS (see, e.g., D'Amico, Kim, and Wei (2008)) may shed additional light on the market price of inflation risk.

[14] To impose the restriction that risk premiums have a reduced rank (the effective number of factors driving expected excess returns is less than $dim(\mathcal{P}_t) = 3$) we proceed as in the literature on block-exogenous VARs. First the reduced rank regression of $\mathcal{P}_{t+1} - K^{\mathbb{Q}}_{\mathcal{PP}}\mathcal{P}_t$ on $[\mathcal{P}'_t, M'_t]$ and a constant gives ML estimates (conditional on $(\lambda^{\mathbb{Q}}, r^{\mathbb{Q}}_{\infty}, \Sigma_X)$) of $(K^{\mathbb{P}}_{0\mathcal{P}}, K^{\mathbb{P}}_{\mathcal{PP}}, K^{\mathbb{P}}_{\mathcal{P}M})$. From these estimates, we then construct the ML estimates $(K^{\mathbb{P}}_{0M}, K^{\mathbb{P}}_{M\mathcal{P}}, K^{\mathbb{P}}_{MM})$ as in the case of a block exogenous VAR. When there is cointegration and $d < 3$, we estimate the model directly by maximum likelihood under the rank constraints on $[K^{\mathbb{P}}_0, K^{\mathbb{P}}_X]$ and $K^{\mathbb{P}}_X - K^{\mathbb{Q}}_X$ imposed directly.

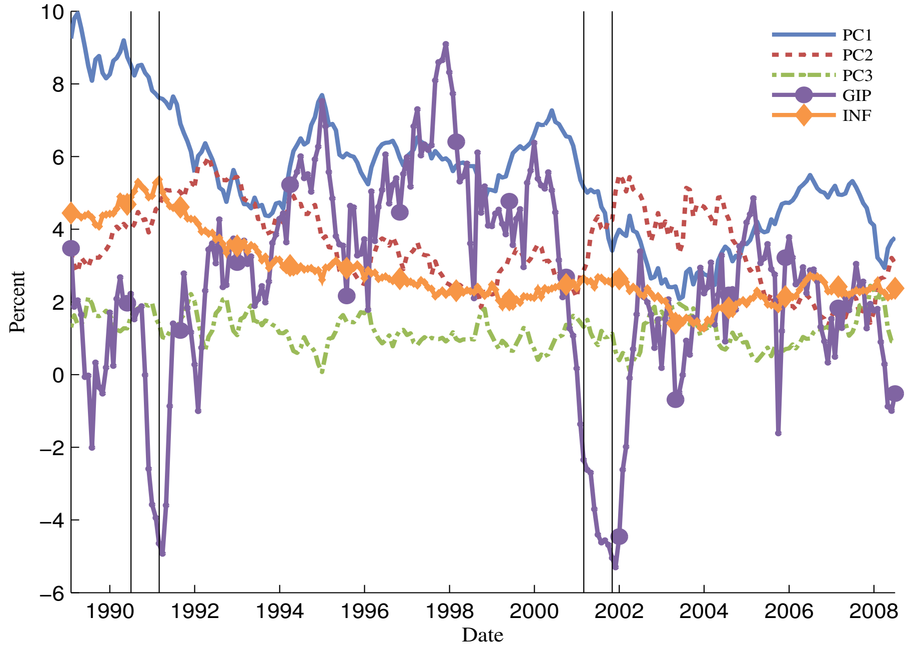


Figure 2: **Term Structure and Macro Variables** This figure plots the time series of $(PC1, PC2, PC3)$ of swap-implied zero yields and smoothed growth in industrial production and CPI inflation. The vertical bars mark $NBER$ recessions.

errors, the properties of their model-implied excess returns are not invariant to a reordering of these steps.

We denote the macro-$DTSM$s examined by $MA_0(5)^{1UR}_{dRP}$, with d denoting the number of risk premium factors and $1UR$ indicating that cointegration is imposed with one common trend. For comparison, we also examine a three-factor, yield-only model $YA_0(3)^{1UR}_{1RP}$ that omits M_t altogether. This is the $A_0(3)$ model estimated with one common trend and one risk premium factor.

5 Risk Premium Accounting: Model Comparison

The state variables used in our analysis of macro $DTSM$s are displayed in Figure 2.[15] Our sample period, which does not extend further back in time owing to the fact that the LIBOR-based swap market largely came into existence in the late 1980's, is

[15]The state vector has been rescaled as described in Appendix D.

Parameter	Estimate		
	$A_0(3)^{1UR}_{1RP}$	$A_0(5)^{1UR}_{1RP} - M$	$A_0(5)^{1UR}_{2RP} - M$
$\lambda_1^{\mathbb{Q}}$	-0.002121	-0.002105	-0.002113
$\lambda_2^{\mathbb{Q}}$	-0.043	-0.04295	-0.04291
$\lambda_3^{\mathbb{Q}}$	-0.1229	-0.1227	-0.1227
$r_\infty^{\mathbb{Q}}$	0.114	0.1144	0.1141

Table III: **Estimates of Risk-Neutral Parameters:** $\lambda^{\mathbb{Q}}$ is the vector of eigenvalues of $K_X^{\mathbb{Q}}$ and $r_\infty^{\mathbb{Q}}$ is the long-run $\mathbb{Q}$-mean of r_t.

characterized by a generally declining level of interest rates ($PC1$). However, there were sizable swings in both the level and slope of the swap curve during this period. Clearly visible in the GIP series are the recessions during the early 1990's and 2001. Inflation was relatively benign over this period.

Maximum likelihood estimates of the parameters governing the $\mathbb{Q}$ distributions of X_t are displayed in Table III. A striking feature of these estimates is how similar they are across the cointegrated models, whether or not X_t includes macro information or is three or five dimensional under $\mathbb{P}$. What this pattern says is that the parameters of the $\mathbb{Q}$ distribution are determined largely by the cross-sectional restrictions on bond yields, and not by their time-series properties under the $\mathbb{P}$ distribution.

The smallest eigenvalue of $K_{\mathcal{PP}}^{\mathbb{Q}}$ (in absolute value) is small, indicating that a linear combination of $\mathcal{P}_t$ has highly persistent behavior under $\mathbb{Q}$. However, unlike under $\mathbb{P}$, we do not impose cointegration under $\mathbb{Q}$.

Table VI in Appendix D displays the components of the singular value decomposition $\mathcal{K}$ that, along with the vector $\lambda^{\mathbb{Q}}$, determine the parameters $(K_0^{\mathbb{P}}, K_1^{\mathbb{P}})$ of the drift of X_t under the historical distribution. We begin our exploration of the implications of these estimates for bond market risk premiums by examining the time series behavior of expected excess returns on portfolios of bonds that represent pure exposures to the level and slope risks. Toward this end, we form a portfolio whose value changes (locally) one-to-one with changes in $PC1$, but whose value is unresponsive to changes in $PC2$ or $PC3$. Then we construct the portfolio of bonds that is a pure investment in $PC2$ risk. The temporal behavior of the expected excess returns on these PC-mimicking portfolios, $xPCj_t^0$ $(j = 1, 2)$, reveal which of level and slope risks are priced in the swap market and ultimately, as we discuss below, reveal the contributions of (orthogonalized) macroeconomic shocks to variation in premiums associated with these risks.

More precisely, assuming the PCs are constructed from a set of M bonds with maturities $(\tau_1, \tau_2, \ldots, \tau_M)$, we seek an M-vector of portfolio weights ω^j with the property that (locally) changes in the price of this portfolio track changes in PCj, while remaining unchanged in response to changes in PCk, for $j \neq k$. Letting $(\ell_1^j, \ell_2^j, \ldots, \ell_M^j)$ denote the loadings for the j^{th} PC, to track changes in PCj we set $\omega^{j\prime} = (-\ell_1^j/\tau_1, -\ell_2^j/\tau_2, \ldots, -\ell_M^j/\tau_M)$. Equivalently, we purchase $-\ell_i^j/\tau_i/e^{-y_i^0\tau_i}$ in face value of the bond with maturity τ_i, where y_i^0 is the yield on the i^{th} bond at the inception of the investment. Now imagine that the yield curve shifts by the amount $x \times PCk$ (the yield on the i^{th} bond shifts by the amount $x \times \ell_i^k$). Then immediately after this shift the price of our initial investment in bonds becomes

$$P_{jk}(x) = \sum_i -\frac{\ell_i^j}{\tau_i e^{-y_i^0\tau_i}} e^{-(y_i^0 + x\ell_i^k)\tau_i}.$$

