# Dynamic Stochastic General Equilibrium Models In a Liquidity Trap and Self-organizing State Space Modeling

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## Abstract

This paper estimates new Keynesian, dynamic stochastic general equilibrium models in a liquidity trap (the non-negativity constraint on short term nominal interest rates) using the Monte Carlo particle filter, proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998). This method is a natural extension of Yano (2009). In our method, we estimate the parameters of the models using the time-varying-parameter approach, which is often used to infer invariant parameters in practice. Moreover, natural rates of macroeconomic data, time-varying parameters, and unknown states are estimated simultaneously using self-organizing state space modeling. Adopting our method creates the great advantage that the structural changes of parameters are detected naturally. In empirical analysis, we estimate new Keynesian DSGE models in a liquidity trap using Japanese macroeconomic data which includes the "zero-interest-rate" period (1999-2006). The analysis shows that the target rate of inflation is too low in the 1990s and the 2000s, and it causes deflation in the Japanese economy.

**Keywords:** dynamic stochastic general equilibrium model, monetary policy, non-negativity constraint on short term nominal interest rate, liquidity trap, Monte Carlo particle filter

**JEL Classification Codes:** C11, C13, E32

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# 1 Introduction

In recent years, Japan's long-term stagnation in the 1990s and deflation from the late 1990s to the early 2000s are the hot topics in the economy. The 1990s are often called "a lost decade" because the real growth of the Japanese economy suddenly slowed down and the economy experienced a long-term recession at the time. Furthermore, deflation was observed in the economy from 1994 to the early 2000s. To fight against deflation, the Bank of Japan adopted a zero-interest-rate policy from 1999 to 2006 and a quantitative-easing policy from 2001 to 2006. The reasons behind the lost decade have been actively debated. Is it caused by aggregate supply factors (such papers as Hayashi and Prescott (2002), Hayashi (2003) and Miyao (2006)), or aggregate demand factors (such papers as Kuttner and Posen (2001) and Kuttner and Posen (2002)) [1]? Hayashi and Prescott (2002) point out that the slowdown of total factor productivity growth in the 1990s and the reduction of the work-week length cause the long-term recession. Thus, Hayashi (2003) proposes structural reforms of the Japanese economy to escape from long-term stagnation. Krugman (1998), however, emphasizes the importance of monetary factors. He points out that the economy is "trapped" by the non-negativity constraint on short-term nominal interest rates because of deflation, and calls the situation a liquidity trap [2]. To escape from the trap and long-term stagnation, he proposes adopting inflation targeting in the Japanese economy. The two seminal papers beget a great number of papers, for example, McCallum (2000), Svensson (2001), Orphanides and Wieland (2000), Eggertsson and Woodford (2003), Jung et al. (2005), Baba et al. (2005), Auerbach and Obstfeld (2005), Adam and Billi (2006), Braun and Waki (2006), Braun and Shioji (2006), Eggertsson and Pugsley (2006), Christiano (2004), and Nakajima (2008). Ugai (2007) is a survey on the zero-interest-rate policy and the quantitative-easing policy of the Bank of Japan, and many related papers are cited therein.

In recent years, new Keynesian, dynamic stochastic general equilibrium models of monetary analysis have been rapidly developing. The early works of Kimball (1995), Roberts (1995), and Yun (1996) beget the subsequent many papers (see McCallum and Nelson (1999), Clarida et al. (1999), Gali (2002), and related literatures which are referred therein) [3]. "Middle-size" new Keynesian models are developed by Christiano et al. (2005) and Smets and Wouters (2003), and their models are often adopted by practitioners in the government and the central bank. The fit performances of their models are discussed by Fout (2005), Trabandt (2006), and Del Negro et al. (2007). Using the "middle-size" new Keynesian models, Braun and Waki (2006), Christiano (2004), Iiboshi et al. (2005), Sugo and Ueda (2008), and Ichiue et al. (2008) analyze the Japanese economy.

Bayesian statistics are now becoming a standard tool to estimate DSGE models. DeJong et al.

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[1] Caballero et al. (2008), Sekine et al. (2003), Kobayashi and Inaba (2002), Hosono and Sakuragawa (2004), and previous studies point out the importance of the non-performing loan problem in the lost decade. See Miyao (2006) and related papers are cited therein. In this paper, we don't make an assertion that the NPL problem is less important in the lost decade. However, it is outside the scope of this paper because our model does not include financial intermediaries. The roles of financial intermediaries and the NPL problem in the decade will be explained in a future study.

[2] Eggertsson (2008) describes that a liquidity trap is defined as a situation in which the short-term nominal interest rate is zero. In this paper, we follow his definition.

[3] See also Walsh (2003), Woodford (2003), Kato (2006), and Gali (2008), and related literatures, which are referred therein.

(2000), Schorfheide (2000), Otrok (2001) Smets and Wouters (2003), Levin et al. (2005), Del Negro et al. (2007), Smets and Wouters (2007), and Hirose and Naganuma (2007) estimate parameters of DSGE models using Markov Chain Monte Carlo methods (MCMC) [4]. Fernandez-Villaverde and Rubio-Ramirez (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007a) have shown that the Monte Carlo particle filter (MCPF) and maximizing likelihood can be successfully applied to estimate DSGE models [5]. An and Schorfheide (2007) is an excellent survey on this area, and see references cited therein [6]. Using MCMC, Iiboshi et al. (2005), Sugo and Ueda (2008), and Ichiue et al. (2008) estimate DSGE models for Japan in the "pre-zero-interest-rate" period (1970[1981]-1995) [7]. They avoid using data from the "zero-interest-rate" period (1999-2006) because it is necessary to estimate the nonlinear Taylor rule with the non-negativity constraint on short-term nominal interest rates. However, the periods are a matter of serious concern for the long-term stagnation and the deflation in the 1990s. Thus, there exists a need to estimate DSGE models for the Japanese economy including the "zero-interest-rate" period (1999-2006).

This paper proposes a new method to estimate parameters of dynamic general equilibrium models in a liquidity trap based on the Monte Carlo particle filter, proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998) [8]. This method is a natural extension of Yano (2009). Our method is based on Bayesian statistics and nonlinear, non-Gaussian, and non-stationary state space modeling (NNNSS) to estimate unknown parameters and states. Furthermore, in our method, we estimate the parameters using the time-varying-parameter approach, which is often used to infer invariant parameters in practice. In most previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, is a framework to analyze how stable structural parameters are. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. Additionally, we would like to stress that the novel feature of our method is that we are able to estimate DSGE models in a liquidity trap (Krugman (1998)) because it is based on nonlinear and non-stationary state space modeling. In the other words, it is able to estimate DSGE models with the nonlinear Taylor rule. Furthermore, in our method, the fit of a DSGE model is evaluated using the log-likelihood of it. Thus, we are able to compare the fits of DSGE models. Moreover, we estimate time-varying trends of macroeconomic data: natural output, a inflation rate, and a real interest rate. In practice, the Hodrick and Prescott (1997) filter is often used to estimate the natural output of the Japanese economy. However, it is an open question whether the HP filter and the magic number, which is suggested in Hodrick and Prescott (1997), are appropriate for estimation of Japanese natural output. Urasawa (2008) uses the Baxter and King (1999) filter to provide the stylized facts of

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[4] Altug (1989), McGrattan et al. (1997), Kim (2000), Ireland (2001), and Ireland (2004) estimate parameters in DSGE models using maximizing the likelihood of the Kalman filter.

[5] Amisano and Tristani (2007) estimates a small DSGE model on euro area data, using the conditional particle filter to compute the model likelihood.

[6] Canova (2007) and Dejong and Dave (2007) are comprehensive introductions to Bayesian macroeconometrics.

[7] To estimate DSGE models for Japan, Fuchi et al. (2005) use GMM and Fujiwara (2007) uses maximum likelihood estimation.

[8] Introductions to Monte Carlo particle filters are Gordon et al. (1993), and Doucet et al., eds (2001), Ristic et al. (2004). Yano (2008b) and Yano and Yoshino (2007) propose time-varying structural vector autoregressions based on the Monte Carlo particle filter and a self-organizing state space model. Time-varying structural vector autoregressions based on Markov chain Monte Carlo methods are proposed by Primiceri (2005) and Canova and Gambetti (2006).

the Japanese business cycles [9]. Our method, based on Yano (2009), is an alternative to these filters, and it is "DSGE-based" estimation of time-varying economic trends and natural rates. In empirical analysis, we estimate new Keynesian DSGE models in a liquidity trap using Japanese macroeconomic data, which includes the "pre-zero-interest-rate" period (1980-1998), the "zero-interest-rate" period (1999-2006), and the "post-zero-interest-rate" period (2007-2008). One restriction on our method, however, exists. We assume that the timings of when the economy is trapped in a liquidity trap and its subsequent escape are given. In other words, these timings are exogenous.

In most previous papers on DSGE models, structural parameters of them are assumed to be "deep (invariant)." Our method, however, is a framework to analyze how stable structural parameters are. The time-varying-parameter approach is practically often used in state space modeling to estimate parameters, for example, Kitagawa (1998) and Liu and West (2001). Even if we assume the random walk priors, which are described in section 3, it does not indicate that the deep parameters of DSGE models are "time-varying." Our framework is just a practical one to estimate deep parameters. Adopting our method creates the great advantage that the structural changes of parameters are detected naturally. Thus, it is suitable to analyze how stable structural parameters are. The second advantage of our method is that we are able to estimate new Keynesian DSGE models in a liquidity trap (Krugman (1998)) because NNNSS allows model switching.

Braun and Waki (2006) point out that the presence of the zero nominal interest rate bound on monetary policy creates two difficulties. First it complicates the solution of the model since the policy function is not well approximated by a linear function. The second difficulty is that the zero nominal interest rate bound alters the stability properties of the model as pointed out by Benhabib et al. (2001). They find that there are two steady-states; one where the nominal interest rate is zero and one with a positive nominal interest rate. There are infinitely many equilibria that converge to the former steady-state and a unique convergent path to the latter one. Braun and Waki (2006) confront these two issues by approximating the Taylor rule with the piece-wise linear function and focusing on a particular class of equilibria. Following Braun and Waki (2006), our attention is restricted to equilibria in which the zero nominal interest rate constraint binds once for a finite number of periods, and other equilibria in which the zero constraint might bind for a while, cease to bind and then start to bind again are ruled out. Moreover, Braun and Waki (2006) develop an algorithm for computing perfect foresight equilibria in situations where the nominal interest rate is zero over some interval of time. In this paper, we adopt the algorithm in our estimation method [10].

Our paper is closely related with Fernandez-Villaverde and Rubio-Ramirez (2007b) [11]. However, there exist several large differences between our paper and theirs. The first point is that they focus on the stabilities of "structural" parameters of the Taylor rule and Calvo pricing. In contrast we estimate any

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[9] See Christiano and Fitzgerald (2003) also. Smets and Wouters (2007) estimate invariant trends of macroeconomic data.

[10] In appendix, we outline the algorithm of Braun and Waki (2006).

[11] Canova (2006) evaluates the stability of policy parameters of a small new Keynesian model using MCMC and the Kalman filter. Justiniano and Primiceri (2008) estimate DSGE models allowing for time variation in the volatility of the structural innovations using MCMC. Bjornland et al. (2008) estimate the time-varying natural rate of interest and output and the implied medium-term inflation target for the US economy based on DSGE models using MCMC and the Kalman filter. Hatano (2004) estimates structural parameters of a overlapping generations model using the Kalman filter.

parameters using the TVP approach. The second point is that they use MCPF to estimate the second-order approximation of DSGE models, whereas, we focus on the nonlinearity of the Taylor rule of the economy in a liquidity trap. The third point is that they use maximizing the likelihood of MCPF to estimate parameters, while, we adopt a self-organizing state space model for parameter estimation. Yano (2008a) reports that the variances of the estimates of a self-organizing state space model are smaller than the ones of the maximizing-likelihood approach. The fourth point is that we estimate a time-varying trend of real output, a time-varying inflation target, and a time-varying equilibrium real interest rate.

This paper is structured as follows. In section 2, we describe a new Keynesian DSGE model. In section 3, we explain our method based on the Monte Carlo particle filter and a self-organizing state space model. In section 4, we show the results of our empirical analysis. In section 5, we describe conclusions and discussions.

