Monetary Policy Regime Shifts under the Zero Lower Bound: An Application of a Stochastic Rational Expectations Equilibrium to a Markov Switching DSGE Model

Hirokuni Iiboshi

1Tokyo Metropolitan University and ESRI, Cabinet Office

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   • Previous Works

2 Advantages and Summary

3 Model
   • NK Model with MS Taylor Rule under ZLB
   • Expectations Function
   • Static One-Period Problem of a MS-DSGE subject to ZLB
   • Stochastic, Rational-Expectations Equilibrium (SREE)

4 Calibration Methods
   • Procedure of Calculating Policy Functions
   • Calibration Parameters

5 Calibration Results
   • Policy and Impulse Response Functions
   • Monte Carlo Study
   • Policy Implications

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Motivation

- We can observe regime switching monetary policy in terms of two aspects
  1. zero interest rate (vs. non-zero interest rate) $\rightarrow$ depending on levels of last shocks and state variables
  2. Passive Monetary Policy (vs. Active Monetary Policy) $\rightarrow$ Markov Switching (depending only on last regime)

- $\rightarrow$ indeterminacy (multiple equilibria) $\rightarrow$ unstable economy
- $\rightarrow$ lack of a control variable

Questions

- Can we have the unique equilibrium (determinacy) of the economy including periods with both aspects?
- **To what extent impact** of both aspects on the economy?
- How to play a role of expectations for fluctuating the economy?
- What policy makes the economy better?
Previous Works

1. MS-Linear-DSGE model
   - General Theory (Unique and Stable Conditions, Solving Method)
     - Farmer, Waggoner and Zha (2009, 2011)
   - Markov Switching Monetary Policy Rule
     - Davig and Leeper (2007)
   - Empirical Studies
     - Schorfheide (2005), Davig and Doh (2008) and Liu, Waggoner and Zha (2011)

2. ZLB constraint (nonlinear Rational Expectation (RE) Model)
     - Stochastic Expectations using Numerical Methods of DP
Features and Advantages of Methods (Stochastic Rational Expectations EQ)

- Dynamic Model can be solved as **Simple Static one-period problem** using policy function and expectations functions.
- Finding out a equilibrium of stochastic rational expectations as well as perfect foresight.
- It is applicable to any model such as a non-linear DSGE model or a MS DSGE.
Contributions

- Numerical Method finding out unique equilibrium (fixed point) of a MS DSGE model subject to the ZLB
- Comparison between stochastic rational expectations and perfect foresight of the model
Summary

- Under the ZLB
  1. Small difference in dropped level of output and inflation between Active (or Aggressive) and Passive policy regimes.
  2. Non-negligible gap between Stochastic Rational Expectations and Perfect Foresight
      - Perfect Foresight make output and inflation biased upward.
  3. Intensifying uncertainty (or bigger variance of shocks) would deepen further declines of output and inflation even for the same size shocks, regardless of monetary policy regimes.
      - a policy forming expectations would play an important role of recovering an economy by mitigating uncertainty of aggregate demand shock, rather than monetary policy regime should remain aggressively.
      - the means of Output and Inflation are biased downward from their steady state.
Motivation

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New Keyesian Model

Euler equation

\[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + g_t \]  \quad (1)

NK Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t \]  \quad (2)

Aggregate Demand Shock

\[ g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \]  \quad (3)

Aggregate Supply Shock

\[ u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \]  \quad (4)

where \( y_t \) is output, \( \pi_t \) is inflation. These variables are percentage deviations from the zero inflation steady state, in the case of output, from a trend path.
Markov Switching Monetary Policy Rule

\[ i_t(S_t) \geq -r^*, \quad \text{for } S_t = 1, 2, \tag{5} \]

where \( r^* \) is the real rate of the deterministic zero inflation steady state, or \( r^* = 1/\beta - 1 \). and \( i_t = l_t - r^* \), \( (i_t \) denotes the nominal interest rate expressed as deviation from \( r^* \).