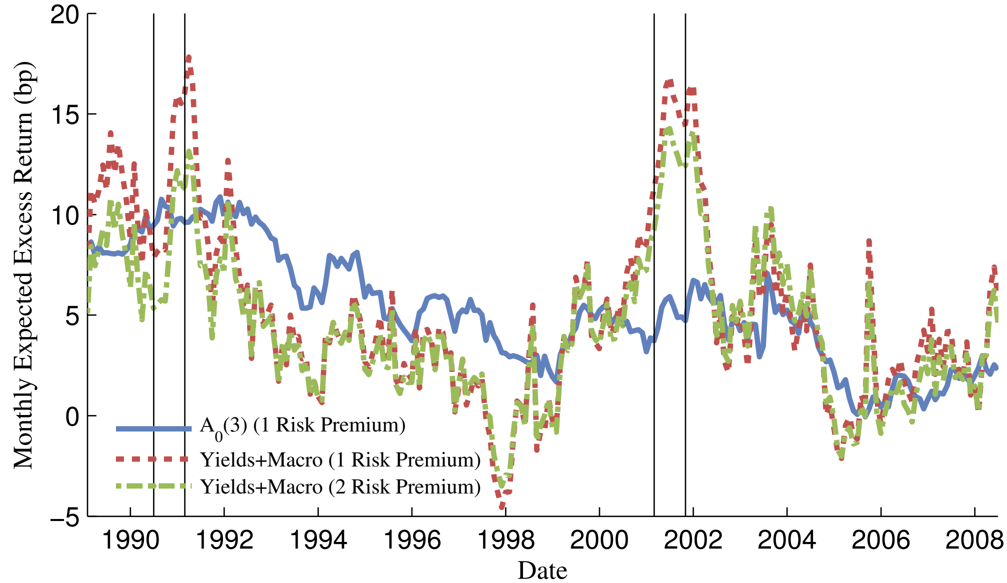
It follows that local changes in the value of our portfolio are $P'_{jk}(0) = \sum_i \ell_i^j \ell_i^k$, which is zero if $j \neq k$ by the orthogonality of the PCs.[16]

Moreover, with the state rotated to $X'_t = (\mathcal{P}'_t, M'_t)$, the instantaneous expected excess return $xPCj_t^0$ on the portfolio that mimicks movements in PCj is equal to the j^{th} entry of the difference $\mu_{\mathcal{P}}^{\mathbb{P}}(X_t) - \mu_{\mathcal{P}}^{\mathbb{Q}}(\mathcal{P}_t)$.[17] Economically significant variation in $xPCj_t^0$ means that the j^{th} PC risk is priced in swap markets. The finding that two or more of the PC risks are priced does not, by itself, imply that two or more risk-premium factors underlie expected excess bond returns (i.e., that $d > 1$). Conversely, if only one PC risk is priced, then there can only be one risk-premium factor, namely the risk premium for this PC. In general, the value of d is revealed by the rank of D in the SVD of $(K_{\mathcal{P}X}^{\mathbb{P}} - K_{\mathcal{P}X}^{\mathbb{Q}})$, and fixing d at a value less than three does not force the variation in any of the entries of $(K_{\mathcal{P}X}^{\mathbb{P}} - K_{\mathcal{P}X}^{\mathbb{Q}})X_t$ to zero.

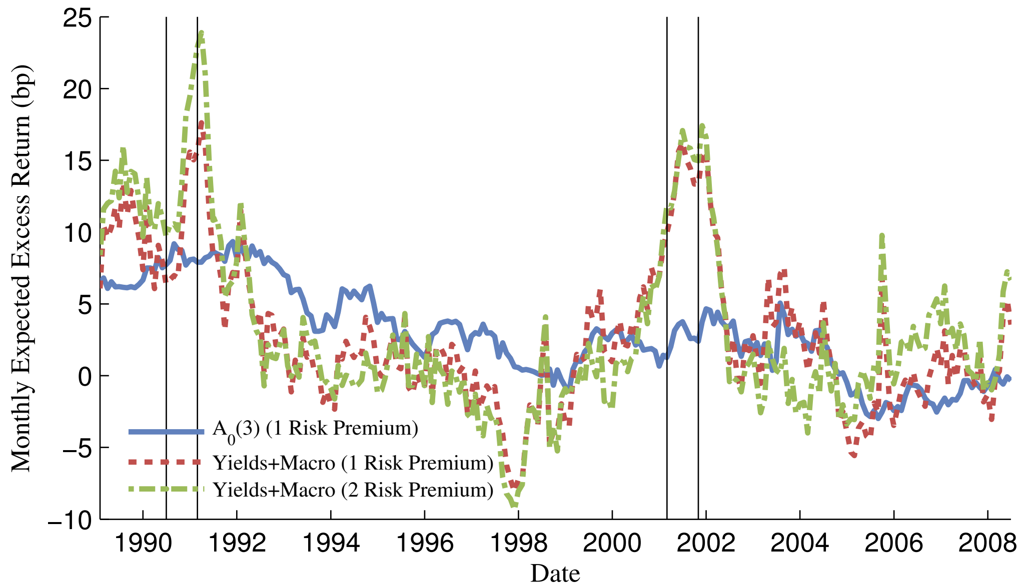
Using our factor-replicating portfolios and the maximum likelihood estimates of the parameters in Table III and Table VI, we computed the model-implied instantaneous expected excess returns $xPC1_t^0$ and $xPC2_t^0$ for both yield-only and macro-$DTSM$s. Focusing first on the level risk premium Figure 3(a), we see that $xPC1_t^0$ from the yield-only model $YA_0(3)_{1RP}^{1UR}$ shows much less cyclicality than its counterparts from

[16]For the monthly sampling frequency used in this study, we found that rebalancing our factor-mimicking portfolios, in order to better track changes in the associated principal components, had negligible effects on the results.

[17] From equation (13) in Section 4, when an amount proportional to ℓ_i^j/τ_i is invested in bond i, the expected excess return will be equal to $B_{pc}(j) \cdot (\mu_{\mathcal{P}}^{\mathbb{P}}(X_t) - \mu_{\mathcal{P}}^{\mathbb{Q}}(\mathcal{P}_t))$, where $B_{pc}(j) = \sum_{i=1}^M \ell_i^j B(\tau_i)$ is the vector of weights that define the affine mapping between $PCj(t)$ and X_t. Since the state is rotated so that the first three entries of X are $\mathcal{P}$, $B_{pc}(j)$ is the selection vector with a one in the j^{th} entry and zeros elsewhere.



(a) Excess Return on Level-Mimicking Portfolio



(b) Excess Return on Slope-Mimicking Portfolio

Figure 3: Expected excess returns on the (negative of the) level- and the slope-mimicking portfolios implied by various models.

the macro-$DTSM$s. $xPC1_t^0$ from the two macro-$DTSM$s, $MA_0(5)_{dRP}^{1UR}$ with $d = 1$ or 2, track each other quite closely.

The differences between the yields-only and macro models are comparably large for the case of slope risk Figure 3(b). Again model $YA_0(3)_{1RP}^{1UR}$ without macro variables produces a risk premium with much less variability over the business cycle than what is produced by models $MA_0(5)_{dRP}^{1UR}$ ($d = 1$ or 2).[18] The latter are at their highest levels during the two recessions, and achieve their lowest values (during our sample period) around the time of the Asian crisis in 1997-98. The slope of the swap curve ($PC2$) increased substantially in these recessions, especially in early 2000, and the curve was relatively flat around the time of the Asian crisis (see Figure 2).

6 Forward Term Premiums

Excess holding period returns on portfolios of individual bonds reflect the risk premiums for every segment of the yield curve up to the maturity of the underlying bond. A different perspective on market risk premiums comes from inspection of the forward term premiums, the differences between forward rates for a q-period loan to be initiated in p periods and the expected yield on a q-period bond purchased p periods from now. Figure 4 displays the forward term premiums based on the point estimates of model $MA_0(5)_{2RP}^{1UR}$ for "in-p-for-1" loans (one-year loans initiated in p years) for $p = 2, 5$, and 9. These premiums are increasing in p.

The levels of our long-term forward term premiums lie somewhat below those typically obtained from yield-only models. For instance, Rudebusch, Sack, and Swanson (2007) compare "in-10-for-1" forward term premiums from the $YA_0(3)$ models of Bernanke, Reinhart, and Sack (2004) (updated) and Kim and Wright (2005), the model-free forward-factor regressions from Cochrane and Piazzesi (2005), and the $DSGE$ motivated $DTSM$ in Rudebusch and Wu (2007). For the overlapping segment of our sample periods, these forward rates tend to fluctuate between two and four percent (see Figure 4 in Rudebusch and Wu (2007)).

A more direct comparison for swap data and our sample period is provided in Figure 5, which displays the "in-9-for-1" forward term premiums for the three models we are comparing. Throughout most of the period between 1989 and 2001 the forward term premium from model $MA_0(5)_{2RP}^{1UR}$ lies below those implied by the other models. This is a manifestation, we believe, of the fact that expectations of future spot rates

[18] When we estimated a yield-only with X_t comprised of the of first five PCs of bond yields and $d = 1$, our counterpart to the model examined by Cochrane and Piazzesi (2008), the implied $xPC2_t^0$ is virtually constant over our sample period. This reinforces our finding that conditioning on macro information gives new insights into the cyclical behavior of bond risk premiums.