# 2 The Model

## 2.1 Households

In the economy, there is a continuum of households indexed by $j \in (0, 1)$. The households consume and provide labor. The utility of the household $j$ is given by

$$E_0^j \sum_{t=0}^{\infty} \beta^t \Big[\log(C_{j,t} - hC_{t-1}) + \Upsilon\big(\frac{M_{j,t}}{P_t}\big) - \frac{\Psi_L}{1+\sigma_L} L_{j,t}^{1+\sigma_L}\Big], \tag{1}$$

where $E_0^j$ is the expectation operator, conditional on household $j$'s information at time 0, $C_{j,t}$ is household $j$'s consumption, $C_{t-1}$ is past aggregate consumption, $M_{j,t}/P_t$ is the household $j$'s real money balances, $L_{j,t}$ is household $j$'s labor hours, $t$ is a time index, and $h$, $\chi$, $\Psi_L$, and $\eta$ are constants. Following Braun and Waki (2006), we assume satiation of utility from real balances, i.e. there exists $\bar{m}$ such that $\Upsilon'(m) > 0$ for all $m < \bar{m}$ and $\Upsilon'(m) = 0$ for all $m \geq \bar{m}$, where $m$ is a real balance [12]. The constraint condition of the household $j$ is given by

$$C_{j,t} + I_{j,t} + \frac{M_{j,t}}{P_t} + \frac{B_{j,t}}{P_t} \leq W_t L_{j,t} + \frac{M_{j,t-1}}{P_t} + r_t^K K_{j,t} + (1 + i_{t-1})\frac{B_{j,t-1}}{P_t} + \Pi_{j,t}, \tag{2}$$

where $I_{j,t}$ is investment by household $j$, $B_{j,t}$ is household $j$'s domestic bonds, $W_t$ is the average real wage, $i_t$ is the short-term nominal interest rate, $K_{j,t}$ is household $j$'s capital, $r_t^K$ is the rental rate of $K_{j,t}$, and $\Pi_{j,t}$ is the profit of the firm $j$. In addition to Eq. (2), we assume the households are subject to the no-Ponzi condition.

$$\lim_{T\to\infty} E_0[(\prod_{t=0}^{T} \frac{1}{1+i_t})B_{j,T}] = 0. \tag{3}$$

## 2.2 Capital Accumulation and Adjustment Cost

The time evolution of Capital, $K_{j,t}$ is given by

$$K_{j,t} = (1-\delta)K_{j,t-1} + \Big[1 - s\big(\frac{I_t}{I_{t-1}}\big)\Big], \tag{4}$$

where $\delta$ is the depreciation cost of capital, $K_{j,t}$, and $s(\cdot)$ is a adjustment cost function. We restrict the function $s(\cdot)$ to satisfy the following properties: $s(1) = s'(1) = 0$ and $s''(1) = \nu > 0$.

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[12] If $\Upsilon'(m) > 0$ for all $m$, then the zero interest rate bound never binds. See Braun and Waki (2006).

## 2.3 Final Good Sector

In the final good sector, a single final good is produced by a perfectly competitive, representative firm. The final good is produced using a continuum of intermediate good, $Y_{j,t}$, indexed by $j \in (0, 1)$. The final good, $Y_t$, is produced using the aggregate technology.

$$Y_t = \Big[\int_0^1 (Y_{j,t})^{\frac{1}{1+\lambda_p}} dj\Big]^{1+\lambda_p}, \tag{5}$$

where $Y_{j,t}$ is the quantity of intermediate good $j$, $\lambda_p$ is a constant. The demand curve for $Y_{j,t}$ is given by

$$Y_{j,t} = \Big(\frac{P_{j,t}}{P_t}\Big)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t, \tag{6}$$

where $P_{j,t}$ is the price of intermediate good $j$ and $P_t$ is the aggregate price of the final good. The aggregate price is given by

$$P_t = \Big[\int_0^1 (P_{j,t})^{-\frac{1}{\lambda_p}} dj\Big]^{-\lambda_p}. \tag{7}$$

## 2.4 Intermediate Goods Firms

In the intermediate goods sector, monopolistic competitive domestic firms produce intermediate goods which is indexed by $j \in (0, 1)$. The firm $j$'s production function is given by

$$Y_{j,t} = Z_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}, \tag{8}$$

The aggregate technology level, $Z_t$, is given by

$$\log Z_t = (1 - \xi_Z) \log \bar{Z} + \xi_Z \log Z_{t-1} + \epsilon_{Z,t}, \tag{9}$$

where $\epsilon_{Z,t} \sim N(0, \sigma_{Z,t}^2)$ and $\bar{Z}$ and $\xi_Z$ are constants. Solving the cost minimization of the firm $j$, the first order condition becomes

$$\frac{W_t}{r_t^K} = \frac{1-\alpha}{\alpha}\frac{K_{j,t}}{L_{j,t}} \tag{10}$$

The firms $j$'s real marginal cost is given by

$$MC_t = \frac{1}{Z_t}\Big(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} W_t^{1-\alpha} (r_t^K)^{\alpha}\Big) \tag{11}$$

In the sticky prices model, proposed by Calvo (1983), a fraction $1 - \xi_p$ of all firms re-optimize their nominal prices while the remaining $\xi_p$ fraction of all firms do not re-optimize their nominal prices. Following Christiano et al. (2005), firms that cannot re-optimize their price index to lagged inflation are as follows.

$$P_{j,t} = \pi_{t-1} P_{j,t-1}, \tag{12}$$

where $\pi_t = P_t/P_{t-1}$. We call this price setting “lagged inflation indexation.” The firm $j$ chooses $P_{j,t}$ to maximize

$$\begin{gathered} E_t \sum_{l=0}^{\infty} (\beta\xi_p)^l \Big[\frac{P_{j,t}}{P_{t+l}} X_{tl} - MC_{t+l}\Big] Y_{j,t+l}, \\ \text{subject to } Y_{j,t} = \Big(\frac{P_{j,t}}{P_t}\Big)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t, \end{gathered} \tag{13}$$

where $X_{tl}$ is

$$X_{tl} = \begin{cases} \pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+l-1} \text{ for } l \geq 1 \\ 0 \text{ for } l = 0. \end{cases} \tag{14}$$

The aggregate price index of sticky prices and inflation indexation is obtained by

$$P_t = \left[(1-\xi_p)(\tilde{P}_t)^{\frac{1}{1-\lambda_p}} + \xi_p(\pi_{t-1}P_{t-1})^{\frac{1}{1-\lambda_p}}\right]^{1-\lambda_p}. \tag{15}$$

## 2.5 Monetary Policy

The monetary authority is assumed to determine the nominal interest rate according to the Taylor rule with non-negativity constraint on the short-term nominal interest rate (the nonlinear Taylor rule) [13].

$$i_t = \max\big[r_0, (i_{t-1})^{\rho_i}(Y_t^{\phi_Y}\pi_t^{\phi_\pi})^{1-\rho_i} e^{\epsilon_t^i}\big], \tag{16}$$

where $r_0 \geq 0$ is the lower bound of the nominal interest rate, $\phi_Y$ and $\phi_\pi$ are constants, and $\epsilon_{i,t} \sim N(0, \sigma_{i,t}^2)$. In ordinary cases, $r_0$ is zero or nearly equal to zero.

## 2.6 Market Clearing

In the final market equilibrium, the final good production is equivalent to the households' demand for consumption, investment, and the expenditure of the government.

$$Y_t = C_t + I_t + G_t, \tag{17}$$

where $Y_t = \left[\int_0^1 (Y_{j,t})^{\frac{1}{1+\lambda_p}} dj\right]^{1+\lambda_p}$, $C_t = \left[\int_0^1 (C_{j,t})^{\frac{1}{1+\lambda_p}} dj\right]^{1+\lambda_p}$, $I_t = \left[\int_0^1 (I_{j,t})^{\frac{1}{1+\lambda_p}} dj\right]^{1+\lambda_p}$, and $G_t$ is a government expenditure.

## 2.7 Linearized Model

We linearize the model described above around the non-stochastic steady state. The linearized model consists of the hybrid new IS curve (HNISC), the hybrid new Keynesian Phillips curve (HNKPC), the nonlinear Taylor rule (NTR), and several equations. HNISC is obtained as follows [14].

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} - \frac{1}{1+h}E_t\big[\hat{i}_t - \hat{\pi}_{t+1}\big] + \epsilon_{C,t}. \tag{18}$$

HNKPC is obtained as follows.

$$\hat{\pi}_t = \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p(1+\beta)}\big[(1-\alpha)\hat{W}_t + \alpha\hat{r}^K - \hat{Z}_t\big] + \epsilon_{\pi,t}. \tag{19}$$

The other equations are

$$\hat{W}_t = \sigma_C(\hat{Y}_t - h\hat{Y}_{t-1}) + \sigma_L\hat{L}_t, \tag{20}$$

$$\hat{L}_t = -\hat{W}_t + \hat{r}^K + \hat{K}_t, \tag{21}$$

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}\hat{I}_{t+1} + \frac{\nu}{1+\beta}\hat{Q}_t + \epsilon_{I,t}, \tag{22}$$

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[13] See Taylor (1993), Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006), and Braun and Waki (2006).

[14] In this paper, a hat over a variable indicates the percentage deviation from its steady state value.

$$\hat{Q}_t = -E_t[\hat{i}_t - \pi_{t+1}] + \frac{1-\delta}{1-\delta+\bar{r}^K} E_t\hat{Q}_{t+1} + \frac{\bar{r}^K}{1-\delta+\bar{r}^K} E_t + \epsilon_{Q,t}, \tag{23}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t, \tag{24}$$

$$\hat{Y}_t = \Psi_C\hat{C}_t + \Psi_I\hat{I}_t + \Psi_G\hat{G}_t, \tag{25}$$

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{L}_t, \tag{26}$$

$$\hat{G}_t = \rho_G\hat{G}_{t-1} + \epsilon_{G,t}, \tag{27}$$

and

$$\hat{Z}_t = \xi_Z\hat{Z}_{t-1} + \epsilon_{Z,t}, \tag{28}$$

where $\epsilon_{C,t} \sim N(0,\sigma^2_{C,t})$, $\epsilon_{\pi,t} \sim N(0,\sigma^2_{\pi,t})$, $\epsilon_{I,t} \sim N(0,\sigma^2_{I,t})$, $\epsilon_{Q,t} \sim N(0,\sigma^2_{Q,t})$, $\epsilon_{G,t} \sim N(0,\sigma^2_{G,t})$, and $\epsilon_{Z,t} \sim N(0,\sigma^2_{Z,t})$. Following Braun and Waki (2006), we focus on the equilibria in which the zero nominal interest rate constraint in Eq. (16) binds once for a finite number of periods. In other words, the constraint binds for all $t$ such that $S < t \leq T$, and a short-term nominal interest rate is positive for all $t$ such that $0 \leq t \leq S$ or $t > T$. Thus, the linearized NTR is given by

$$\hat{i}_t = \max\left[-(r^s+\pi^s),\ \ \rho_i\hat{i}_{t-1} + (1-\rho_i)(\phi_Y\hat{Y}_t + \phi_\pi\hat{\pi}_t) + \epsilon_{i,t}\right], \tag{29}$$

where $r^s$ is an equilibrium real rate and $\pi^s$ is the target rate of inflation.