\[ i_t(S_t) = \max \{-r^*, \phi_{\pi}(S_t)\pi_t + \phi_y(S_t)y_t\}. \tag{6} \]

\[ \phi_{\pi}(S_t) = \begin{cases} \phi_{\pi 1} > 1 & \text{for } S_t = 1; \text{ Aggressive Policy Regime} \\ \phi_{\pi 2} \leq 1 & \text{for } S_t = 2; \text{ Passive Policy Regime} \end{cases} \]

\[ \phi_y(S_t) = \begin{cases} \phi_{y 1} > \phi_{y 2} & \text{for } S_t = 1; \text{ Aggressive Policy Regime} \\ \phi_{y 2} & \text{for } S_t = 2; \text{ Passive Policy Regime} \end{cases} \]

a two-state Markov chain with transition probability matrix

\[ P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix} \]

where \( p_{11} = \Pr(S_t = 1|S_{t-1} = 1) \), and \( p_{22} = \Pr(S_t = 2|S_{t-1} = 2) \).
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Solving this Model indicates Finding out Policy Functions.

- **Policy Functions:** \( x_t = x(z_t) \)

1. **Control Variables**

   \[ x(z_t) = \{ y_t, \pi_t, i_t \} \]

   where \( y_t, \pi_t \) and \( i_t \) are output, inflation, and nominal interest rate.

2. **State Variables**

   \[ z_t = \{ g_t, u_t \} \]

   where \( g_t \) and \( u_t \) are aggregate demand and supply shocks.

3. **The law of motions of state variables**

   \[ z_{t+1} = h(z_t, x(z_t), \varepsilon_{t+1}), \text{ or } z_{t+1} = h(z_t, \varepsilon_{t+1}) \]

   - If a model is linear, then \( x \) is a matrix and \( z_t \) is a vector. And using **Minimal State Variable (MSV) Solution**, we can get a policy function.
   - But, if a model is not, we use a numerical method to obtain the function.
Figure: Policy Functions under ZLB

(a) Output Gap

(b) Nominal Interest Rate
• **Expectations Functions** of standard DSGE model (Adam and Billi 2007)
  
  • **Perfect Foresight** derived from Policy Functions for \{ y_t, \pi_t, i_t \}

\[
E_t y_{t+1}(u_t, g_t) = y( E_t u_{t+1}, E_t g_{t+1}) = y(\rho_u u_t, \rho_g g_t)
\]  

(7)

In this way, Expectations Functions are function of states \( u_t \) and \( g_t \) at current period \( t \).
Discretization of Expectation Function of Perfect Foresight using Policy Function for \{ y_t, \pi_t, i_t \}

\[ y^e(g(i), u(j)) = y(u^e(i), g^e(j)), \]

\[ u^e(i) = \rho_u u(i), \]

\[ g^e(j) = \rho_g g(j). \]

for \( i, j = 1, 2, \cdots, N \)

where, \( g(i) \) and \( u(j) \) are the value of exogenous shocks at \( i \)–th and \( j \)–th interpolation nodes, respectively. \( N \) is \# of grids.
Expectations Function of a **MS-DSGE model**

- **Perfect Foresight** derived from Policy Functions for \{ \( y_t, \pi_t, i_t \) \}

\[
E_t y_{t+1}(u_t, g_t, S_t = s) = \sum_{r=1}^{L} p_{sr} y(u_{t+1}, g_{t+1}, S_{t+1} = r),
\]

for \( s, r = 1, \cdots, L \), and \( p_{sr} = \Pr( S_{t+1} = r \mid S_t = s ) \).