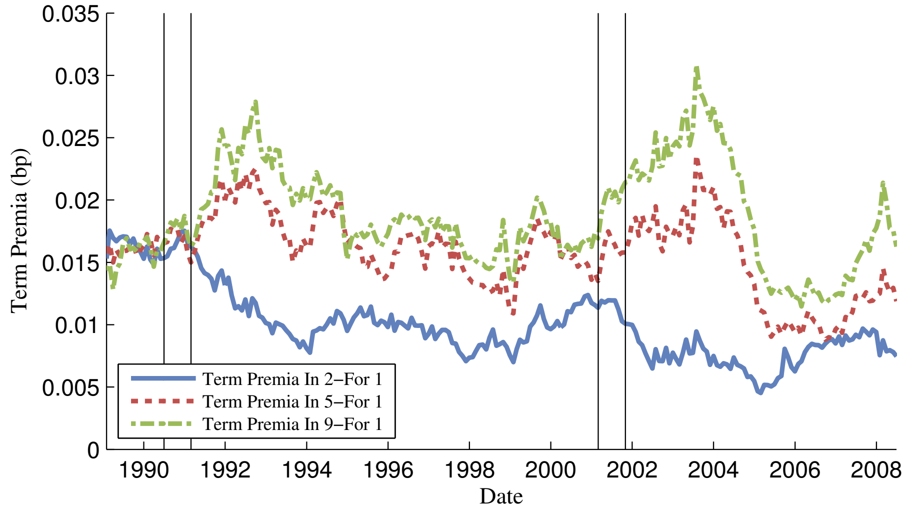


Figure 4: **Forward Term Premiums in Model** $MA_0(5)^{1UR}_{2RP}$ **:** The figure plots the difference between the forward rate on a one-year loan starting in p years and the expected spot rate $E^P_t[y^{12}_{t+p*12}]$, the "in-p-for-1" forward term premiums.

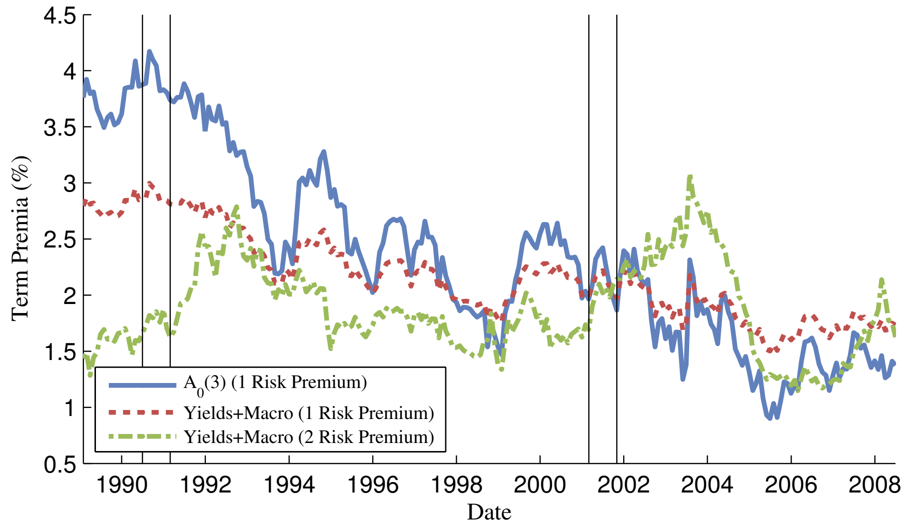


Figure 5: **"in-9-for-1" Forward Term Premiums:** The figure plots the difference between the forward rate on a one-year loan starting in 9 years and the expected spot rate $E^P_t[y^{12}_{t+108}]$, the "in-9-for-1" forward term premiums, for models $YA_0(3)^{1UR}_{1RP}$, $MA_0(5)^{1UR}_{1RP}$, and $MA_0(5)^{1UR}_{2RP}$.

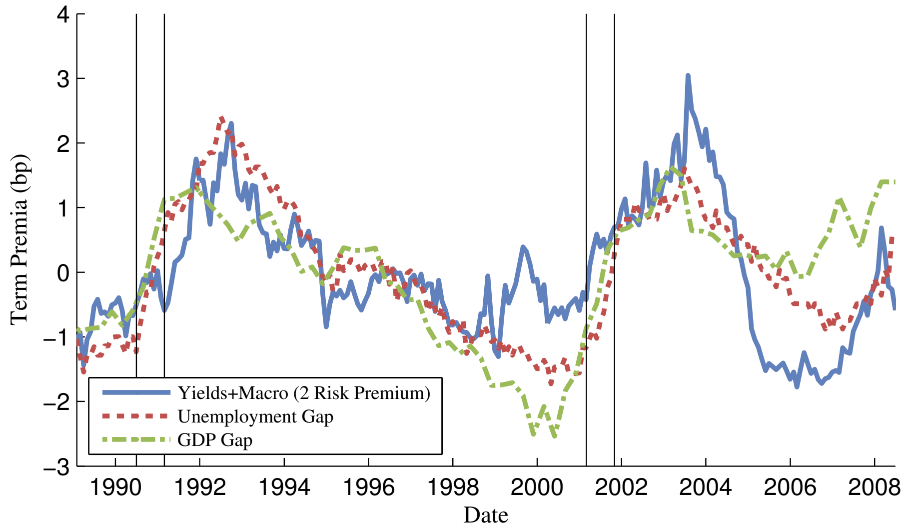


Figure 6: **Term Premiums and Macro Variables:** "in-9-for-1" term premiums for model $MA_0(5)^{1UR}_{2RP} - M$, along with standardized measures of the output gap (based on GDP) and unemployment gap.

play a more prominent role in shaping yields on long-term bonds in model $MA_0(5)^{1UR}_{2RP}$ owing to the conditioning on macroeconomic information and the accommodation of two risk-premium factors. With $d = 2$, more of the variation in forward rates is being attributed to variation in expectations of future spot rates.

Equally notable is that, comparing across the macro-$DTSM$s, the forward term premium with $d = 2$ shows more cyclical variation than with $d = 1$, particularly prior to 2001. We view these patterns in Figure 5 as providing compelling evidence that the dimensionality of expected excess returns in swap markets is at least two.

Additional insight into the properties of the term premium in model $MA_0(5)^{1UR}_{2RP}$ comes from overlaying various measures of (the negative of) the output gap and the unemployment rate on the model-implied term premium. The close tracking in Figure 6 of the forward term premium and the GDP output gap and unemployment rate, neither of which was used in the estimation of our macro-$DTSM$s, is striking. From a comparison of Figure 5 and Figure 6 it is apparent that the yields-only model $YA_0(3)^{1UR}_{1RP}$ fails to replicate the cyclical fluctuation in the forward premiums in the early 1990's. Furthermore, the relatively delayed peak in the early 2000's of premiums in model $MA_0(5)^{1UR}_{2RP}$, relative to the NBER dating of the 2001 recession, matches well with the delayed peak in unemployment and trough in the output gap.

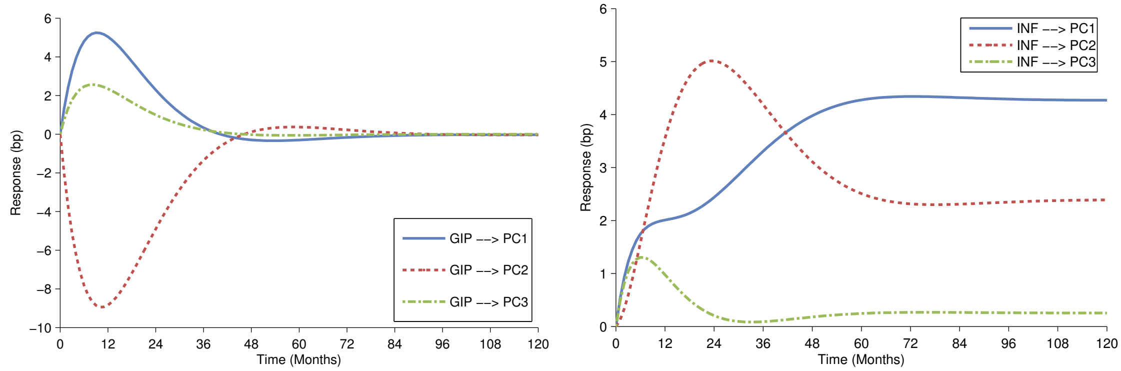
At the shorter end of the maturity spectrum, the "in-2-for-1" forward term premium implied by model $MA_0(5)^{1UR}_{2RP}$ exhibits comparable high-frequency (i.e., shorter than business cycle frequency) variation as the "in-2-for-0.25" forward term premium computed by Kim and Orphanides (2005). Their premium was inferred from an $YA_0(3)^{0UR}_{3RP}$ model estimated using survey forecasts of future interest rates. Professional forecasters are conditioning (at least) on similar macro information as that embodied in GIP and INF, and so we find it reassuring that our implied forward term premiums show similar patterns.