## 2.8 State Space Model

Structural linear rational expectations models are given by

$$\begin{cases} \boldsymbol{\Gamma}_0\boldsymbol{x}_t = \boldsymbol{\Gamma}_1\boldsymbol{x}_{t-1} + \boldsymbol{\Psi}\boldsymbol{z}_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t + \boldsymbol{C}, & \text{if } 0 \leq t \leq S \text{ or } t > T \\ \boldsymbol{\Gamma}'_0\boldsymbol{x}_t = \boldsymbol{\Gamma}'_1\boldsymbol{x}_{t-1} + \boldsymbol{\Psi}'\boldsymbol{z}'_t + \boldsymbol{\Pi}\boldsymbol{\eta}_t + \boldsymbol{C}, & \text{if } S < t \leq T, \end{cases} \tag{30}$$

where $\boldsymbol{x}_t = [E_t\hat{C}_{t+1}, E_t\hat{\pi}_{t+1}, E_t\hat{r}^K_{t+1}, E_t\hat{I}_{t+1}, E_t\hat{Q}_{t+1}, \hat{Y}_t, \hat{C}_t, \hat{i}_t, \hat{\pi}_t, \hat{W}_t, \hat{r}^K_t.\hat{Z}_t, \hat{L}_t, \hat{K}_t, \hat{I}_t, \hat{Q}_t, \hat{G}_t]^t$,
$\boldsymbol{z}_t = \left(\epsilon_{C,t}, \epsilon_{\pi,t}, \epsilon_{I,t}, \epsilon_{Q,t}, \epsilon_{i,t}, \epsilon_{G,t}, \epsilon_{z,t}\right)^T \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_t)$
with $\boldsymbol{\Sigma}_t = diag((\sigma_{C,t})^2, (\sigma_{\pi,t})^2, (\sigma_{I,t})^2, (\sigma_{Q,t})^2, (\sigma_{i,t})^2, (\sigma_{G,t})^2, (\sigma_{z,t})^2)$, $\boldsymbol{z}'_t = \left(\epsilon_{C,t}, \epsilon_{\pi,t}, \epsilon_{I,t}, \epsilon_{Q,t}, \epsilon_{G,t}, \epsilon_{z,t}\right)^T$ $\sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_t)$ with $\boldsymbol{\Sigma}_t = diag((\sigma_{C,t})^2, (\sigma_{\pi,t})^2, (\sigma_{I,t})^2, (\sigma_{Q,t})^2, (\sigma_{G,t})^2, (\sigma_{z,t})^2)$, $\boldsymbol{\Pi} = \boldsymbol{0}$, and $\boldsymbol{C} = \boldsymbol{0}$ [15]. Sims (2002) proposes the solution of linear rational expectations models using QZ decomposition [16]. Following Sims (2002), reduced linear rational expectations models are obtained by

$$\begin{cases} \boldsymbol{x}_t = \boldsymbol{\Theta}_1\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{1,t}, & \text{if } 0 \leq t \leq S \text{ or } t > T \\ \boldsymbol{x}_t = \boldsymbol{\Theta}'_1\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}'_{1,t}, & \text{if } S < t \leq T, \end{cases} \tag{31}$$

where $\boldsymbol{\epsilon}_{1,t} = \boldsymbol{\Theta}_0\boldsymbol{z}_t$ and $\boldsymbol{\epsilon}'_{1,t} = \boldsymbol{\Theta}'_0\boldsymbol{z}'_t$. The symbols, $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_0$, $\boldsymbol{\Theta}'_1$, and $\boldsymbol{\Theta}'_0$ are described in Sims (2002).

The measurement equation of the model is

$$\boldsymbol{Y}_t = \boldsymbol{Y}^s + \boldsymbol{H}\boldsymbol{x}, \tag{32}$$

where $\boldsymbol{Y}_t = [YGR_t, CGR_t, IGR_t, WGR_t, INFL_t, LGR_t, INT_t]^t$, $\boldsymbol{Y}^s = [Y^s, Y^s, Y^s, Y^s, \pi^s, L^s, r^s+\pi^s]^t$, and $\boldsymbol{v}_t = \left(\epsilon^v_{Y,t}, \epsilon^v_{C,t}, \epsilon^v_{\pi,t}, \epsilon^v_{I,t}\epsilon^v_{W,t}, \epsilon^v_{L,t}, \epsilon^v_{i,t}\right)^T \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_{v,t})$

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[15] We set $\boldsymbol{\Pi}$ to $\boldsymbol{0}$ to rule out the indeterminacy and sunspot equilibria, which are discussed in Sims (2002), Lubik and Schorfheide (2003), and Hirose (2007).

[16] In empirical analysis, we use Sims's gensys.R and related codes. See http://sims.princeton.edu/yftp/gensys/

with $\mathbf{\Sigma}_{v,t} = diag((\sigma^v_{Y,t})^2(\sigma^v_{C,t})^2, (\sigma^v_{I,t})^2, (\sigma^v_{\pi,t})^2, (\sigma^v_{W,t})^2, (\sigma^v_{L,t})^2, (\sigma^v_{i,t})^2)$ [17]. The growth rate of real variables, $YGR_t$, $CGR_t$, $IGR_t$, $WGR_t$, and $LGR_t$, are a log difference of real GDP per capita, real consumption per capita, real investment per capita, real average wage, and average labor hours, respectively, rate of inflation, $INF_t$, is the a log difference of GDP deflator, and nominal interest rate, $INT_t$, is the uncollateralized overnight call rate. Any observations are annualized. The symbols, $Y^s$, $\pi^s$, $L^s$ and $r^s$ are the trend of real output, the target rate of inflation, the trend of labor, and the trend of real interest rates, respectively.

In our method, we estimate the parameters of Eq. (18)- (32) using the TVP approach, which is explained in section 3. Thus, we define the vector of time-varying parameters as follows.

$$\begin{aligned}\tilde{\theta}_t = [&h_t, \xi_{p,t}, \sigma_{L,t}, \nu\xi_{Z,t}, \Psi_{C,t}, \Psi_{I,t}, \Psi_{G,t}, \rho_{i,t}, \phi_{Y,t}, \phi_{\pi,t}, \rho_{G,t}, \sigma_{C,t}, \sigma_{\pi,t}, \sigma_{I,t}, \sigma_{Q,t}, \sigma_{i,t}, \sigma_{G,t}, \sigma_{Z,t}, \\ &Y^s_t, \pi^s_t, L^s_t, r^s_t, \sigma^v_{Y,t}, \sigma^v_{C,t}, \sigma^v_{\pi,t}, \sigma^v_{I,t}, \sigma^v_{W,t}, \sigma^v_{L,t}, \sigma^v_{i,t}].\end{aligned} \tag{33}$$

Note that we calibrate four parameters:$\beta$, $\alpha$, $\delta$, and $\bar{r}^K = 1/\beta - 1 + \delta$ (see section 4.1). Reduced linear rational expectations models are also redefined by

$$\begin{cases} \boldsymbol{x}_t = \mathbf{\Theta}_{1,t}\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{1,t}, & \text{if } 0 \le t \le S \text{ or } t > T \\ \boldsymbol{x}_t = \mathbf{\Theta}'_{1,t}\boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}'_{1,t}, & \text{if } S < t \le T, \end{cases} \tag{34}$$

where $\boldsymbol{\epsilon}_{1,t} = \mathbf{\Theta}_{0,t}\boldsymbol{z}_t$ and $\boldsymbol{\epsilon}'_{1,t} = \mathbf{\Theta}'_{0,t}\boldsymbol{z}_t$.

In previous papers on DSGE models, structural parameters of them are assumed to be “deep (invariant).” Our method, however, analyzes how stable structural parameters are. The time-varying-parameter approach is often used in state space modeling to estimate invariant parameters, for example, Kitagawa (1998) and Liu and West (2001). Even if we assume the random walk priors, which are described in section 3, it does not indicate that the deep parameters are “time-varying.” Our framework is just a practical one to estimate deep parameters. Adopting our framework creates the great advantage that the structural changes of parameters are detected naturally. Thus, it is suitable to analyze how stable structural parameters are. The second advantage of our method is that we are able to estimate new Keynesian DSGE models in the liquidity trap (Krugman (1998)) because NNNSS, which is described in section 3, allows model switching.

# 3 Estimation Method

To estimate a state vector $\boldsymbol{x}_t$ and a time-varying-parameter vector, $\tilde{\boldsymbol{\theta}}_t$, we adopt the Monte Carlo Particle Filter (MCPF), proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998).

## 3.1 Nonlinear, Non-Gaussian, and Non-stationary State Space Model

In this subsection, we describe a nonlinear, non-Gaussian, and non-stationary state space model and a self-organizing state space model (MCPF is described in the next subsection).

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[17] This equation is a modified version of the measurement equation of An and Schorfheide (2007) and Hirose and Naganuma (2007).

A nonlinear, non-Gaussian, and non-stationary state space model for the time series $\boldsymbol{Y}_t,\ t = \{1, 2, \cdots, T\}$ is defined as follows.

$$\begin{aligned} \boldsymbol{x}_t &= f_t(\boldsymbol{x}_{t-1}, \boldsymbol{\epsilon}_{1,t}, \boldsymbol{\xi}_s), \\ \boldsymbol{Y}_t &= h_t(\boldsymbol{x}_t, \boldsymbol{v}_t, \boldsymbol{\xi}_o), \end{aligned} \tag{35}$$

where $\boldsymbol{x}_t$ is an unknown $n_x \times 1$ state vector, $\boldsymbol{\epsilon}_{1,t}$ is $n_\epsilon \times 1$ system noise vector with a density function $q(\boldsymbol{\epsilon_1}|\cdot)$ [18], $\boldsymbol{v}_t$ is $n_v \times 1$ observation noise vector with a density function $r(\boldsymbol{v}|\cdot)$. The function $f_t : \boldsymbol{R}^{n_x} \times \boldsymbol{R}^{\epsilon_v} \rightarrow \boldsymbol{R}^{n_x}$ is a possibly nonlinear time-varying function and the function $h_t : \boldsymbol{R}^{n_x} \times \boldsymbol{R}^{n_v} \rightarrow \boldsymbol{R}^{n_y}$ is a possibly nonlinear time-varying function. The first equation of (35) is called a system equation and the second equation of (35) is called an observation equation. We would like to emphasize the functions, $f_t$ and $h_t$, are possibly time dependent. A system equation depends on a possibly unknown $n_s \times 1$ parameter vector, $\boldsymbol{\xi}_s$, and an observation equation depends on a possibly unknown $n_o \times 1$ parameter vector, $\boldsymbol{\xi}_o$. This NNNSS specifies the two following conditional density functions.

$$\begin{aligned} &p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_s), \\ &p(\boldsymbol{Y}_t|\boldsymbol{x}_t, \boldsymbol{\xi}_o). \end{aligned} \tag{36}$$

We define a parameter vector $\boldsymbol{\theta}$ as follows.

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\xi}_s \\ \boldsymbol{\xi}_o \end{bmatrix}. \tag{37}$$

We denote that $\theta_j,\ (1 \leq j \leq J)$ is the $j$th element of $\boldsymbol{\theta}$ and $J(= n_s + n_o)$ is the number of elements of $\boldsymbol{\theta}$. This type of state space model (35) contains a broad class of linear, nonlinear, Gaussian, or non-Gaussian time series models. In state space modeling, estimating the state space vector $\boldsymbol{x}_t$ is the most important problem. For the linear Gaussian state space model, the Kalman filter, which is proposed by Kalman (1960), is the most popular algorithm to estimate the state vector $\boldsymbol{x}_t$. For nonlinear or non-Gaussian state space models, there are many algorithms. For example, the extended Kalman filter (Jazwinski (1970)) is the most popular algorithm; other examples are the Gaussian-sum filter (Alspach and Sorenson (1972)), the dynamic generalized model (West et al. (1985)), and the non-Gaussian filter and smoother (Kitagawa (1987)). In recent years, MCPF for NNNSS has been a popular algorithm because it is easily applicable to various time series models [19].

In econometric analysis, generally, we don't know the parameter vector $\boldsymbol{\theta}$. In our framework, the unknown parameter vectors are $\boldsymbol{\xi}_o$ and $\boldsymbol{\xi}_s$ [20]. In traditional parameter estimation, maximizing the log-likelihood function of $\boldsymbol{\theta}$ is often used. The log-likelihood of $\boldsymbol{\theta}$ in MCPF is proposed by Kitagawa (1996). However, MCPF is problematic to estimate the parameter vector $\boldsymbol{\theta}$ because the likelihood of the filter contains errors from the Monte Carlo method. Thus, you cannot use nonlinear optimizing algorithm like Newton's method [21]. To solve the problem, Kitagawa (1998) proposes a self-organizing state space model. In Kitagawa (1998), an augmented state vector is defined as follows.

$$\boldsymbol{z}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{\Theta_t} \end{bmatrix}, \tag{38}$$

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[18] The system noise vector is independent of past states and current states.

[19] Many applications are shown in Doucet et al., eds (2001).

[20] Details of $\boldsymbol{\xi}_o$ and $\boldsymbol{\xi}_s$ are discussed in the next subsection.