- **Discretization** of **Perfect Foresight**

\[
y^e(g(i), u(j), S = s) = \sum_{r=1}^{L} p_{sr} y(u^e(i), g^e(j), S^e = r)
\]

for \( i, j = 1, 2, \cdots, N \), and \( s, r = 1, \cdots, L \)

where \( u^e(i) = \rho_u u(i) \), and \( g^e(i) = \rho_g g(i) \).
- **Expectations Functions** of standard DSGE model (Adam and Billi 2007)

- **Stochastic Rational Expectations** derived from Policy Functions for \( \{ y_t, \pi_t, i_t \} \)

\[
E_t y_{t+1}(u_t, g_t) = \int_{\varepsilon_u} \int_{\varepsilon_g} y(u_{t+1}, g_{t+1}) f(d\varepsilon_{g,t+1}|\varepsilon_{u,t+1}) f(d\varepsilon_{u,t+1})
\]

\[
= \int_{\varepsilon_u} \int_{\varepsilon_g} y(\rho_u u_t + \varepsilon_{u,t+1}, \rho g g_t + \varepsilon_{g,t+1}) f(d\varepsilon_{g,t+1}|\varepsilon_{u,t+1}) f(d\varepsilon_{u,t+1}),
\]

(10)

where \( f(\cdot) \) is probability density function of the shock innovation.
**Discretization** of Expectation Function of Stochastic Rational Expectations

\[
y^e(g(i), u(j)) = \sum_{k=1}^{M} \sum_{l=1}^{M} \omega(k) \omega(l) y(u^e(k, i), g^e(l, j)), \quad (11)
\]

\[
g^e(l, j) = \rho_g g(j) + \varepsilon_g(l),
\]

\[
u^e(k, i) = \rho_u u(i) + \varepsilon_u(k),
\]

for \(i, j = 1, 2, \ldots, N\), and \(k, l = 1, 2, \ldots, M\).

where \(\omega(k)\), \(\omega(l)\) and \(\varepsilon(k)\), \(\varepsilon(l)\) are \(k\)-th and \(l\)-th weights and \(k\)-th and \(l\)-th nodes (deviations) of future shock innovations \(\varepsilon_{u,t+1}\) and \(\varepsilon_{g,t+1}\) calculated by **Gaussian-Hermite quadrature** scheme, respectively.
Expectations Function of a **MS-DSGE model**

- **Stochastic Rational Expectations** derived from Policy Functions for \{ \( y_t, \pi_t, i_t \) \}

\[
E_t y_{t+1}(u_t, g_t, S_t = s) = \sum_{r=1}^{L} p_{sr} \int_{\varepsilon_{gt+1}} \int_{\varepsilon_{ut+1}} y(u_{t+1}, g_{t+1}, S_{t+1} = r) f(d\varepsilon_{u,t+1}) f(d\varepsilon_{g,t+1}).
\]

for \( s, r = 1, \cdots, L \) and \( p_{sr} = \Pr(S_{t+1} = r | S_t = s) \).

- **Discretization of Stochastic Rational Expectations**

\[
y_e(g(i), u(j) S = s) = \sum_{r=1}^{L} p_{sr} \sum_{k=1}^{M} \sum_{l=1}^{M} \omega(k) \omega(l) y(\varepsilon_{u,t+1}, \varepsilon_{g,t+1}) f(d\varepsilon_{u,t+1}) f(d\varepsilon_{g,t+1}).
\]

\[
\omega(k) \omega(l) y(u^e(k, i), g^e(l, j), S^e = r)
\]

for \( i, j = 1, 2, \cdots, N, k, l = 1, \cdots, M, \) and \( s, r = 1, \cdots, L \)

where \( \omega(k), \omega(l) \) and \( \varepsilon(k), \varepsilon(l) \) are \( k \)-th and \( l \)-th weights and \( k \)-th and \( l \)-th nodes of future shock innovations \( \varepsilon_{u,t+1} \) and \( \varepsilon_{g,t+1} \) calculated by **Gaussian-Hermite quadrature scheme**, respectively.
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Static One-Period Problem of a standard DSGE Model subject to the ZLB, using Policy Functions and Expectations Functions (Billi, 2013)