The forward term premiums in model $MA_0(5)^{1UR}_{2RP}$ are not perfectly tracked by macroeconomic variables tied to the real business cycle. Two periods stand out when there were particularly large differences between the premiums and the standardized macro variables: around the peak of the dot-com equity market bubble and the period of the bond market "conundrum" during 2005 - 2006. Several authors have attributed the behavior of long-term rates during the latter period to sharp declines in forward term premiums.[19] The evidence in Figure 6 is consistent with this, but it also suggests that falling forward term premiums were not driven by considerations related to output growth or unemployment alone. Wright (2008) conjectures that the relatively steep decline in forward term premiums during the conundrum period may be attributable in part to declining uncertainty about future inflation rates. An interesting question for future research is whether, when viewed through the lens of a macro-$DTSM$ that accommodates time-varying volatility, changing second moments of macroeconomic variables do in fact explain the gaps we see in Figure 6.

7 More on Macro Risks and the Term Structure

To delve more deeply into the effects of macroeconomic information on the shape of the swap curve we examine impulse responses to shocks to M_t. In interpreting the following results, two points about these responses warrant emphasis. First, the ordering of the variables in the model-implied VAR for X_t has the PCs preceding the macro variables, exactly as they enter in (4). It follows that the orthogonalized responses represent shocks to $GIP^{\perp}$ and $INF^{\perp}$. Second, these responses are not identical to those implied by an unconstrained VAR estimated outside of our macro-$DTSM$. As discussed in Joslin, Singleton, and Zhu (2009), setting d (the dimensionality of expected excess returns) less than three implies restrictions across the risk-neutral and

[19] Recent papers on this issue, using both reduced-form and structural pricing models, include Rudebusch, Swanson, and Wu (2006), Cochrane and Piazzesi (2008), Bandholz, Clostermann, and Seitz (2007), and Backus and Wright (2007).



(a) $GIP \rightarrow PC$'s (b) $INF \rightarrow PC$'s

Figure 7: Impulse responses of PCs to shocks to GIP and INF within the estimated $\mathbb{P}$-distribution of X_t implied by model $MA_0(5)^{1UR}_{2RP}$.

physical distributions of bond yields. This, in turn, implies that maximum likelihood estimates of the parameters in (3)– and hence the impulse responses implied by these estimates– are affected by the no-arbitrage constraints associated with our macro-$DTSM$ pricing model.

7.1 Responses of PCs to Innovations in M_t

We begin by examining the responses of the PCs to shocks in GIP and INF implied by model $MA_0(5)^{1UR}_{2RP}$. From Figure 7(a) it is evident that shocks to GIP have cyclical effects on the PCs that peak at about one year and then die out after about three years. The largest effects are on the slope $PC2$ with increases in GIP tending to induce flattening in the swap curve. This is why Figure 1 shows a strong positive comovement between the excess return on the slope-mimicking portfolio and minus the rate of output growth.

Shocks to INF have permanent long-run effects on the level and slope of the swap curve owing to the cointegration among $(PC1, PC2, INF)$ (Section 2). The largest effects in the long run are on $PC1$. Over business cycle frequencies, INF induces an increase in $PC2$ (yield-curve steepening). These patterns are consistent with $DSGE$s that emphasize the importance of long-run inflation expectations and central bank inflation targets for the shape of the yield curve (e.g., Hordahl and Tristani (2007), Rudebusch and Wu (2008)).

7.2 Priced "Macro" Risks in Model $MA_0(5)^{1UR}_{2RP}$

Expanding upon our discussion of priced risks in Subsection 4.1, only the prices of risk associated with the components of GIP and INF spanned by $\mathcal{P}_t$ (say $SGIP$ and $SINF$) are identified in model $MA_0(5)^{1UR}_{2RP}$.[20] To illustrate the implications of this observation, it is instructive to revisit Chairman Bernanke's assertion that "a substantial portion of the decline in distant-horizon forward rates of recent quarters can be attributed to a drop in term premiums. ... the decline in the premium since last June 2004 appears to have been associated mainly with a drop in the compensation for bearing real interest rate risk."[21] His assessment is based on Kim and Wright (2005) and, as we saw in Subsection 3.2, their framework presumes that macro information has no predictive content for future $\mathcal{P}_3$ or inflation, after conditioning on current $\mathcal{P}_t$. Relaxing these restrictions on the predictive content of M_t, we also find that risk premiums fell substantially in 2004 (Figure 6), though in our model the decline started earlier than the middle of 2004.

To examine the *source* of the decline in forward term premiums, we computed the expected excess returns on the portfolios of bonds with payoffs that are (locally) perfectly correlated with movements in $SGIP$ and $SINF$, $xSGIP_t$ and $xSINF_t$, both of which are weighted sums of the excess returns on the PC-mimicking portfolios. We see from Figure 8 that $xSGIP_t$ and the "in-9-for-1" forward term premium are highly correlated, with a sample correlation of 0.71. This high correlation is anticipated by the patterns in Figure 6 as we would expect a high $xSGIP$ when output growth is near a trough after a recession.

$xSINF$ is quite small throughout our sample period until its trough in 2003. Notably, Chairman Bernanke gave speeches on deflation risk in November, 2002 and and again in July, 2003 expressing concerns about possible deflation in the U.S.[22] We see that, during this window of time, expected excess returns to bearing spanned inflation risk fell sharply into negative territory.

Also notable about Figure 8 is that, during late 2003 and 2004 when forward term premiums fell sharply, there were simultaneous moves, in opposite directions, in $xSGIP_t$ and $xSINF_t$. The rise in $xSINF_t$ during this period coincides with a

[20] The fitted values from the projections of GIP and INF onto $\mathcal{P}_t$ are constructed using the weights $(-0.21, -0.53, -1.71)$ and $(-0.41, -0.32, 0.69)$, respectively. Recall from Section 2 that the R^2s from these regressions are 0.14 and 0.80.

[21] See his speech before the Economic Club of New York on March 20, 2006 titled "Reflections on the Yield Curve and Monetary Policy."

[22] See Chairman Bernanke's remarks "Deflation: Making Sure 'It' Doesn't Happen Here" before the National Economists Club on November 21, 2002 and "An Unwelcome Fall in Inflation?" before the Economists Roundtable on July 23, 2003.

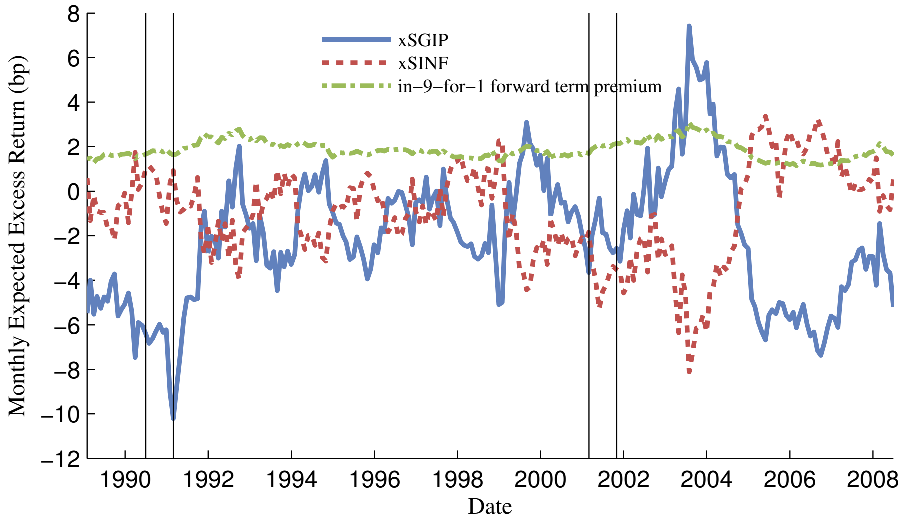
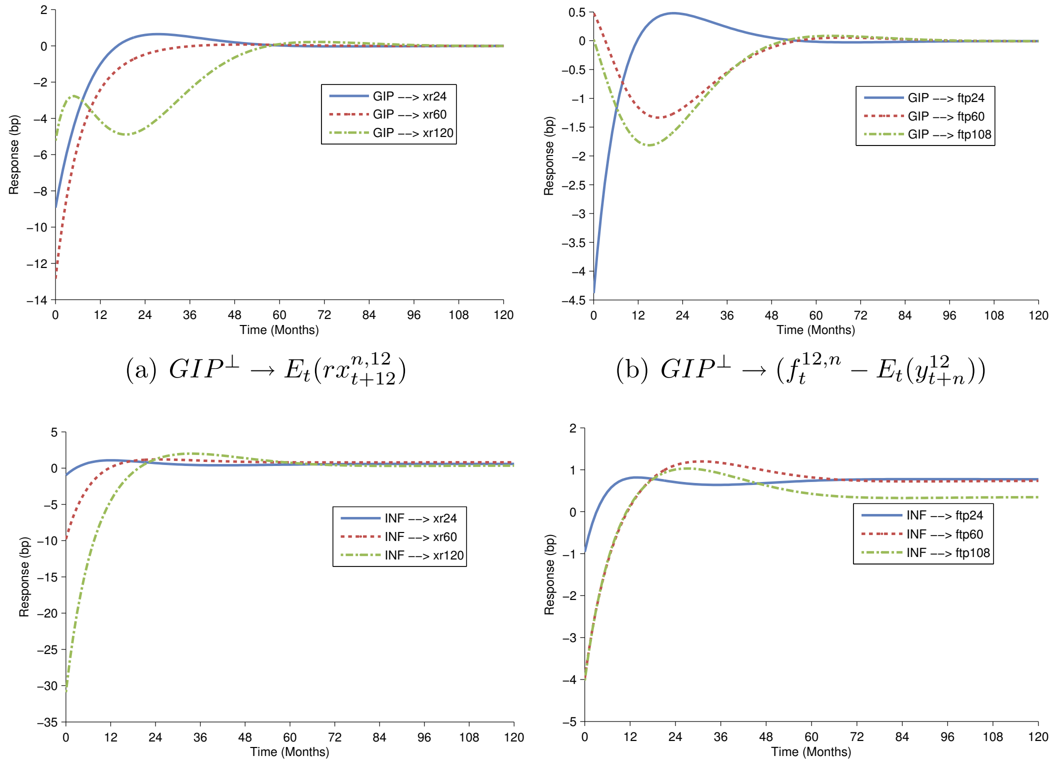