[21] See Yano (2008a).

where $\boldsymbol{\Theta_t} = (\tilde{\boldsymbol{\theta}}_t, \boldsymbol{\theta})^t$, $\tilde{\boldsymbol{\theta}}_t$ is a vector of time-varying parameters, and $\boldsymbol{\theta}$ is a vector of invariant parameters. Note that $\tilde{\boldsymbol{\theta}}_t = \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{\epsilon}_{2,t}$, with $\boldsymbol{\epsilon}_{2,t}$ a white noise sequence distributed with a density function $p_2(\boldsymbol{\epsilon}_{2,t}|\boldsymbol{\Sigma}_{\xi_s})$. An augmented system equation and an augmented measurement equation are defined as

$$\begin{aligned} \boldsymbol{z}_t &= F_t(\boldsymbol{z}_{t-1}, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_s), \\ \boldsymbol{Y}_t &= H_t(\boldsymbol{z}_t, \boldsymbol{v}_t, \boldsymbol{\xi}_o), \end{aligned} \tag{39}$$

where

$$F_t(\boldsymbol{z}_{t-1}, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_s) = \begin{bmatrix} f_t(\boldsymbol{x}_{t-1}, \boldsymbol{\epsilon}_{1,t}, \boldsymbol{\xi}_s) \\ \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{\epsilon}_{2,t} \\ \boldsymbol{\theta} \end{bmatrix}$$

and

$$H_t(\boldsymbol{z}_t, \boldsymbol{v}_t, \boldsymbol{\xi}_o) = h_t(\boldsymbol{x}_t, \boldsymbol{v}_t, \boldsymbol{\xi}_o)$$

where $\boldsymbol{\epsilon}_t = (\boldsymbol{\epsilon}_{1,t}, \boldsymbol{\epsilon}_{2,t})^t$. This NNNSS is called a self-organizing state space (SOSS) model. In our method, we stress that states, time-varying parameters, and invariant parameters are estimated simultaneously. Therefore, our problem is how to estimate $\boldsymbol{z}_t$.

### 3.2 Monte Carlo Particle Filter

The Monte Carlo particle filter is a variant of sequential Monte Carlo algorithms. In MCPF, the expectation of a posterior distribution are approximated using "particles" that have weights.

$$E[p(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t})] \simeq \frac{1}{\sum_{m=1}^{M} w_t^m} \sum_{m=1}^{M} w_t^m \delta(\boldsymbol{z}_t - \boldsymbol{z}_t^m), \tag{40}$$

where $w_t^m$ is the weight of a particle $\boldsymbol{z}_t^m$, $M$ is the number of particles, and $\delta$ is the Dirac's delta function [22]. Weights $w_t^m$ $m = \{1, 2, \cdots, M\}$ are defined as follows.

$$w_t^m = r(\psi(\boldsymbol{y}_t, \boldsymbol{z}_t^m)) \Big| \frac{\partial \psi}{\partial \boldsymbol{y}_t} \Big|, \tag{41}$$

where $\psi$ is the inverse function of the function $h$ [23]. The right hand side of Eq. (41) is the likelihood function of an NNNSS model. In the standard algorithm of MCPF, the particles $x_t^m$ are resampled with sampling probabilities proportional to $w_t^1, \cdots, w_t^M$. Resampling algorithms are discussed in Kitagawa (1996). After resampling, we have $w_t^m = 1/M$. Therefore, Eq. (40) is rewritten as

$$E[p(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t})] \simeq \frac{1}{M} \sum_{m=1}^{M} \delta(\boldsymbol{z}_t - \hat{\boldsymbol{z}}_t^m), \tag{42}$$

where $\hat{\boldsymbol{z}}_t^m$ are particles after resampling. Particles $\boldsymbol{x}_t^m$ $m = \{1, 2, \cdots, M\}$ are sampled from a system equation:

$$\boldsymbol{z}_t^m \sim p(\boldsymbol{z}_t|\boldsymbol{z}_{t-1}^m, \boldsymbol{\xi}_s). \tag{43}$$

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[22] The Dirac delta function is defined as

$$\begin{aligned} &\delta(x) = 0, \text{ if } x \neq 0, \\ &\int_{\infty}^{\infty} \delta(x) dx = 1. \end{aligned}$$

[23] See Kitagawa (1996).

Kitagawa (1996) shows that the log-likelihood of $\boldsymbol{\theta}$ is approximated by

$$l(\boldsymbol{\theta}) \simeq \sum_{t=1}^{T} \log(\sum_{m=1}^{M} w_t^m) - T \log M, \tag{44}$$

where $T$ is the number of observations. Using Eq. (44), we can compare the fits of DSGE models. In self-organizing state space modeling, the augmented state vector is estimated using MCPF. Thus, states and parameters are estimated simultaneously without maximizing the log-likelihood of Eq. (39) because the parameter vector $\boldsymbol{\theta}$ in Eq. (39) is approximated by particles and it is estimated as the state vector in Eq. (38) [24].

On a self-organizing state space model, however, Hürseler and Künsch (2001) points out a problem: determination of initial distributions of parameters for a self-organizing state space model. The estimated parameters of a self-organizing state space model comprise a subset of the initial distributions of parameters. We must know the posterior distributions of parameters to estimate parameters adequately. However, the posterior distributions of the parameters are generally unknown. Parameter estimation fails if we do not know their appropriate initial distributions. Yano (2008a) proposes a method to seek initial distributions of parameters for a self-organizing state space model using the simplex Nelder-Mead algorithm to solve the problem. In this paper, we use uniform distributions for initial distributions of time-varying parameters because most time-varying parameters are restricted to be more than zero and less than unity.

## 3.3 Time-varying Parameters

In this paper, we assume the "symmetric" random walk prior (the Litterman prior) to estimate time-varying parameters (see Doan et al. (1984)) [25]. The random walk prior is given by

$$\tilde{\boldsymbol{\theta}}_t = \tilde{\boldsymbol{\theta}}_{t-1} + \boldsymbol{\epsilon}_{2,t}, \tag{45}$$

where $\boldsymbol{\epsilon}_{2,t} \sim q(\boldsymbol{\epsilon}_{2,t}|\Sigma_{\boldsymbol{\xi}_s})$, $q(\boldsymbol{\epsilon}_{2,t}|\Sigma_{\boldsymbol{\xi}_s})$ is a Gaussian distribution, and $\Sigma_{\boldsymbol{\xi}_s}$ is a diagonal matrix. In general, the diagonal components, $\{\xi_{1,s}, \xi_{2,s}, \cdots, \xi_{L,s}\}$, of $\Sigma_{\boldsymbol{\xi}_s}$ are different. In this paper, however, to reduce computational complexity, we define time evolution of a coefficient as follows:

$$\tilde{\theta}_{i,t} = \tilde{\theta}_{i,t-1} + |\xi_{s,\cdot}|\epsilon_{2,i,t}, \tag{46}$$

where $\epsilon_{2,i,t} \sim N(0, |\xi_{s,2}|)$ if $h$, $\xi_p$, $\sigma_L$, $\nu$, $\xi_Z$, $\Psi_C$, $\Psi_I$, $\Psi_G$, $\phi_Y$, $\phi_\pi$, and $\rho_G$ and $\epsilon_{2,i,t} \sim |\xi_{s,1}| \times t(df = 25)$ if otherwise. Note that $\sigma_{L,t}$, $\xi_{Z,t}$, $\phi_{Y,t}$, $\phi_{\pi,t}$, $\sigma_{Y,t}$, $\sigma_{\pi,t}$, $\sigma_{i,t}$, $\sigma_{Z,t}$, $\sigma^v_{Y,t}$, $\sigma^v_{\pi,t}$, $\sigma^v_{i,t}$ are restricted to be positive and $h_t$, $\sigma_{C,t}$, $\xi_{p,t}$, and $\rho_{i,t}$ are restricted to be more than zero and less than unity. The particles that violate these restrictions are numerically discarded before resampling.

## 3.4 Algorithm

In our method, we adopt not a smoothing algorithm but a filtering algorithm because the rational expectations hypothesis is consistent with the latter. If we use a smoothing algorithm to estimate time-

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[24] The justification of an SOSS model is described in Kitagawa (1998).

[25] See also a traditional approach, proposed by Cooley and Prescott (1976). The smoothness priors proposed by Kitagawa (1983) is a generalization of the random walk priors.

varying parameters, the estimates of them include the information at times $t+1, t+2, \cdots$ which is not known at time $t$. Our method to estimate time-varying parameters of DSGE models is summarized as follows:

1. In time $t$, generate $\boldsymbol{z}_t$ based on the results at time $t-1$.
2. Using particles, the linear rational expectations system is solved to obtain the state transition equation Eq. (31).
3. If a particle implies indeterminacy (or non-existence of a stable rational expectations solution), then the weight of the particle, $\boldsymbol{w}_t^m$, is set to zero.
4. If $\boldsymbol{\Theta}_1$ or $\boldsymbol{\Theta}_1'$ is not invertible, the particle is discarded (See Braun and Waki (2006) Algorithm in appendix).
5. If a unique stable solution exists, then the weight of the particle is calculated using Eq. (41).
6. Resampling particles with sampling probabilities proportional to $w_t^1, \cdots, w_t^M$.
7. Replace $t$ with $t+1$.
8. Go to 1.

# 4 Empirical Analysis

We use data from 1981:Q1 up to 2007:Q4 [26]. We assume the Japanese economy was trapped in a liquidity trap (the non-negativity constraint on nominal short-term interest rates) at 1999:Q1. Moreover, we suppose the economy escapes from the trap at 2006:Q4 because the quantitative-easing policy and the zero-interest-rate policy of the BOJ are ended at 2006:Q1 and 2006:Q3, respectively.

## 4.1 Preliminary Setting

Following Sugo and Ueda (2008), we calibrate four parameters: $\beta = 0.99$, $\alpha = 0.3$, $\delta = 0.06$, and $\bar{r}^K = 1/\beta - 1 + \delta$. For preliminary setting, we estimate our DSGE model using Dynare, developed by Juillard (1996) [27]. In Table 1, the estimates of Dynare are shown.

[Table 1 about here.]

For our method, we determine the prior distributions of time-varying parameters based on Table 1. The other simulation settings are described in appendix B.

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[26] We remove data from 1980:Q1 to 1980:Q4 to avoid the influences of the second oil shock. The details of the data are described in appendix A.

[27] For the Dynare MCMC estimation, all Japanese data from 1998:Q1 to 1998:Q4 are detrended by the Hodrick-Prescott filter. The prior distributions of the parameters for Dynare are determined following Sugo and Ueda (2008).

## 4.2 Preliminary Estimation: Calvo Parameter and Taylor Parameters

First, we estimate the time-varying Calvo parameter when the other parameters, which are estimated by Dynare, are fixed. To compare the result of Fernandez-Villaverde and Rubio-Ramirez (2007b), data are detrended by the Hodrick-Prescott filter. To avoid the "zero-interest rate" period, data from 1981:Q1 to 1998:Q4 are used in this estimation because the HP filter is not suitable to detrend the zero nominal interest rate. Fig. 1 shows that the Calvo parameter fluctuates from 1985:Q1 to 1998:Q4. This result is consistent with Fernandez-Villaverde and Rubio-Ramirez (2007b).

[Figure 1 about here.]

Second, we estimate the time-varying Taylor parameters when the other parameters, which are estimated by Dynare, are fixed. To compare the result of Fernandez-Villaverde and Rubio-Ramirez (2007b), data are also detrended by the Hodrick-Prescott filter. Data from 1981:Q1 to 1998:Q4 are also used in this estimation. Fig. 2 shows that the Taylor parameters fluctuate from 1985:Q1 to 1998:Q4. These results are consistent with Fernandez-Villaverde and Rubio-Ramirez (2007b).

[Figure 2 about here.]

Fernandez-Villaverde and Rubio-Ramirez (2007b) suggest that these fluctuations of structural parameters cause serious doubts on Calvo pricing and new Keynesian DSGE models. We, however, document different suggestions in the following subsections.

