ToTEM II: An Updated Version of the Bank of Canada’s Quarterly Projection Model

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Introduction

- ToTEM has served as the Bank’s main projection and policy analysis model since December 2005.

- An updated version of the model (ToTEM II) replaced ToTEM in June 2011.

- The model has been improved along a number of dimensions (model structure, parameterization)
What’s new in ToTEM II: Overview

1. Model Structure now allows for:
   a. Term premium and risk spreads
   b. Role for net wealth in household consumption
   c. Housing and Inventory Investment
   d. Simpler way to achieve strategic complementarities
   e. More procyclical labor productivity
   f. Rule of Thumb price and wage setters

2. Parameterization:
   a. Majority of parameters are estimated using ML
   b. Estimation has significantly improved in-sample goodness of fit relative to ToTEM
Outline

1. Model structure
2. Parameterization
3. Some model properties
4. Application: The impact of shocks to interest rate spreads during the crisis
Multiple Assets: Framework

- Long term assets
  - Assumptions:
    - Short (90 day) and long-term (5 year) assets are imperfect substitutes.
    - Subset of households participate only in the long-term market.
  - Implications:
    - Time varying term premium
    - Independent role for long term rates for consumption, housing and business investment.

- Risk premia
  - Exogenous and time varying.
  - On short and long-term household and firm rates.
Multiple Assets: Interest Rate Equations

- Long term risk free rate

\[
\hat{R}_{t}^{20} = \frac{1}{20} E_t \sum_{j=0}^{19} \hat{R}_{t+j}^1 + t\phi_t
\]

- Short and long term rates faced by households

\[
\hat{R}_{t}^{1,H} = \hat{R}_{t}^1 + s\phi_{stsp_t}
\]

\[
\hat{R}_{t}^{20,H} = \hat{R}_{t,20} + l\phi_{ltsp_t}
\]
Multiple Assets: Consumption Euler Equation

- For simplicity, assume:
  - Separable utility in consumption and leisure
  - No habit formation
  - No role for financial wealth

- Then, the lifetime household consumption is given by:

\[
c_t = E_t c_{t+20} - \omega \sigma \sum_{j=0}^{19} (\hat{R}_{t+j}^{1,H} - \hat{\pi}_{t+j+1}^{cpi}) - (1 - \omega) \sigma \left( 20 \cdot \hat{R}_{t}^{20,H} - \sum_{j=0}^{19} \hat{\pi}_{t+j+1}^{cpi} \right)
\]

- Term premium and risk spreads affect consumption.
Household Net Wealth Closure Condition

- Open Economy models require a condition to ensure that NFA is stationary.

- ToTEM I: Country specific risk premium depends on NFA gap

- ToTEM II: Household discount factor depends on household net wealth, which includes NFA.
  - Model consistent HH Financial Wealth = Housing Wealth + Govt Debt + Shadow value of capital + NFA
Household Net Wealth in the Euler Equation

- For simplicity, assume:
  - Separable utility in consumption and leisure
  - No habit formation
  - Just one asset
- Then, the lifetime household consumption is given by:
  \[
  \hat{c}_t = E_t \hat{c}_{t+1} - \sigma (\hat{R}_{t+j}^{1,H} - E_t \hat{\pi}^{cpi}_{t+1}) - \hat{\beta}_t
  \]
- In ToTEM II, we assume \( \hat{\beta}_t = -\psi (f_{W_t} - \bar{f}_W) \). Households become more patient when their net wealth falls below steady state.
- Note that movements in house prices affect consumption.
House price shocks and household debt

- In general, higher levels of debt may amplify the effects of house price shocks on important macroeconomic variables such as inflation, output and consumption.
- How can we take this into account in ToTEM II?
- We have explored two ways:
  - Higher levels of debt imply higher sensitivity of consumption to financial wealth (i.e. higher $\psi$)
  - Higher levels of SS debt imply higher levels of SS housing value (increase in debt is due to mortgage debt). Notice that:

\[
(fw_t - fw) = \frac{P^H_H Y^H}{Y^d} (\hat{P}^H_t + \hat{H}^H_t - \hat{Y}^d_t + ...)
\]
Residential and Inventory Investment

- ToTEM I: and QPM:
  - “Consumption” consisted of national accounts consumption + residential investment + inventory investment

- ToTEM II:
  - Housing and Inventory stocks enter HH utility function, which leads to separate demand for national accounts consumption, residential and inventory investment
  - Housing and Inventory stock accumulation
  - Residential investment and inventory investment are subject to adjustment costs.
  - Identical production technologies.
Residential and Inventory Investment: Implications

- Separate model – consistent projection for residential and inventory investment.
- Ability to analyze shocks to each.
- Allow the model to account for differences in time series properties of consumption, residential investment and inventory investment.
- Stock-flow dynamics for housing and inventories.
New Production Technology

- Production technology now allows the model to:
  - Achieve a **high degree of strategic complementarity** (or real rigidities) without assuming firm specific capital.
  - Better replicate **the procyclicality of labour productivity**.