Figure 8: The "in-9-for-1" term premium for model $MA_0(5)^{1UR}_{2RP} - M$ (right scale) and the instantaneous expected excess returns on bond portfolios with payoffs that are (locally) perfectly correlated with the components of GIP ($xSGIP$) and INF ($xSINF$) that are spanned by $\mathcal{P}_t$ (left scale).

prolonged increase in $PC1$ (see Figure 2), and the cointegrating relationship between INF and $PC1$ might well have raised risk premiums associated with exposure to inflation risk. At it turned out, our smoothed measure of realized inflation did enter a prolonged period of gradual increases starting in early 2004. We see, then, that once we condition on GIP and INF (in addition to $\mathcal{P}_t$) changes in forward term premiums cannot be unambiguously attributed to developments related to real growth risk versus inflation risk during this period.

7.3 Responses of Excess Returns to Innovations in M_t

A distinctive feature of our formulation of a macro-$DTSM$ is that there are unspanned components of GIP and INF, $GIP^{\perp}$ and $INF^{\perp}$, that influence expected excess returns on bonds. To compute the responses of expected excess returns to these components we first compute the responses of X_t and then substitute these into the expressions $E^{\mathbb{P}}_t[rx^{n,h}_{t+h}] = a^n + b^n \cdot X_t$, where the loadings a^n and b^n can be computed by recursion from the parameter estimates in Table III. Again the ordering of the variables in X_t is $(\mathcal{P}'_t, GIP, INF)$, so we are computing responses to $GIP^{\perp}$ and



(a) $GIP^{\perp} \rightarrow E_t(rx^{n,12}_{t+12})$ (b) $GIP^{\perp} \rightarrow (f^{12,n}_t - E_t(y^{12}_{t+n}))$

(c) $INF^{\perp} \rightarrow E_t(rx^{n,12}_{t+12})$ (d) $INF^{\perp} \rightarrow (f^{12,n}_t - E_t(y^{12}_{t+n}))$

Figure 9: **Excess Return Impulse Responses:** Each panel plots the responses of expected excess returns on 2 year, 5 year, and 10 year bonds held for one year to shocks in the state variables.

$INF^{\perp}$.[23] We consider $n \in \{24, 60, 120\}$ and $h = 12$ (i.e., an annual holding period). For the case of forward term premiums, we consider "in-p-for-1" forward loans for $p = 2, 5,$ and 9 years.

Figure 9(a) plots the responses of the $E^{\mathbb{P}}_t[rx^{n,12}_{t+12}]$ to a shock to $GIP^{\perp}$ implied by model $MA_0(5)^{1UR}_{2RP}$. The negative responses are consistent with the widely documented counter-cyclical variation in term premiums in bond markets (e.g., Stambaugh (1988)). The initial impact of an innovation in GIP is larger for $E^{\mathbb{P}}_t[rx^{24,12}_{t+12}]$ and $E^{\mathbb{P}}_t[rx^{60,12}_{t+12}]$ than for $E^{\mathbb{P}}_t[rx^{120,12}_{t+12}]$. The responses for the two- and five-year bonds tend to die out monotonically over about two years. In contrast, the responses of $E^{\mathbb{P}}_t[rx^{120,12}_{t+12}]$ to an

[23] More precisely, $INF^{\perp}$ represents the component of the innovation in INF that is orthogonal to innovations in both $\mathcal{P}_t$ and GIP.

output shock follow a "U"-shaped pattern, reaching a trough after about two years and dying out after about four years.

Underlying these patterns are the responses of individual forward term premiums (Figure 9(b)). The "U"-shaped response pattern associated with shocks to GIP shows up at both the five- and nine-year horizons, as anticipated by the patterns in Figure 4. Forward term premiums are evidencing pronounced cyclical patterns by the five-year maturity point along the swap curve, but such patterns are masked in excess returns on five-year bonds by the dominant effect on the latter of the responses of forward premiums on loans initiated at shorter horizons.

The effects of an innovation in $INF^{\perp}$ on expected excess returns and forward term premiums are displayed in Figure 9(c) and Figure 9(d)). The responses are negative, increase (in absolute value) with maturity, and tend to die out relatively quickly, within a year or so. Exactly the same patterns are implied for expected excess *real* holding period returns, since the $E_t^{\mathbb{P}}[rx_{t+12}^{n,12}]$ are invariant to the addition and subtraction of the continuously compounded inflation over the holding period. Interestingly, the pattern in Figure 9(c) is virtually identical to the impulse responses of short-term real yields to inflation shocks reported in Ang, Bekaert, and Wei (2008) based on a regime-switching estimated over a much longer sample period.[24]

Finally, note that for our chosen holding period of one year, the patterns of responses to surprises in M_t differ across the maturities n of the underlying bonds. If twelve-month, expected excess returns on bonds of all maturities have a one-dimensional factor structure– that is, if we can express these returns as

$$E_t^{\mathbb{P}}[rx_{t+12}^{n,12}] = \varsigma_0^n + \varsigma_z^n z_t^{RP}, \tag{14}$$

for a scalar process z_t^{RP}– then the time paths of the responses of the $E_t^{\mathbb{P}}[rx_{t+12}^{n,12}]$, as n varies, must be identical. The empirical evidence that these responses are not the same means that the dimensionality of z^{RP} is at least two.

7.4 Responses of Macro Variables to Term-Premiums Shocks

The existing theoretical and empirical literature has not reached clear-cut conclusions on the relationship between term premiums and economic activity. Bernanke, in his 2006 speech, argues that a higher term premium will depress the portion of spending that depends on long-term interest rates and thereby will have a dampening economic

[24] They assume that agents can infer inflation perfectly from bond yields within each of their regimes. Thus, from the agents' perspective, there is no unspanned inflation risk. However, an econometrician studying the conditional distribution of X_t obtained by integrating out over regimes may detect a component of inflation that is unspanned by bond yields.