## 4.3 Empirical Analysis

Figure 3 shows the annualized estimates of $Y_t^s$, $\pi_t^s$, $L_t^s$, and $r_t^s$ [28]. The black lines in all figures are means of particles, and the green and red lines are 95% confidence intervals, which are calculated using 100 bootstrap samples of particles. From the mid-1980s to the early 1990, $Y_t^s$ is from about 2% to 5%, and the periods are called the "bubble economy." From the mid-1990s to the early 2000s, $Y_t^s$ is relatively small, and the periods are called "a lost decade." In the 2000s, $Y_t^s$ is from 0% to 1%. From the mid-1980s to the mid-1990, $\pi_t^s$ is positive, and it is from 1% to 2%. From the early 1990 to present, $\pi_t^s$ is negative. The results shows the target rate of the inflation of the BOJ is changed in the early 1990s, and the target in the 1990s and 2000s is too low. From the 2006, the BOJ announces "understanding of the price stability," and it states a stable inflation rate is from 0% to 2%, which is measured by consumer price index, excluding food. This low target rate makes $\pi_t^s$ negative because it is well known that CPIs have upward bias. From the mid-1980s to the early 1990s, $r_t^s$ is above 5%, and from the early 1990s to present, it is below 1%. The $r_t^s$ is an estimate of an equilibrium real rate [29]. Krugman (1998) states ERR of the Japanese economy in the late 1990s is negative. However, our estimate of ERR is not negative but quite low in 1997 and 1998. It strongly suggests that the BOJ, which adopted quite low interest rate policy at the time, needed positive inflation rates to stimulate the economy in the late 1990s. Note that the target rate of inflation, $\pi_t^s$, in the 1990s is negative.

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[28] We remove the results from 1981:Q1 to 1984:Q4 to avoid the influences of poor prior distributions.

[29] Laubach and Williams (2003) and Trehan and Wu (2007) estimate time-varying equilibrium real rate using a simple, backward-looking model of the U.S. economy.

[Figure 3 about here.]

Figure 4 shows the estimates of the endogenous variables. The output gap, $\hat{Y}_t$, indicates that the favorable economic situation ends at early 1990s, and serious recessions happen in the early 1990s, 1997-1998, 2000-2001. The inflation rate, $\hat{\pi}_t$, shows that in the mid-1980s and the 1990s negative deviation from the target rate of inflation happen. In particular, the negative deviation in the 1990s is very long, and it indicates the long-term recession of the economy. Interest rate, $\hat{i}_t$, shows the deviation from the equilibrium real interest rate, and it presents the fact that the BOJ made expansionary monetary policy in the late 1980s and the early 1990s. From 1999, the $\hat{i}_t$ is zero because the Japanese economy is in a liquidity trap. The symbol, $\hat{Z}_t$, shows the negative technology shocks that happened in the early 1990s, the late 1990s, and the early 2000s, and they correspond to the recessions from 1985 to 2007.

[Figure 4 about here.]

Fig. 5 shows the estimates of time-varying parameters. These estimates indicate that some "structural" parameters are time-varying. The results indicate that habit persistence, $h$, the Calvo parameter, $\xi_p$, and the coefficient of AR(1) technology process, $\xi_z$, are relatively stable. The parameter, $\rho_G$, is gradually decreasing from 1980 to 2008.

[Figure 5 about here.]

Figure 6 shows the estimates of time-varying parameters of NTR. The inertia term, $\rho_i$, is from 0.2 to 0.5 in most periods. It indicates that the BOJ makes the nominal short-term interest rate smooth. The coefficient of the output gap, $\phi_Y$, is from 0.2 to 0.3 in most periods, and it shows that the BOJ's reaction of output gap is stable from 1985 to present. The coefficient of the inflation rate, $\phi_\pi$, decreases from 1.5 to 1.15.

[Figure 6 about here.]

[Figure 7 about here.]

The time evolutions of standard deviations, $\sigma_C$ and $\sigma_\pi$, are shown in figure 8. These results indicate that there does not exist the "great moderation" in the Japanese economy.

[Figure 8 about here.]

How stable are structural parameters? Our conclusion is a little bit different from the serious doubts of Fernandez-Villaverde and Rubio-Ramirez (2007b). The doubts in their paper are caused by the strong correlation between inflation and the Calvo parameter and the instability of the coefficients of the Taylor rule. We agree with them that there exist some structural changes of structural parameters from 1981:Q1 to 2008:Q4. Our estimates, however, indicate that the structural changes are not strongly correlated with inflation and business cycles. In particular, severe structural changes of $\sigma_L$ and $\nu$ might point out that the models of the perfect competitive labor markets and Tobin's $q$ are imperfect. Additionally, it is commonly known that the Japanese economy is strongly depend on trades, while we estimate the closed economy model. Our results only suggest the necessity of more investigations on new Keynesian DSGE models.

In practice, the Hodrick and Prescott (1997) filter is often used to estimate the natural output of the Japanese economy. However, whether the HP filter and the magic number, which is suggested in Hodrick and Prescott (1997), are appropriate for estimation of Japanese natural output is an open question. Urasawa (2008) uses the Baxter and King (1999) filter to provide the stylized facts of Japanese business cycles. Our method is an alternative to these filters, and it is "DSGE-based" estimation of time-varying economic trends. In Figure 9, we compare our annualized estimates of output gap with estimates of the HP filter and the CF filter. In the upper panel of Figure 9, we show our estimate (the black line) and the estimate of the HP filter (the blue line). From 1985 to the mid-1990s the black line is different from the blue one. The blue line indicates that the output gap is negative in the late 1980s and positive in the early 1990s. In the late 1980s, Japanese economy was in the "bubble" economy, and in the early 1990s, was in the "Heisei" recession. The output gap based on the HP filter is not consistent with these facts, and the one based on our method is consistent with them. Before the mid-1990s, our method is better than the HP filter. The black line coincides with the blue one from the mid-1990s to the 2000s. In the lower panel of Figure 9, we show our estimate (the black line) and the estimate of the CF filter (the green line). The green line is much smoother rather than the black line, and the black one coincides with the green one from the late 1990s to the 2000s. We conclude that our estimate of output gap relatively coincides with the estimates, which are calculated by the HP/CF filters, although our method is totally different from the filters.

[Figure 9 about here.]

Using the log-likelihood of a model, Eq. (44), we compare DSGE models: the model in section 2, the DSGE model without inflation idexation, and the DSGE model without habit formation. The log-likelihoods of models and the estimates of $|\xi_s|$ are shown in Table 2. These results indicate that our model in section 2 is better than the other models. They also show that the inflation inertia and the habit persistence have crucial roles in empirical analysis based on DSGE models, and they are consistent with An and Schorfheide (2007), Hirose and Naganuma (2007), and related studies.

[Table 2 about here.]

# 5 Conclusion and Discussion

This paper proposes a new method to estimate parameters of dynamic stochastic general equilibrium models in a liquidity trap based on the Monte Carlo particle filter and a self-organizing state space model. This method is a natural extension of Yano (2009). Our method analyzes how stable structural parameters are. Adopting it creates the great advantage that the structural changes of parameters are detected naturally. The novel feature of our method is that we are able to estimate parameters of new Keynesian DSGE models in a liquidity trap (Krugman (1998)), because nonlinear, non-Gaussian, and non-stationary state space models allow model switching. Moreover, we estimate time-varying trends of macroeconomic data: real output, inflation rate, and real interest rate. To estimate trends of macroeconomic data, the Hodrick-Prescott filter, proposed by Hodrick and Prescott (1997), is often used. In recent years, the Baxter-King filter, proposed by Baxter and King (1999), and the Christiano-Fitzgerald filter, proposed by Christiano and Fitzgerald (2003) are also often used. Our method is an alternative to these filters, and it is a "DSGE-based" estimation of time-varying economic trends. We conclude that our estimate of output gap relatively coincides with the estimates, which are calculated by the HP/CF filters, although our method is totally different from the filters. In empirical analysis, we estimate new Keynesian DSGE models in a liquidity trap using Japanese macroeconomic data, which include the "zero-interest-rate" period (1999-2006). The analysis shows that the growth rate of natural output declines in the late 1990s but becomes as high as about 0.5% in the mid-2000s. The target rate of inflation is too low in the 1990s and the 2000s, and it causes deflation in the Japanese economy. In the the "zero-interest-rate" period, the impulse responses to technology shocks, aggregate demand shocks, and aggregate supply shocks are more volatile than the other period because the stabilizing effect of monetary policy is lost in the liquidity trap. These results are consistent with Section 4.2, Woodford (2003). Following Eggertsson and Woodford (2003), this problem can be solved by adopting the flexible $2\% - 3\%$ targeted rate of inflation based on GDP deflator.

In a new study, we are estimating new Keynesian, small open economy DSGE models, new Keynesian DSGE models with liquidity-constraint households, Christiano et al. (2005), and second-order approximation of DSGE models. Furthermore, our method can be easily extended to estimate state-dependent-pricing models with random menu costs, proposed by Dotsey et al. (1999) [30]. In policy analysis of DSGE models, impulse response functions are often used. In our framework, the effectiveness of the traditional way is ambiguous because parameters in DSGE models are time-varying. If we calculate impulse response function at time $t$, the results of them may be meaningless because the parameters may have changed at time $t+1$. Canova and Gambetti (2006) proposes the use of generalized impulse response functions in time-varying structural vector autoregressions. However, in time-varying analysis of DSGE models, it is an open question. We assume that the timings of when the economy is trapped in a liquidity trap and its subsequent escaped from it are given. The endogenous timings are our future work.

# A Braun and Waki (2006) Algorithm

Braun and Waki (2006) develop an algorithm for computing perfect foresight equilibria in situations in which the zero nominal interest rate constraint binds once for a finite number of periods. In this section, we outline the algorithm. **Backward Solution Algorithm**

**Case 1:** $t > T$

For all $t > T$, reduced linear rational expectations models are obtained by

$$\boldsymbol{x}_t = \boldsymbol{\Theta}_1 \boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{1,t}. \tag{47}$$

If $\boldsymbol{\epsilon}_{1,t}$ $(t > T)$ and $x_T$ are given, we can obtain the entire sequence of $x_t$ for all $t$ such that $t > T$ by sequential forward substitution of Eq. (47).

**Case 2:** $S < t \leq T$

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[30] Gertler and Leahy (2006) and Bakhshi et al. (2007) derive a Phillips curve equation from a DSGE model with state-dependent pricing.

For all $t$ such that $S < t \leq T$

$$\boldsymbol{x}_t = \boldsymbol{\Theta}_1' \boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{1,t}'. \tag{48}$$

If $\boldsymbol{\Theta}_1'$ is invertible and $\boldsymbol{\epsilon}_{1,t}$ are given, from $x_T$, we can obtain the entire sequence of $x_t$ for all $t$ by sequential backward substitution such as

$$\begin{aligned} \boldsymbol{x}_{T-1} &= (\boldsymbol{\Theta}_1')^{-1}(\boldsymbol{x}_T - \boldsymbol{\epsilon}_{1,T}'), \\ \boldsymbol{x}_{T-2} &= (\boldsymbol{\Theta}_1')^{-1}(\boldsymbol{x}_{T-1} - \boldsymbol{\epsilon}_{1,T-1}'), \\ &\vdots \\ \boldsymbol{x}_S &= (\boldsymbol{\Theta}_1')^{-1}(\boldsymbol{x}_{S+1} - \boldsymbol{\epsilon}_{1,S+1}'). \end{aligned} \tag{49}$$

**Case 3:**
For all $t$ such that $0 \geq t \geq S$, we again have

$$\boldsymbol{x}_t = \boldsymbol{\Theta}_1 \boldsymbol{x}_{t-1} + \boldsymbol{\epsilon}_{1,t}. \tag{50}$$

If $\boldsymbol{\Theta}_1$ is invertible and $\boldsymbol{\epsilon}_{1,t}$ are given, we can obtain the entire sequence of $x_t$ for all $t$ by sequential backward substitution again.

If $S$ and $T$ are given, the equilibrium is computed with their algorithm. Given a level of the capital stock in period $T$ , $k_T$, calculate the equilibrium path for all $t \geq T+1$. Next use the equilibrium values of the variables in period $T$ to solve the system backward for $k_0$. Repeat for different choices of $k_T$ until the implied initial capital stock $k_0$ is equal to its value in Japanese data. Braun and Waki (2006) assume that S occurs in 1997, and then choosing $T$ to be the earliest year where the constraint ceases to bind. In our method, we assume that $S$ and $T$ are given. Thus, we need only to check the invertability of $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_1'$ [31].

# B Data Source

We use quarterly macroeconomic data on the Japanese economy from 1981:Q1 to 2007:Q4.