- Strategic complementarity (i.e. real rigidities)
  - It is needed to obtain an empirically plausible degree of nominal rigidity in the model.
  - ToTEM I assumes firm specific capital. Derivation is extremely complicated and it is done only up to first order approximation.
  - ToTEM II has a role for material inputs in the production. How?
New Production Technology: Material Inputs

Intermediate Output (VA)

\[ IM_t^c = \left( \int_0^1 (M_t^c(j))^{\frac{1}{\gamma}} dj \right)^{\frac{1}{1-\gamma}} \]
The production function for gross output of the finished good is given by:

\[ y_t^c(i) = C_t(i) + M_t^C(i) = \min \left\{ \frac{Y_t^{c,va}(i) - A_t^c \omega_t FC_t^c}{1 - s_m}, \frac{IM_t^c(i)}{s_m} \right\} \]

where:

\[ IM_t^c(i) = \left( \int_0^1 (M_t^C(j))^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{-\frac{\epsilon}{\epsilon-1}} \]

\[ C_t = \left( \int_0^1 (C_t(i))^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{-\frac{\epsilon}{\epsilon-1}} \]
New Production Technology: Material Inputs

- Changes in the price of intermediate output are not fully reflected in the real marginal cost of the final good sector:

\[ \widehat{r_{mc_t}} = (1 - \mu s_m) \widehat{p_{t,va}} \]

- Note that real marginal cost is given by:

\[ r_{mc_t}(i) = s_m + p_{t,va} (1 - s_m) \]

- Relative price of material input is fixed at 1.
The introduction of fixed costs and the use of material inputs make labour productivity more procyclical due to locally increasing returns to scale. How?

Simplest case: only fixed cost and assume $\hat{Y}_{t}^{C,va} = \alpha \hat{L}_{t}, \alpha < 1$. Then, the linearized expression for gross output in terms of labour is given by:

$$\hat{Y}_{t}^{C} = \left(1 + \frac{k}{Y_{C}} \right) \alpha \hat{L}_{t} + \text{shocks}$$

As long as $\left(1 + \frac{k}{Y_{C}} \right) \alpha > 1$, we get locally increasing returns to scale.

Material inputs amplifies the locally increasing returns to scale.
New Price and Wage determination

- ToTEM I: Calvo price and wage-setting with dynamic indexation (Smets and Wouters 2003).

- ToTEM II: Calvo lottery with a fraction of rule of thumb (RoT) price and wage setters (modification of Galí and Gertler 1999).

- ToTEM II setup nests the ToTEM I setup.

- Key difference: Allowing for RoT agents means that we allow for greater discounting of future economic conditions.
Price setting framework: Assumptions

- There are two different groups of price setters: RoT (with weight \( \omega \)) and forward looking (FL, with weight \( 1-\omega \)).
- With probability \( 1-\theta \), RT firms set their price according to the following rule:

\[
p_t^b = p_{t-1} + \gamma \pi_{t-1} + (1 - \gamma) \pi_t + \Theta \hat{\mu}_t
\]

- With probability \( 1-\theta \), FL firms set their price according to the following rule:

\[
p_t^* - p_t = \beta \theta E_t \{p_{t+1}^* - p_{t+1}\} + \{1 - \beta \theta\} \left[ \hat{r}_t \hat{c}_t^0 + \hat{\mu}_t \right] + \beta \theta E_t \{\pi_{t+1} - \pi_{t+1}\}
\]
\[ \pi_t = (1 - \theta) \gamma \omega \phi^{-1} \pi_{t-1} + \beta \theta \phi^{-1} E_t \{ \pi_{t+1} \} + \lambda \tilde{r} mc_t^c. \]

\[ \phi \equiv \theta + \omega (1 - \theta) (1 + \gamma \beta \theta) \]

\[ \tilde{\lambda} \equiv (1 - \omega) (1 - \theta) (1 - \beta \theta) \phi^{-1} \]
New Price Setting Framework: Implications

- It is possible to have a high proportion of RoT agents without large weight on lagged inflation.
- The higher the proportion of RoT agents, the flatter is the NKPC.
- Overdiscounting: presence of RoT agents make expected marginal cost in the distant future to receive a smaller weight relative to near term conditions when compared to the standard Calvo/indexation setup.
- NKPC given $\omega > 0, \gamma = 0, E_t \hat{\mu}_{t+j}^n = 0 \ \forall \ j > 0$:

$$\pi_t = \pi^*_t + \frac{(1 - \omega)(1 - \theta)(1 - \beta \theta) \Theta}{\theta + \omega(1 - \theta)} \sum_{i=0}^{\infty} \left( \frac{\beta \theta}{\theta + \omega(1 - \theta)} \right)^i \widehat{rmc}_t^c + \widehat{\lambda}_t \widehat{\mu}_t^n$$
Parameterization

- Parameters in ToTEM I were chosen using informal methods, such as comparing correlations in the data with those generated by the model.
- Majority of ToTEM II parameters are estimated using ML.
- 41 observables and 41 shocks for the estimation.
- Going forward, ML estimation will save a considerable amount of time when reparameterization is needed due to model changes or major data revisions.
Estimation: some results

- Fraction of agents saving only in long term assets = 0.88
- Fraction of RoT price setters in the consumption sector = 0.48
- Degree of indexation in Consumption Sector = 0.06
- Weight on lagged exchange rate in the UIP = 0.16
- Sensitivity of discount rate to financial wealth ($\psi$) = 0.01
Estimation: Overdiscounting is quantitatively very important
Inflation is less sensitive to most shocks
Exports are less sensitive to exchange rate shocks.
Impact of net wealth changes in consumption is modest, unless it is very persistent.
Investment in residential structure is more sensitive than consumption to interest rate changes.

Reaction of residential investment and consumption to a 100 bps increase in the 5-year mortgage rate.

Percent

Quarters

Residential investment

Consumption
The impact of shocks to interest rate spreads during the crisis

**Chart 1: Business investment**
Deviation from trend

**Chart 2: Residential investment**
Deviation from trend

Source: ToTEM II simulations
Research Agenda

- Term premium and risk spreads
- Model dynamics using higher order approximations
- Welfare analysis