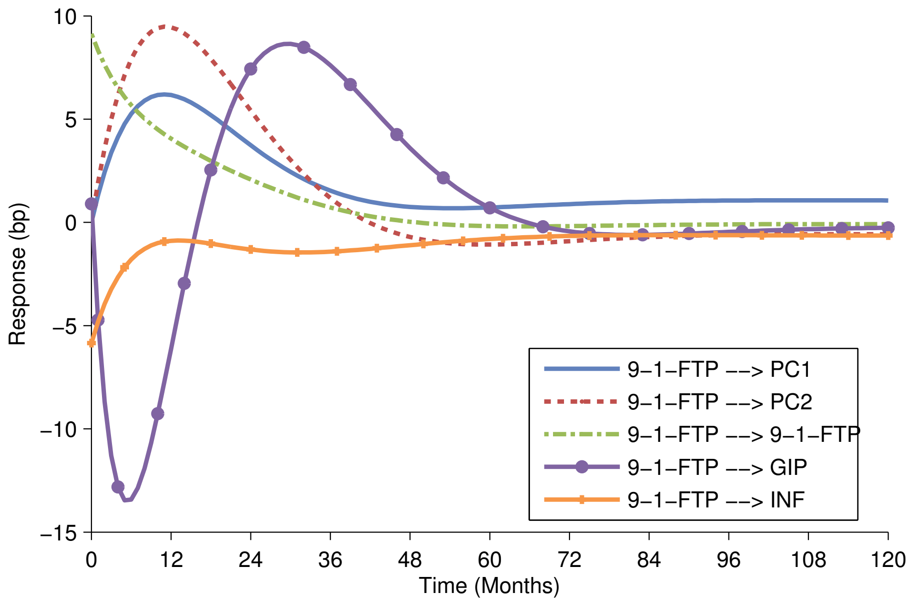


Figure 10: The impulse responses to a one standard deviation shock to the "in-9-for-1" term premium implied by model $MA_0(5)^{1UR}_{2RP}$.

impact. In linearized New Keynesian models in which output is determined by a forward-looking IS equation (such as the model of Bekaert, Cho, and Moreno (2006)), current output depends only on the expectation of future short rates, leaving no role for a term premium effect. A role for a time-varying term premium does arise in models that are linearized at least to the third order (e.g., Ravenna and Seppala (2007b)). Empirically, within the context of their $A_0(3)$ model in which GDP growth is spanned by model-implied bond yields, Ang, Piazzesi, and Wei (2003) report that term premiums are insignificant in predicting future GDP growth. Rudebusch, Sack, and Swanson (2007) (RSS) summarize several studies that report positive associations between term premiums and future output growth. However, in their own analysis they find that a fall in the term premium is associated with higher GDP growth in subsequent quarters.

We examine the response of output growth and inflation to innovations in the "in-9-for-1" year forward term premium in the context of model $MA_0(5)^{1UR}_{2RP}$. Using the fact that the model-implied forward premium $FTP^{9,1}_t \equiv f^{12,108}_t - E_t(y^{12}_{t+108})$ is an affine function of X_t, we derive the model-implied VAR representation of $(PC1, PC2, FTP^{9,1}, GIP, INF)$. As Figure 10 shows, a one standard deviation increase in $FTP^{9,1}$ is followed by a decline in GIP over a period of about 12 to 18 months (consistent with the regression evidence in RSS).

Notably, this initial negative response of GIP is reversed after 18 months, and is then followed by an increase in output growth. The long-term impact of a positive innovation in $FTP^{9,1}$ is close to zero. It is intriguing to relate these patterns to those for shocks to "economic uncertainty" obtained by Bloom (2009) and Bloom, Floetotto, and Jaimovich (2009) in the context of a real business cycle model with adjustment costs in the capital and labor sectors. They find that an increase in economic uncertainty leads to a fall in output growth over about six months that then reverses itself and is followed by an increase in growth over several years, a pattern that is virtually identical to the one in Figure 10. The economic mechanism in their model that induces this pattern is related to the real options that firms face in their investment and hiring decisions. These papers abstract entirely from financial market frictions and a role for an intermediation sector. Our results for swap markets, which are intimately tied to the issuance process for corporate bonds, lead us to wonder whether a different economic mechanism– one related to frictions in the intermediation sector– is at least partially operative.[25]

8 Concluding Observations

This paper develops and estimates an arbitrage-free, Gaussian $DTSM$ in which the state vector X_t includes macroeconomic variables M_t that are not perfectly spanned by contemporaneous bond yields, and in which M_t has significant predictive content for excess returns on bonds over and above the information in bond yields. We show that there is a canonical representation of this model that lends itself to easy interpretation and for which the global maximum to the likelihood function can be attained essentially instantaneously.

To explore the contributions of output growth and inflation to variation in risk premiums in bond markets, we constructed several novel portfolios with payoffs that proved to be informative about the nature of priced risks. Specifically, the expected excess returns on a portfolio of bonds with a payoff that tracked the slope of the swap curve evidenced substantial variation over time, with a notably high correlation with the growth rate of industrial production. Additionally, the responses of risk premiums in the swap market to innovations in macroeconomic information, and vice versa, suggest that financial market developments were central to propagation

[25]See Mendoza and Terrones (2008) and Reinhart and Rogoff (2008) for extensive evidence that credit cycles are central to the propagation of business cycles, and He and Krishnamurthy (2008) for a theoretical model of the effects of frictions in the intermediation sector on asset markets.

mechanisms in business cycles.

Our modeling framework, formally developed in Appendix C, is applicable to any Gaussian pricing setting in which security prices or yields are affine functions of a set of pricing factors $\mathcal{P}_t$ and the relevant state vector embodies information (over and above the past history of $\mathcal{P}$) that is useful for forecasting $\mathcal{P}_t$ under the physical measure $\mathbb{P}$. Accordingly, our framework is well suited to addressing a wide variety of economic questions about characteristics of risk premium in financial markets, including bond and currency markets, as well as equity markets when the latter pricing problems maps into an affine pricing model (e.g., Bansal, Kiku, and Yaron (2007)). The robust means by which we are able to restrict the dimensionality of expected excess returns might be particularly advantageous in multi-market settings, since such restrictions implicitly lead to reductions on the dimensionality of the parameter space. Though neither the state variables nor the pricing factors exhibit time-varying volatility in the settings examined in this paper, our basic framework and its computational advantages extend to affine models with time-varying volatility.

Appendices

A Tests for Unit Roots

Central to much of our analysis is the question of whether the variables included in our econometric model evidence near unit-root behavior. Several researchers have concluded that inflation exhibits near unit root behavior (e.g., Stock and Watson (2007), Henry and Shields (2004)), and under a Fisher-type relationship between inflation and nominal interest rates the level of nominal bond yields will inherit this persistence. We therefore subject each of our variables to unit root tests. The first panel of Table IV reports Dickey-Fuller τ statistics. The specification with 0 lags corresponds to the standard Dickey-Fuller test, whereas the specifications with 1, 3, 6, and 12 lags refer to augmented Dickey-Fuller tests that include lagged differenced terms up to the specified order on the right-hand side of the estimated regression equation. Our selection of lag lengths is guided by the desire to cover a broad range of potential monthly, seasonal, and annual effects. The second panel of Table IV reports the Phillips-Perron statistics. These tests use Newey-West standard errors to account for serial correlation, and the lag length here refers to the number of lags included in the computation of the standard error. Finally, the last line in Table IV shows the statistics from Breitung's (2002) non-parametric unit root test. For each of the three tests, the null hypothesis is that of a unit root process (without drift), against the alternative of a level stationary process.[26] A significant statistic implies that we can reject the null hypothesis of a unit root.

The conclusion we draw from these tests – which is remarkably robust across the different specifications – is that $PC1$, $PC2$, and INF exhibit behavior that we cannot statistically distinguish from a unit root process, whereas $PC3$ and GIP appear to be stationary. The stationarity of GIP is consistent with the way in which consumption growth is specified in a number of recent equilibrium models: consumption growth is *i.i.d.* in the models of Campbell and Cochrane (1999) and Wachter (2006), while it follows a persistent but nevertheless stationary path in the long-run risks literature (Bansal and Yaron (2004), Bansal and Shaliastovich (2007)). These specifications, in turn, are in line with empirical evidence in Lopez and Reyes (2007) suggesting that consumption growth is stationary, at least after accounting for the possibility of structural breaks.

[26] The qualitative conclusions remain unchanged if we instead test for a unit root with drift against a trend stationary process.

Lags	$PC1$	$PC2$	$PC3$	GIP	INF
Augmented Dickey-Fuller Unit Root Tests (τ Statistic)					
0	-2.04	-1.97	-4.28^{***}	-2.94^{**}	-1.75
1	-2.59^{*}	-2.11	-4.81^{***}	-2.58^{*}	-1.64
3	-2.34	-2.05	-4.98^{***}	-3.01^{**}	-1.60
6	-1.83	-2.25	-3.77^{***}	-3.04^{**}	-1.70
12	-2.53	-2.26	-3.86^{***}	-2.35	-1.92
Phillips-Perron Unit Root Tests (Z_ρ Statistic)					
0	-4.99	-7.74	-34.46^{***}	-17.31^{**}	-3.87
1	-5.71	-8.31	-38.51^{***}	-15.90^{**}	-3.61
3	-6.08	-7.91	-41.65^{***}	-17.41^{**}	-3.30
6	-6.17	-8.64	-36.77^{***}	-18.72^{**}	-3.41
12	-6.84	-9.73	-33.76^{***}	-18.08^{**}	-3.70
Breitung's Non-Parametric Unit Root Test ($\hat{\varrho}_T/T$ Statistic)					
n/a	0.05	0.03	0.01^{**}	0.01^{**}	0.07

Table IV: Unit Root Tests. The $(*, **, ***)$ indicate statistical significance at the $(10\%, 5\%, 1\%)$ level.