- Uncollateralized overnight call rate (Bank of Japan): uncollateralized overnight call rate, monthly average (July 1985-December 2007) and collateralized overnight call rate, monthly average (January 1981 - July 1985) are linked at July 1985. All data are averaged over three months.
  http://www.boj.or.jp/en/theme/research/stat/market/index.htm

- Seasonally-adjusted real/nominal GDP, private consumption, private non-residential investment (Cabinet Office): quarterly estimates of GDP, chained, (1994:Q1-2006:Q3, Reference-year = 2000) and quarterly estimates of GDP, fixed-based, (1981:Q1-1994:Q1, Base-year = 1995) are linked at 1994:Q1.
  http://www.esri.cao.go.jp/en/sna/menu.html
  http://www.esri.cao.go.jp/en/sna/qe081-2/gdemenuea.html
  http://www.esri.cao.go.jp/en/sna/qe052-2/gdemenuebr.html

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[31] This algorithm is easily extended for computing perfect foresight equilibria in situations in which the zero nominal interest rate constraint binds twice.

- Seasonally-adjusted GDP deflator (Cabinet Office): the deflator is calculated from seasonally-adjusted real/nominal GDP.
- Seasonally-adjusted labor force population (Ministry of Internal Affairs and Communications): January 1981 - December 2007 (averaged over three months.)
  http://www.stat.go.jp/english/data/roudou/lngindex.htm
- Seasonally-adjusted real wage index, establishments with 30 employees or more, industries covered (Ministry of Health, Labor and Welfare): January 1981 - December 2007 (averaged over three months.)
  http://www.mhlw.go.jp/english/database/db-l/index.html
- Seasonally-adjusted hours worked index, establishments with 30 employees or more, industries covered (Ministry of Health, Labor and Welfare): January 1981 - December 2007 (averaged over three months.)
  http://www.mhlw.go.jp/english/database/db-l/index.html

# C Simulation Setting

We use uniform distributions for initial prior distributions of states, time-varying parameters, and parameters: $uniform(-1,1)$ for states, $uniform(0,1)$ for time-varying parameters, and $uniform(0,0.2)$ for parameters. The number of particle is 10,000 at time $t$. Thus, we generate 270,000 random variables at time $t$. In Eq. (30), we set $\boldsymbol{C}$ to zero.

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# List of Figures

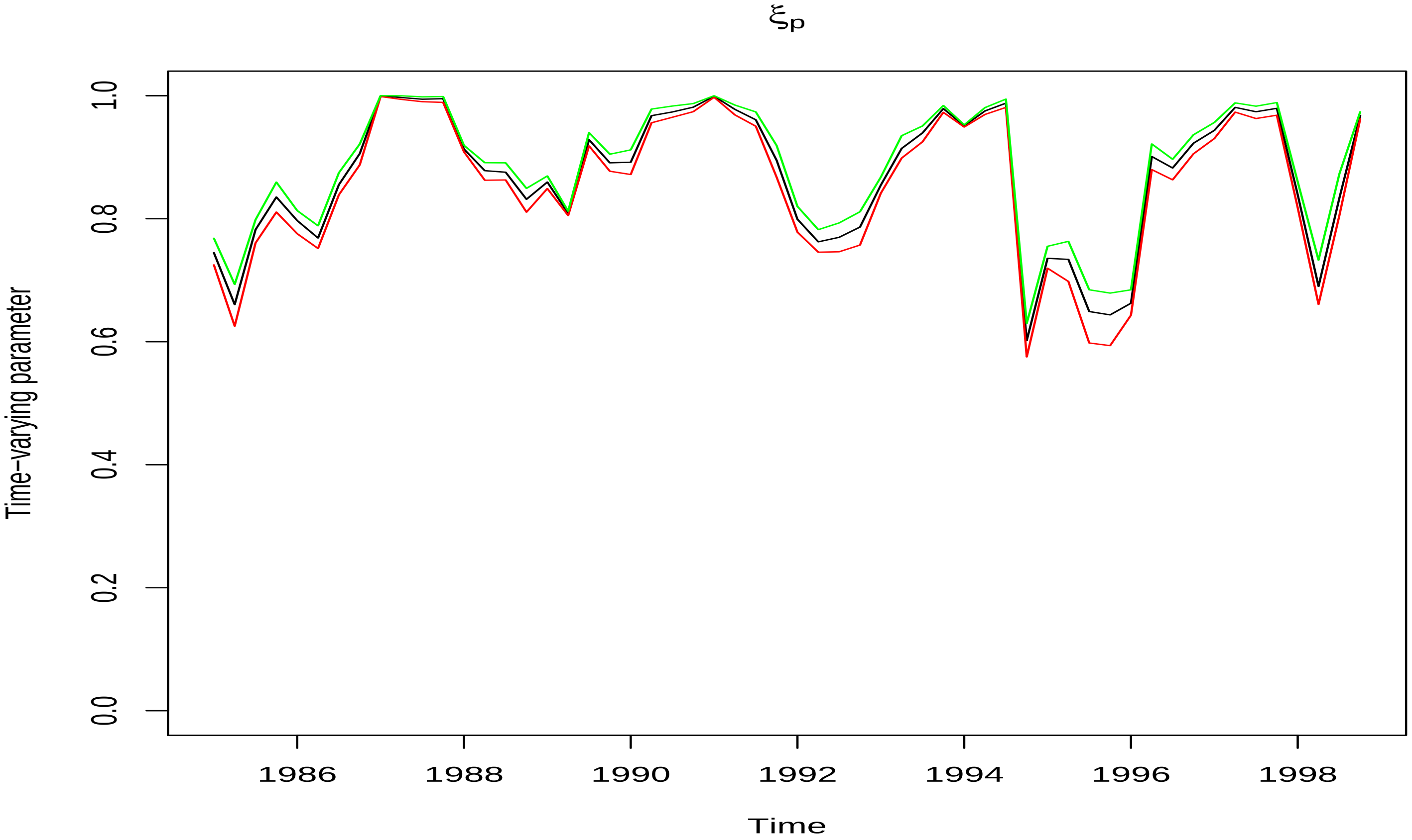

Figure 1: Time-varying Calvo Parameter

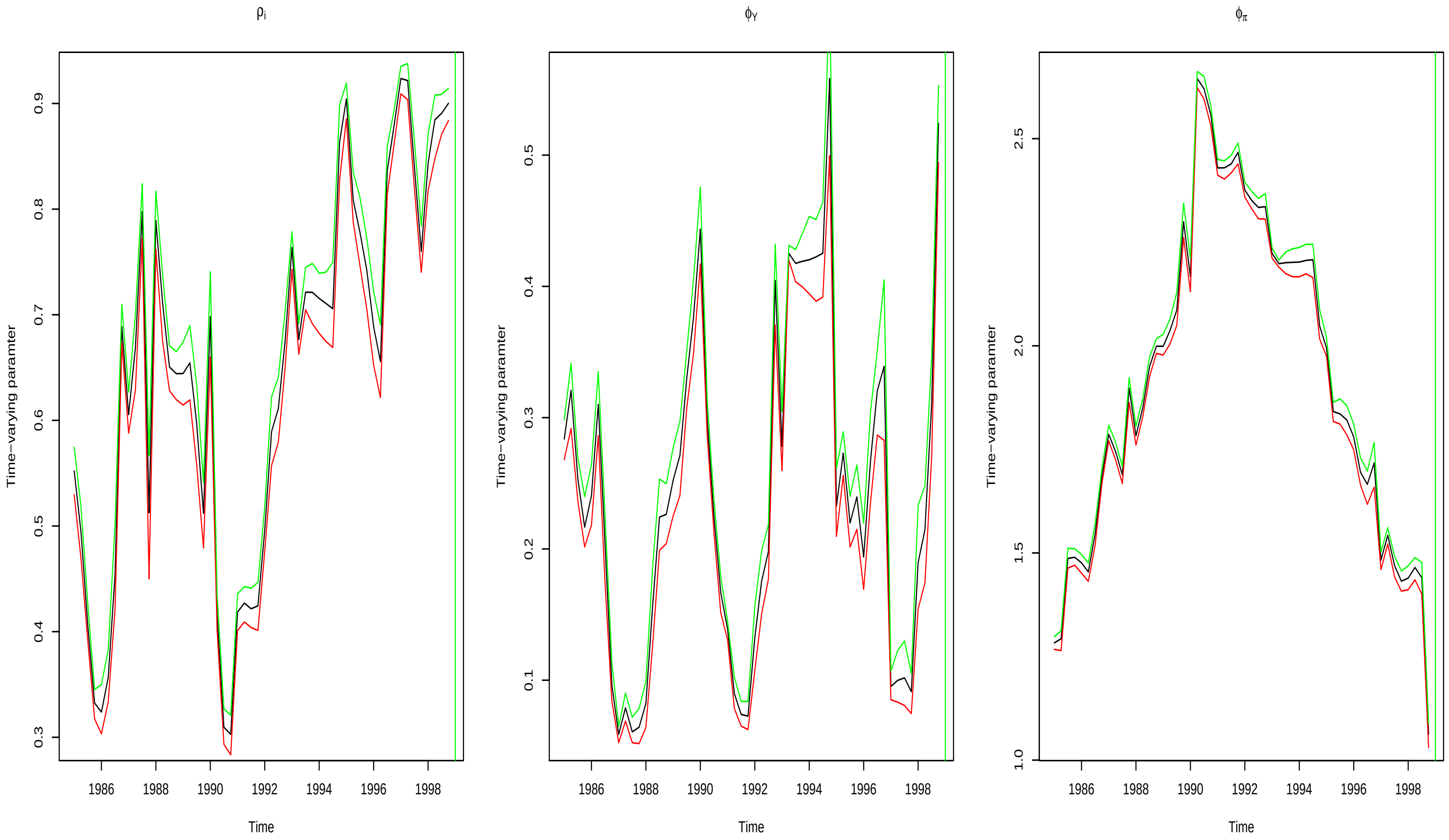

Figure 2: Time-varying Taylor Parameters

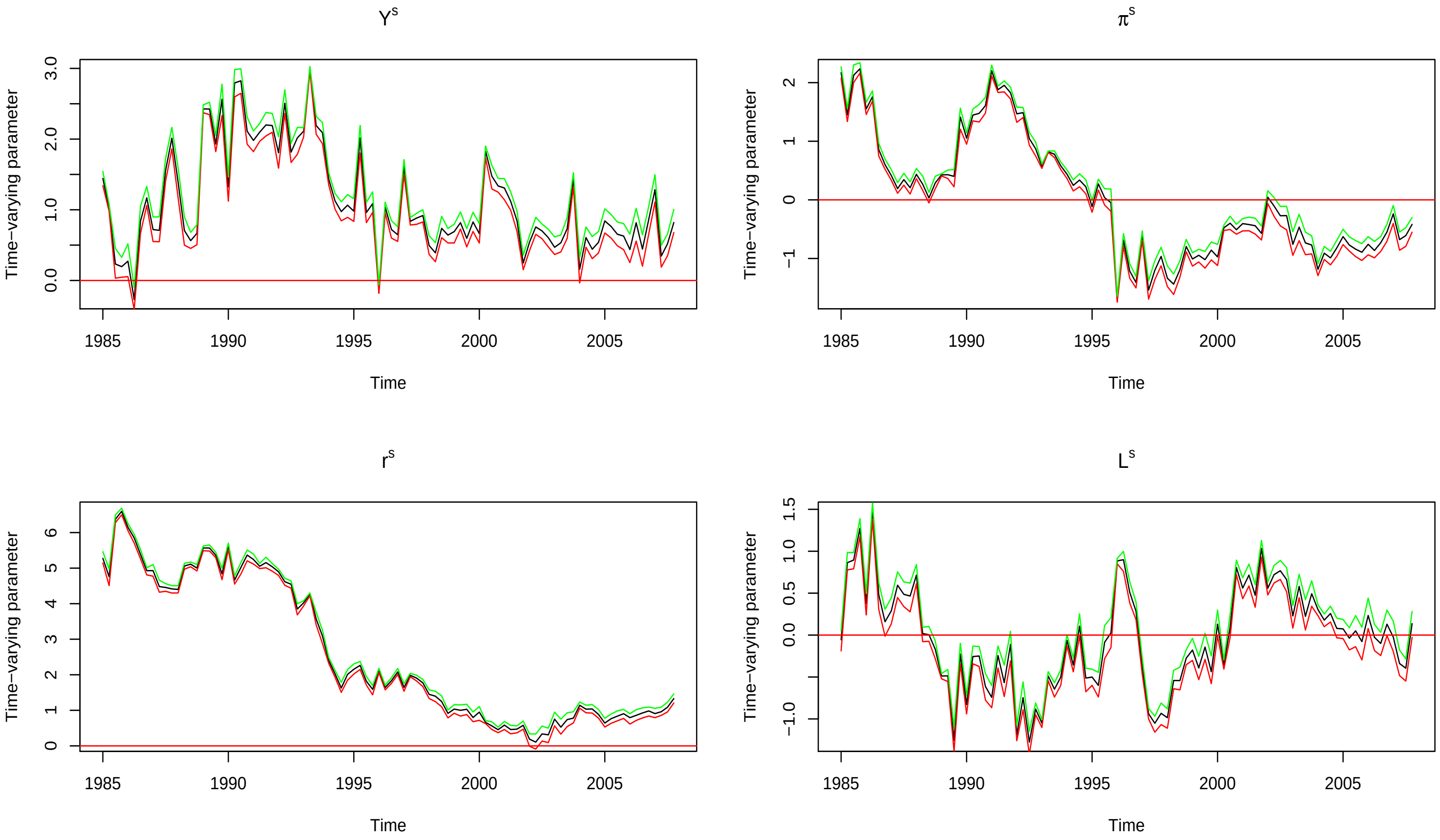

Figure 3: Time-varying trends and targets

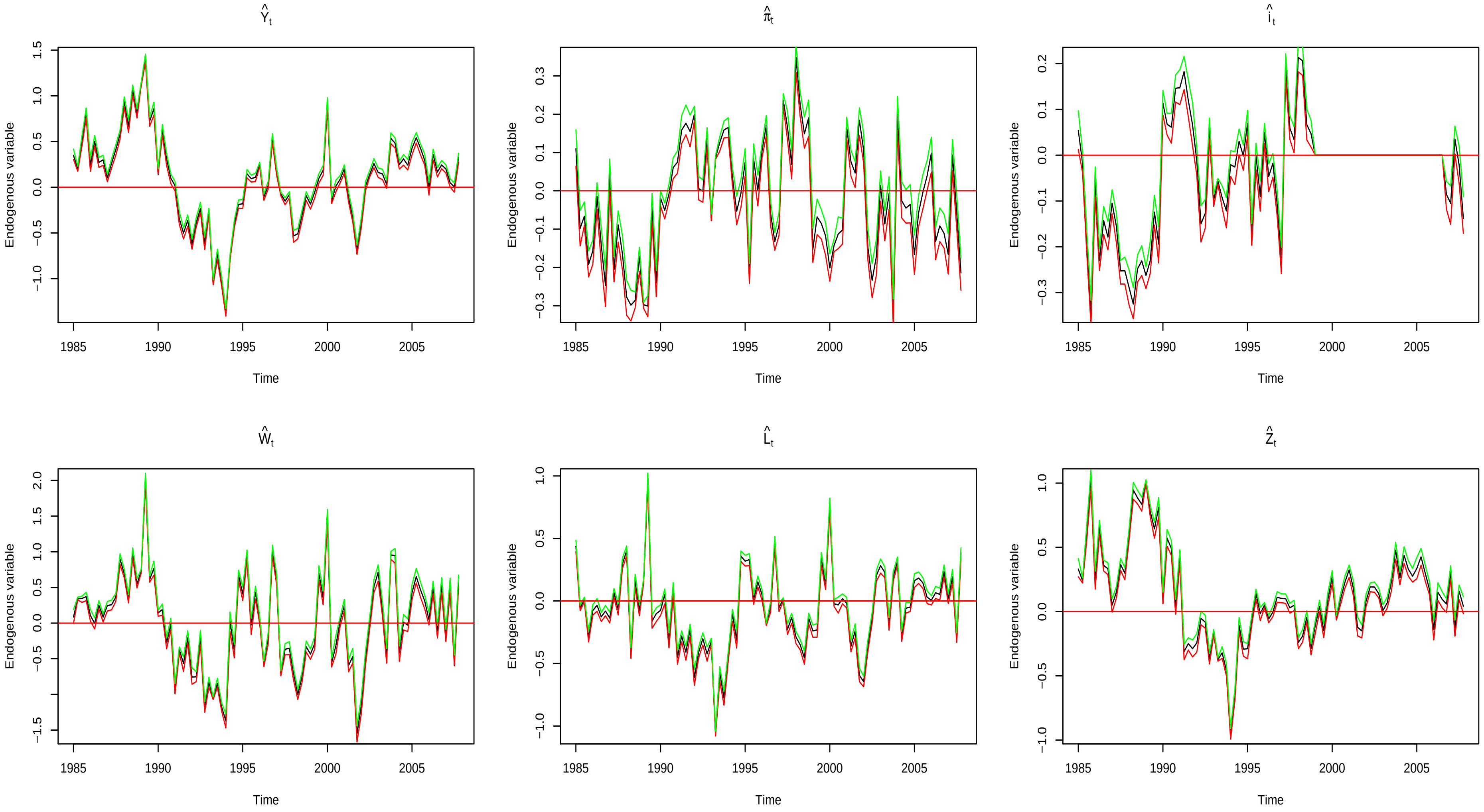

Figure 4: Endogenous variables

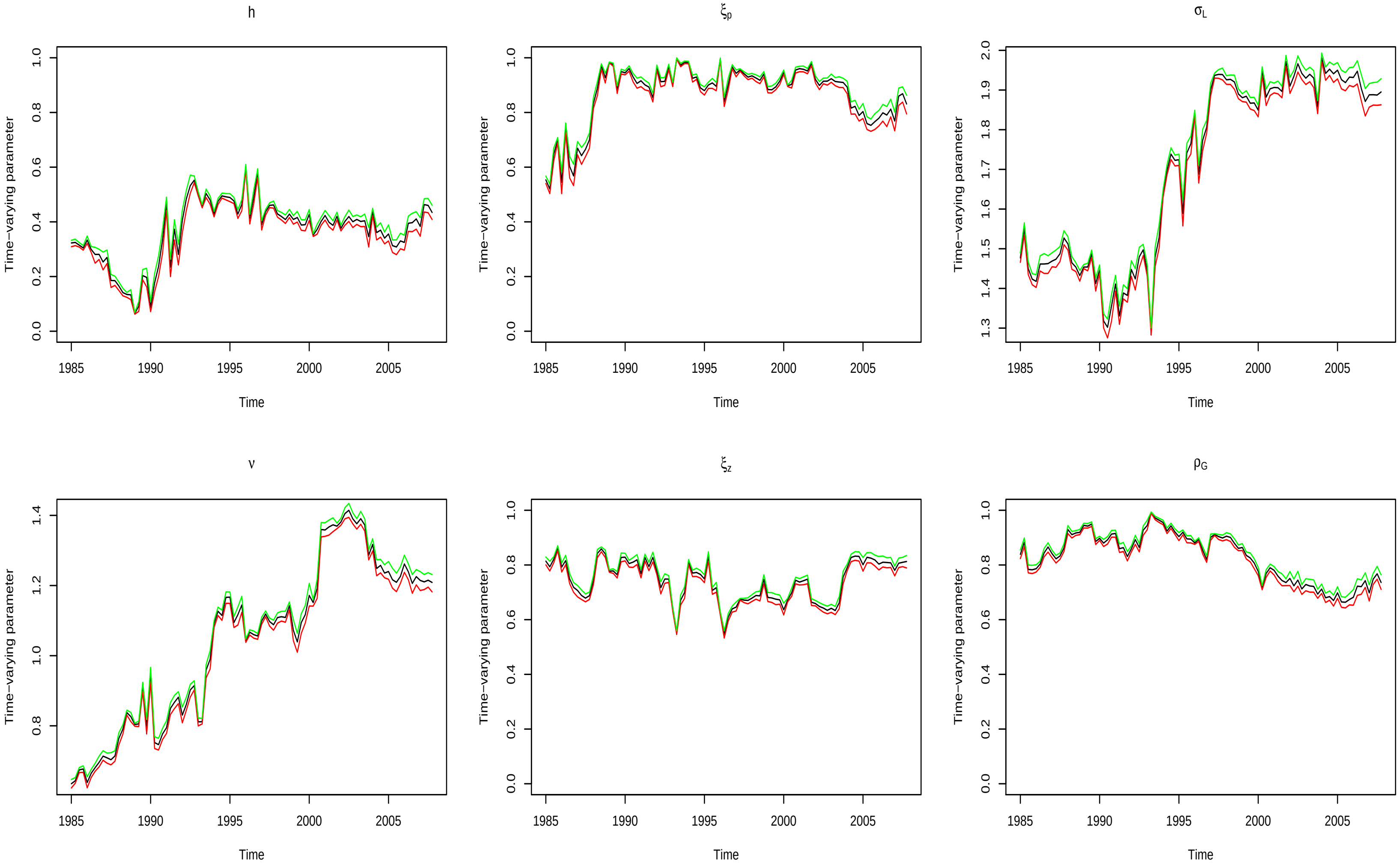

Figure 5: Time-varying parameters

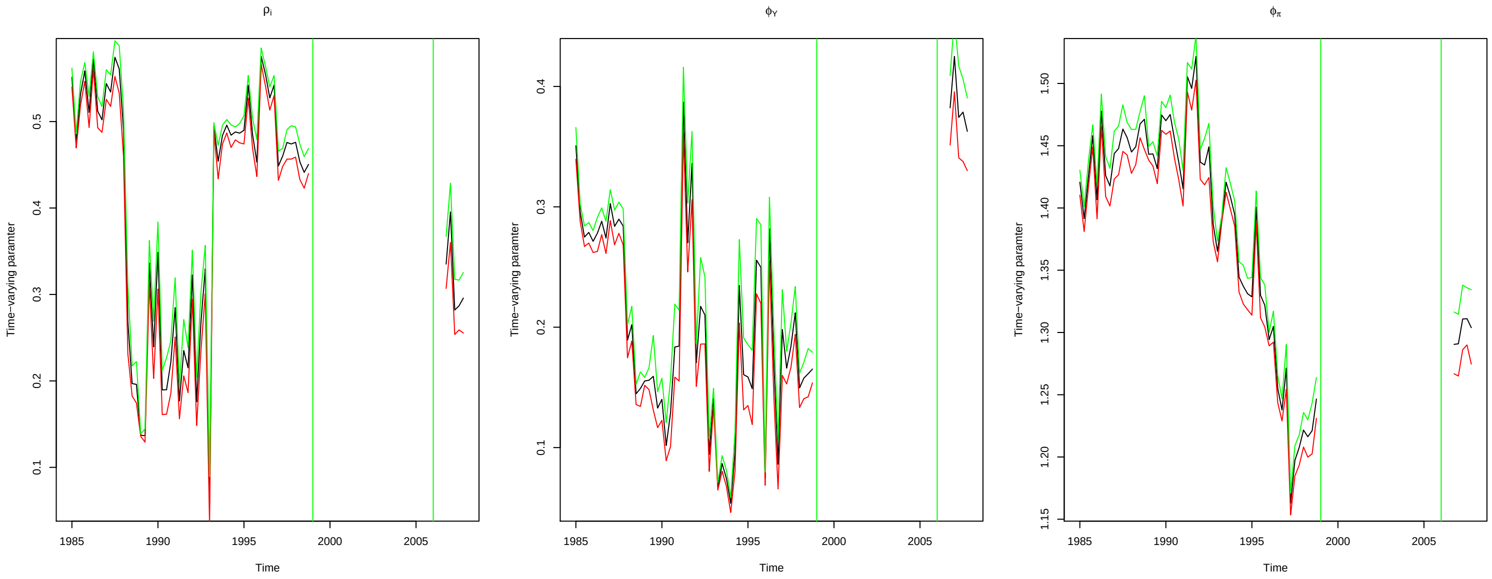

Figure 6: Time-varying parameters of the Taylor rule

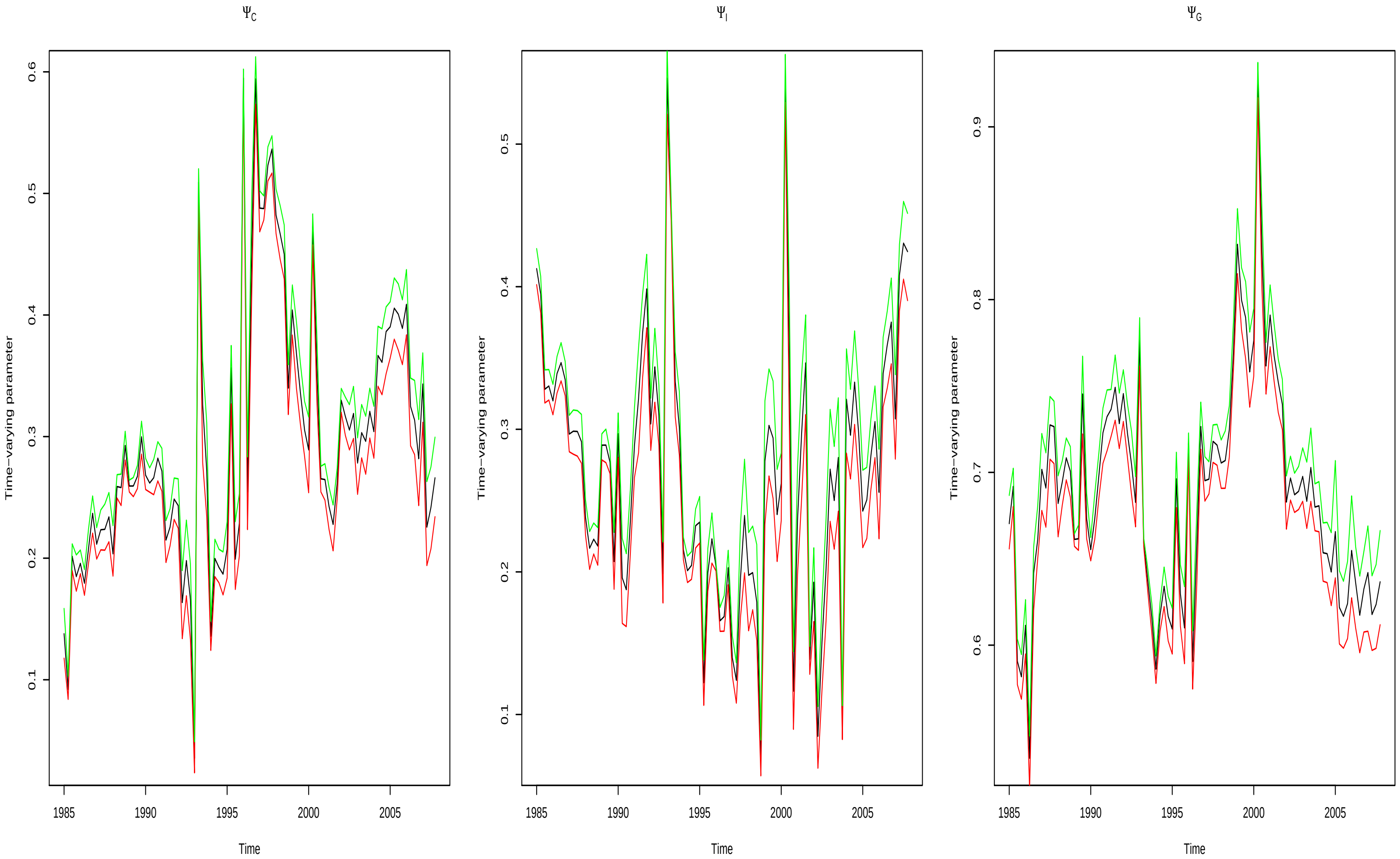

Figure 7: Time-varying parameters

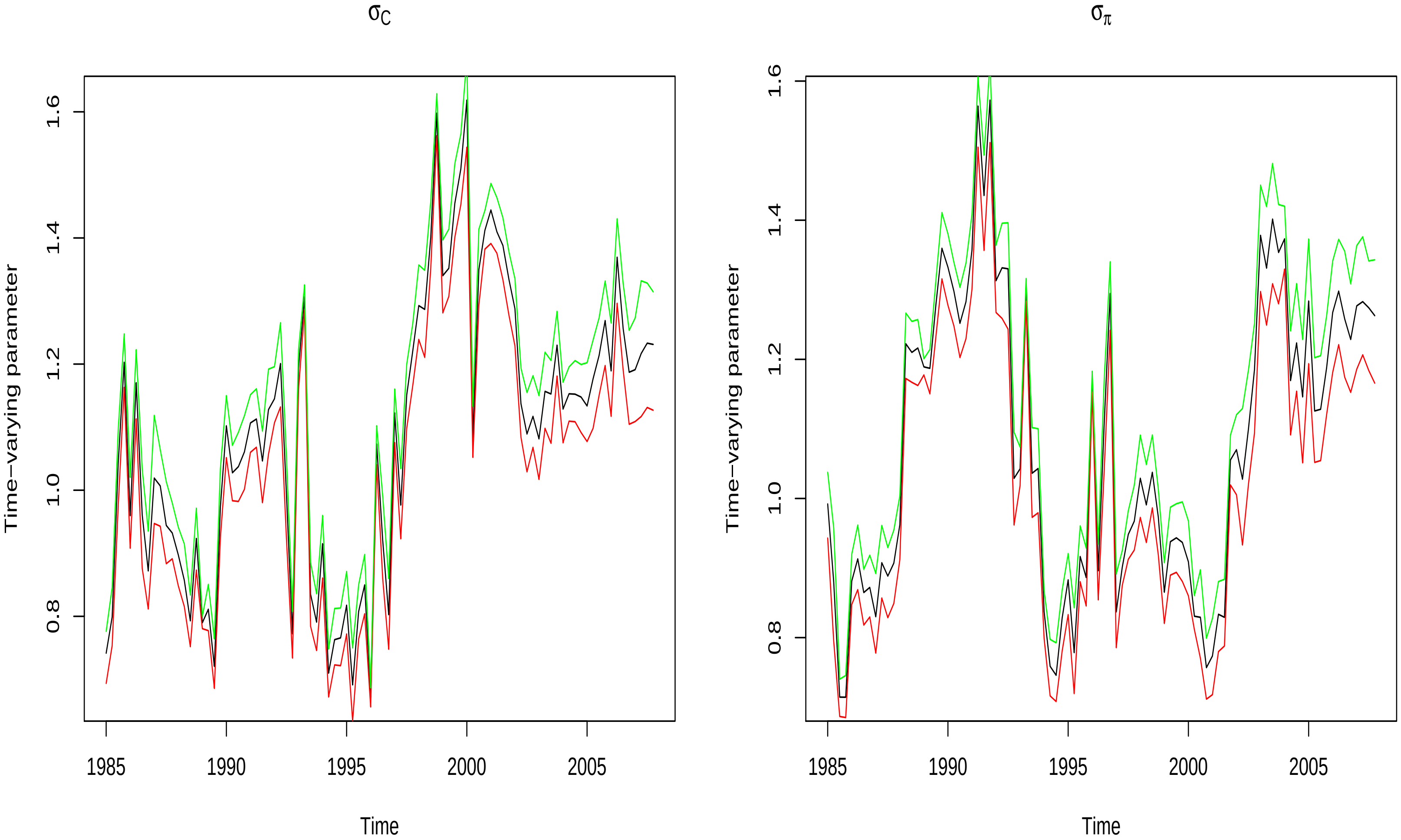

Figure 8: Time-varying parameters

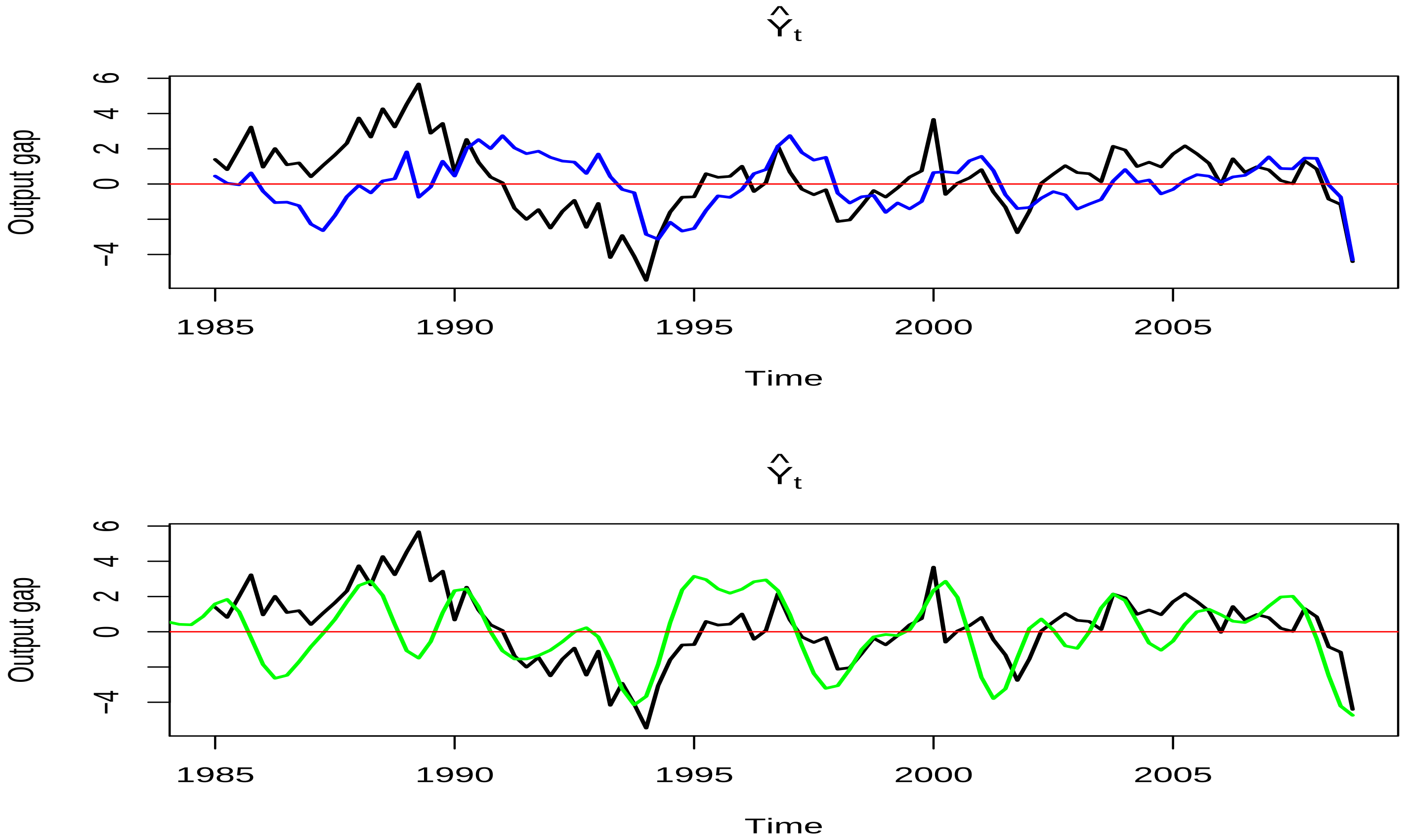

Figure 9: Output gap: Comparing filtering methods