B Order Selection of Autoregressive Models

We compute three well-known information criteria based on unrestricted VARs with one through twelve lags (the maximum lag length was chosen so that both seasonal and annual effects would be captured): Akaike's information criterion (AIC), Hannan and Quinn's information criterion (HQIC), and Schwarz's Bayesian information criterion (SBIC). When additional lags are included as explanatory variables, the in-sample fit improves; the information criteria trade off this gain in likelihood against the additional number of parameters introduced. The recommended lag length is that at which the information criteria attain their minimum values. As Table V shows, all three criteria are minimized at a lag length of 1, suggesting that a first-order Markov structure fits our data well.[27]

[27]HQIC and SBIC are consistent, i.e. they will lead to the correct order choice asymptotically, while the AIC asymptotically overestimates the true VAR order with positive probability. This holds both when the true process is stationary and when it contains unit roots, as is discussed in Lutkepohl (2005), especially Propositions 4.2 and 8.1.

Lags	AIC	HQIC	SBIC
0	-41.6785	-41.6475	-41.6019
1	-54.1559^{*}	-53.9703^{*}	-53.6961^{*}
2	-54.1031	-53.7628	-53.2601
3	-54.0842	-53.5892	-52.858
6	-53.8453	-52.8861	-51.4695
9	-53.6811	-52.2579	-50.1558
12	-53.3595	-51.4721	-48.6846

Table V: Assessments of Lag Length in VAR Models. The criteria-impled optimal lag lengths are indicated by '$*$'.

C Derivation of Results in Subsection 3.1

We define a (discrete-time) *Gaussian macro-dynamic term structure model* as a model where there is a Markov process, X_t, that has affine dynamics under both P and Q, such that the short rate and the macro-variables are affine in X_t. More precisely, we define a Gaussian macro-DTSM to be a probability space $(\Omega, P, \mathcal{F})$ with a discrete filtration $\{\mathcal{F}_t\}_{t\in\mathbb{N}}$, and a processes $\{r_t\}$, the short rate process, and macro-variable $\{M_t^i\}_{t,i}$ if there exists an adapted Markov process $\{X_t\}$ such that

1. $r_t = \rho_0 + \rho_1 \cdot X_t$ for some ρ_0, ρ_1,
2. for each macro-variable, M_t^i, $M_t^i = f_i(\delta_0^i + \delta_1^i \cdot X_t)$, for some function f_i and some parameters δ_0^i, δ_1^i,
3. X_t follows a VAR(1):

$$\Delta X_t = K_0^{\mathbb{P}} + K_X^{\mathbb{P}} X_t + \sqrt{\Sigma_X}\epsilon_{Xt}^{\mathbb{P}},$$

   where $\epsilon_{Xt}^{\mathbb{P}}$ are $\mathcal{F}_{t+1}$-measurable standard normal random variables that are independent of $\mathcal{F}_t$,
4. there is a measure $\mathbb{Q}$, equivalent to $\mathbb{P}$, such that X_t follows a VAR(1) under $\mathbb{Q}$ (that is, there is an essentially affine market price of risk) and so that for any price process P_t , $\{e^{-\int_0^t r_s ds} P_t\}$ is a Q-martingale. In particular, the price of a T-year zero coupon bonds is $E_t^{\mathbb{Q}}[e^{-\int_t^{t+T} r_s ds}]$.

Additionally, we will focus on *observable* macro-DTSMs, where we define a macro-DTSM to be observable in the case that the information in current fixed income prices and current macro-variables fully embody all information about the state of the economy. More precisely, if we let $\mathcal{M}_t \equiv \sigma(\{M_t^i\}_i)$ and $\mathcal{P}_t$ to be the information in fixed income security prices[28], we require $\sigma(\mathcal{M}_t \cup \mathcal{P}_t) = \mathcal{F}_t$. Without loss of generality, we let $\mathcal{F}_t = \sigma(X_t)$. It is worth noting that the proposition below generalizes with similar arguments to characterize both non-observable macro-DTSMs as well as affine jump diffusion macro-DTSMs. Additionally, we suppose that the macro-variables are non-spanning in the sense that $\sigma(M_t^i) \subsetneq \sigma(\mathcal{P}_t \cup M_t^1, \ldots, M_t^{i-1}, M_t^{i+1}, \ldots M_t^I)$ and $\mathcal{P}_t \subsetneq \mathcal{M}_t$.

Recalling that two models are said to be observationally equivalent if the likelihood of any data set is the same for both models (and thus the models are identical from the viewpoint of the econometrician), we have the following result:

Proposition 1: Every *observable non-spanning Gaussian Macro-dynamic term structure model* is observationally equivalent to a Gaussian macro-DTSM such that $X_t = \langle \mathcal{P}_t, M_t \rangle$, where the vector $\mathcal{P}$ is the first $N^{\mathbb{Q}}$ principal components of the zero curve and $r_t = \rho_0 + \rho_1^{\mathcal{P}} \cdot \mathcal{P}_t$. The $\mathbb{P}$ and $\mathbb{Q}$ representations of X_t are:

$$\Delta \begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & K_{\mathcal{P}M}^{\mathbb{P}} \\ K_{M\mathcal{P}}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_X} \epsilon_{Xt}^{\mathbb{P}}, \tag{15}$$

and

$$\Delta \begin{bmatrix} \mathcal{P}_t \\ M_t \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{Q}} \\ * \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}} & 0 \\ * & * \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_X} \epsilon_{Xt}^{\mathbb{Q}}. \tag{16}$$

This representation is unique in the sense that every Gaussian macro-DTSM is observationally equivalent to exactly one such model. Moreover, for any $K_0^{\mathbb{P}}, K_1^{\mathbb{P}}$, positive definite Σ_X, ordered non-negative $\mathbb{Q}$-eigenvalues $(\lambda_1^{\mathbb{Q}}, \ldots, \lambda_{N^{\mathbb{Q}}}^{\mathbb{Q}})$ and $r_\infty^{\mathbb{Q}}$ (the long-run mean of the short rate under $\mathbb{Q}$), there exists a unique such macro-DTSM.[29] That is, $(K_0^{\mathbb{P}}, K_1^{\mathbb{P}}, \Sigma_X, \vec{\lambda}^{\mathbb{Q}}, r_\infty^{\mathbb{Q}})$ parametrize a canonical form for observable non-spanning Gaussian macro-DTSMs.

See the proof for the explicit construction of $(\rho_0, \rho_1^{\mathcal{P}}, K_{0\mathcal{P}}^{\mathbb{Q}}, K_{\mathcal{P}\mathcal{P}}^{\mathbb{Q}})$ from $(\vec{\lambda}^{\mathbb{Q}}, r_\infty^{Q})$. To emphasize, for strict identification we require that $\vec{\lambda}^{\mathbb{Q}}$ be ordered.

Proof: From Joslin, Singleton, and Zhu (2009) we know that, for any $A_0^{\mathbb{Q}}(3)$ pricing model with distinct, real eigenvalues of the feedback matrix of the risk factors,

[28] Formally, we can define the information in fixed income prices at time t to be the σ-field generated by $\{P(X_t) = E_t^Q[g(r_{t+t_1}, r_{t+t_2}, \ldots, r_{t+t_n})] : g \in C_0\}$, which is easily seen to contain any reasonable fixed income price.

[29] One could also choose different salient observable features of the yield curve. Implicit also is the assumption that the eigenvalues of $K_{\mathcal{P},\mathcal{P}}^{\mathbb{Q}}$ are unique and negative. See Joslin, Singleton, and Zhu (2009) for a discussion of both of these points and alternative when these assumptions are violated.

there exists a three-dimensional, latent state vector Y_t such that $r_t = r^{\mathbb{Q}}_{\infty} + \vec{1} \cdot Y_t$ and

$$\Delta Y_t = \text{diag}(\lambda^{\mathbb{Q}})Y_{t-1} + \sqrt{\Sigma^0_Y}\epsilon^{\mathbb{Q}}_t$$

for some 3×3 matrix Σ^0_Y and vector $\lambda^{\mathbb{Q}}$ of eigenvalues of the feedback matrix governing Y, with $\epsilon^{\mathbb{Q}}_t \sim N(0, I)$.

To derive a canonical version of (3), let $B^0(\tau)$ be the loadings given by $\dot{B}^0 = \text{diag}(\lambda^{\mathbb{Q}})B^0 - \vec{1}$, $B^0(0) = 0$. Let $B^0_{PC}(i) = \sum -\ell_i B^0(\tau_i)/\tau_i$, where PC-$i$ has loading ℓ_i on yield maturity τ_i. Let B be the 3×3 matrix with i^{th} row given by $B^0_{PC}(i)$. It follows that the covariance matrix of the innovations to the PCs is $B\Sigma^0_Y B^\top$. In order that (3) is satisfied, it must be that $\Sigma^0_Y = (B^\top)^{-1}\Sigma_{\mathcal{P}}B^{-1}$.