# List of Tables

Table 1: Preliminary Parameter Estimation Based on Dynare

| | prior density | posterior mean | confidence interval | |
|---|---|---|---|---|
| h | beta | 0.3679 | 0.2694 | 0.4609 |
| $\xi_P$ | beta | 0.6785 | 0.5962 | 0.7644 |
| $\sigma_L$ | norm | 1.2608 | 0.9421 | 1.6748 |
| $\nu$ | norm | 0.7134 | 0.3194 | 1.0602 |
| $\xi_Z$ | beta | 0.9124 | 0.8734 | 0.9638 |
| $\rho_G$ | beta | 0.9752 | 0.9547 | 0.9966 |
| $\rho_i$ | norm | 0.6313 | 0.5415 | 0.7163 |
| $\phi_Y$ | norm | 0.036 | -0.0024 | 0.0777 |
| $\phi_\pi$ | norm | 1.3285 | 1.1502 | 1.4825 |

Table 2: Log-likelihood of model

| Model | Log-likelihood | $\|\xi_{s,1}\|$ | $\|\xi_{s,2}\|$ |
|---|---|---|---|
| Standard Model | -2488.100 | 0.318 | 0.068 |
| Model without inflation indexation | -2567.952 | 0.288 | 0.089 |
| Model without habit formation | -2518.378 | 0.345 | 0.098 |