Now let $A^0(t)$ solve $\dot{A}^0 = \frac{1}{2}(B^0)^\top\Sigma^0_Y B^0 - r^{\mathbb{Q}}_{\infty}$, $A^0(0) = 0$. Define $A^0_{PC}(i) = \sum -\ell_i A^0_y(\tau_i)/\tau_i$. Let a be the 3×1 vector with i-th entry $A^0_{PC}(i)$. Then $\mathcal{P}_t = a + BX^0_t$. From an invariant affine transformation it follows that: $K^{\mathbb{Q}}_X = B(\text{diag}(\lambda^{\mathbb{Q}})B^{-1}$, $K^{\mathbb{Q}}_0 = -(K^{\mathbb{Q}}_1)^{-1}a$, $\rho_0 = r^{\mathbb{Q}}_{\infty} - \vec{1}^\top B^{-1}a$, and $\rho_1 = (B^\top)^{-1}\vec{1}$.

Since (3) is an invariant transformation of an identified, canonical model, we know that (3) is also identified and canonical. The underlying parameters are $(r^{\mathbb{Q}}_{\infty}, \lambda^{\mathbb{Q}}, L)$, where L is the Cholesky factorization of Σ_X.

D Details of Our Estimation Strategy for the $A_0(5)$ Models

To address the indeterminacy in models with two risk-premium factors, we proceed as follows. First we rescale the PCs so that (1) $\sum_{i=1}^{8} \ell_{1,i}/8 = 1$, (2) $\ell_{2,10y} - \ell_{2,6m} = 1$, and (3) $\ell_{3,10y} - 2\ell_{3,2y} + \ell_{3,6m} = 1$. This puts all the PCs on similar scales. We then convert INF and GIP to an annual scale. Now all variables take on values between $[-5\%, 10\%]$.

After rescaling, consider the first three rows $A \equiv K^{\mathbb{P}}_X - K^{\mathbb{Q}}_X$. For the case of two risk-premium factors and $A_0(5)$ pricing model, there are many observationally equivalent factorizations of A as $\alpha\beta$, where α is a 3×2 matrix and β is a 2×5 matrix. Rows of β correspond to which linear combination of X_t drive risk premia. The i-th column of α describes how each PC risk premium varies with the ith risk-premium factor. In our estimation we require that the covariance matrix of $\beta' X_t$ be the identity matrix. This makes the risk-premium factors equally variable and conditionally independent. If we write $A = U_0 D_0 V_0'$ as in SVD, then $\alpha_0 = U_0 D_0 H_0^{1/2}$ $\beta_0 = H_0^{-1/2} V_0'$ has unit covariance. If $\alpha_1 = U_1 D V_1'$, then $\beta_1 = V'\beta_0$ will maintain the unit covariance and $\alpha = U_1 D_1$ will be such that the first row has maximal length.

Normalize the conditional covariance matrix of $\beta' X_t$ to the identity matrix, so that the risk premia factors are equally variable and orthogonal. Letting Σ_V denote the covariance matrix of the innovations in $V' X_t$, the conditional covariance matrix of $\Sigma_V^{-1/2} V' X_t$ is the identity matrix. For any orthogonal 2×2 matrix W, $W \Sigma_V^{-1/2} V' X_t$ also has the identity matrix as its conditional covariance matrix. We choose W so that $U D \Sigma_V^{1/2} W'$ gives most of the variation in the overall risk premium to the first of the two factors.

Parameter	Estimate			
	$YA_0(3)^{1UR}_{1RP}$	$YA_0(5)^{1UR}_{1RP}$	$MA_0(5)^{1UR}_{1RP}$	$MA_0(5)^{1UR}_{2RP}$
$U_{1,1}$	-0.6593	-0.9777	-0.6426	-0.7503
$U_{2,1}$	0.7428	0.1143	0.7418	-0.04269
$U_{3,1}$	-0.1167	0.1761	-0.1921	-0.6597
$U_{1,2}$				0.1233
$U_{2,2}$				-0.9894
$U_{3,2}$				-0.07621
D_1	0.04073	0.6097	0.07703	0.0736
D_2				0.03373
$V_{1,1}$	0.3313	0.01574	0.5067	0.3739
$V_{2,1}$	0.763	0.04868	0.4224	0.4231
$V_{3,1}$	0.555	0.01988	0.1945	0.3491
$V_{4,1}$		0.8676	-0.3211	-0.1863
$V_{5,1}$		-0.4942	-0.6511	-0.7243
$V_{1,2}$				-0.4995
$V_{2,2}$				-0.1096
$V_{3,2}$				0.5668
$V_{4,2}$				0.6121
$V_{5,2}$				-0.2062
$K^{\mathbb{P}}_{0,1}$	0.001063	0.001679	0.0005935	0.0006012
$K^{\mathbb{P}}_{0,2}$	-0.001704	-0.0005982	-0.001238	-0.000562
$K^{\mathbb{P}}_{0,3}$	0.001037	0.0004382	0.0009302	0.001407
$K^{\mathbb{P}}_{0,4}$		0.0003334	0.003541	0.003217
$K^{\mathbb{P}}_{0,5}$		0.0002689	-0.0006233	-0.0008224
$K^{\mathbb{P}}_{X,41}$		-0.001441	0.03951	0.04189
$K^{\mathbb{P}}_{X,42}$		-0.004214	0.1498	0.1513
$K^{\mathbb{P}}_{X,43}$		-0.007227	-0.4037	-0.401
$K^{\mathbb{P}}_{X,44}$		-0.1023	-0.09203	-0.09301
$K^{\mathbb{P}}_{X,45}$		0.1034	-0.1372	-0.1314
$K^{\mathbb{P}}_{X,51}$		3.042e-005	0.04338	0.045
$K^{\mathbb{P}}_{X,52}$		0.0003098	0.01393	0.01596
$K^{\mathbb{P}}_{X,53}$		0.002704	0.03179	0.03286
$K^{\mathbb{P}}_{X,54}$		0.03649	-0.006859	-0.006502
$K^{\mathbb{P}}_{X,55}$		-0.3375	-0.09069	-0.08939

Table VI: **Estimates of $K^{\mathbb{P}}_X$, the drift of X under the historical distribution.** The first $N^{\mathbb{Q}}$ rows of $K^{\mathbb{P}}_1$ are $K^{\mathbb{Q}}_X + UDV^{\top}$, where $K^{\mathbb{Q}}_X$ is the $N^{\mathbb{Q}} \times N$ whose first $N^{\mathbb{Q}} \times N^{\mathbb{Q}}$ block is the unique matrix with the prescribed eigenvalues such that the $PC1 - PC3$ are the factors as described in the text.

Parameter	Estimate			
	$YA_0(3)^{1UR}_{1RP}$	$YA_0(5)^{1UR}_{1RP}$	$MA_0(5)^{1UR}_{1RP}$	$MA_0(5)^{1UR}_{2RP}$
σ_{PC1}	27.06	26.62	26.97	27.05
σ_{PC2}	28.43	28.28	27.94	27.85
σ_{PC3}	23.35	23.33	23.3	23.33
σ_4		4.862	104.6	104.6
σ_5		4.189	13.1	13.13
$\rho_{2,1}$	0.4971	0.5068	0.5286	0.5299
$\rho_{3,1}$	-0.6284	-0.6256	-0.6406	-0.645
$\rho_{3,2}$	-0.4563	-0.4579	-0.4539	-0.4625
$\rho_{4,1}$		0.04618	0.1735	0.174
$\rho_{4,2}$		0.1836	0.08362	0.08402
$\rho_{4,3}$		-0.05011	-0.1042	-0.1054
$\rho_{5,1}$		0.04026	0.01243	0.01786
$\rho_{5,2}$		0.03764	0.0767	0.08148
$\rho_{5,3}$		0.03143	-0.02572	-0.0294
$\rho_{5,4}$		-0.2951	0.00182	0.002532

Table VII: **Covariance Estimates**

Estimated model parameters. σ_{PCi} is the standard deviation (in basis points) of the innovation to PCi. (σ_4, σ_5) are the volatilities of innovations to either $(PC4, PC5)$ or (GIP, INF). $PC1$ is scaled so that a 1bp increase will increase the average rate by 1bp. $PC2$ is scaled so that a 1bp increase will increases the spread between 10-year zero and 6-month zero by 1bp. $PC3$ is scaled so that 1bp increase will $\big($-(10-year) + 2 $\times$ (2-year) - (6-month)$\big)$ by 1bp. $PC4$ and $PC5$ are scaled to have loadings vector have length 1. GIP and INF are annualized exponential weighted moving averages.

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