\[ y(g(i), u(j)) = y^e(g(i), (j)) - \sigma \left( i(g(i), u(j)) - \pi^e(g(i), u(j)) \right) + g(i) \quad (14) \]

\[ \pi(g(i), u(j)) = \beta \pi^e(g(i), u(j)) + \kappa y(g(i), u(j)) + u(j) \quad (15) \]

\[ i(g(i), u(j)) = \max \left\{ -r^*, \phi_\pi \pi(g(i), u(j)) + \phi_y y(g(i), u(j)) \right\}, \quad (16) \]

for \( i, j = 1, 2, \cdots, N \)

we numerically find out a fixed-point (an equilibrium) of each grid in the discrete space, \( N \otimes N \), of policy functions; \( y(\cdot), \pi(\cdot), \) and \( i(\cdot) \), using \textit{global numerical procedure (GNP)} described in Appendix I of Billi (2011, AEJ Macro).
Static One-Period Problem of a MS-DSGE Model subject to the ZLB, using Policy Functions and Expectations Functions

\[
y(g(i), u(j), S = s) = y^e(g(i), u(j), S = s) - \sigma \left( i(g(i), u(j), S = s) - \pi^e(g(i), u(j), S = s) \right) + g(i),
\]

(17)

\[
\pi(g(i), u(j), S = s) = \beta \pi^e(g(i), u(j), S = s) + \kappa y(g(i), u(j), S = s) + u(j),
\]

(18)

\[
i(g(i), u(j), S = s) = \max \quad -r^*, \quad \phi_\pi \pi(g(i), u(j), S = s) + \phi_y y(g(i), u(j), S = s) ,
\]

(19)

for \( i, j = 1, 2, \cdots, N, \) and \( s = 1, \cdots, L \)

we numerically find out a fixed-point (an equilibrium) of each grid in the discrete space, \( N \otimes N \otimes L, \) of policy functions; \( y(\cdot), \pi(\cdot), \) and \( i(\cdot), \) using \textit{global numerical procedure} described in Appendix I of Billi (2011, AEJ Macro).
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Stochastic, Rational-Expectations Equilibrium (SREE)

- Fixed Points obtained from eq.(14) through (16) or from eq. (17) through (19) using GNP can be defined as Stochastic, Rational-Expectations Equilibrium (SREE).

**Definition**

(SREE) A *stochastic, rational-expectations equilibrium* is given by a *policy function* \( x(z_t) \) and *corresponding expectation functions* \( E_t x_{t+1}(z_t) \), respectively, which satisfy the equilibrium conditions such as the eq. (1) through (6)
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Global Numerical Procedure (Billi, 2011, AEJ Macro)

A fixed point in the space of policy functions is found with an iterative update rule

\[ \hat{y}^{k+1} = \hat{y}^{k} + \iota^{k}(\hat{y}^{*,k+1} - \hat{y}^{k}), \text{ from step } k \text{ to } k + 1 \]

- Step 1. Assign interpolation nodes and make an initial guess \( \hat{y}^{0} \).
- Step 2. Update the state, evaluate the expectations function, and apply update rule above to derive a new guess \( \hat{y}^{+1} \).
- Step 3. Stop if \( \max_{n=1,...,N} |\hat{y}^{k+1} - \hat{y}^{k}| < \tau \) where \( \tau > 0 \) is convergence tolerance. Otherwise, repeat step 2.
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## Calibration Parameters

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<th>Parameter</th>
<th>Economic Interpretation</th>
<th>Assigned Value</th>
</tr>
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<tr>
<td>( \beta )</td>
<td>quarterly discount factor</td>
<td>( 0.9913 = (1 + \frac{3.5%}{4})^{-1} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>real rate elasticity of output</td>
<td>6.25</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>slope of the Phillips curve</td>
<td>0.024</td>
</tr>
<tr>
<td>( \phi_{\pi 1} )</td>
<td>reaction coefficient of inflation under <strong>Aggressive regime</strong></td>
<td>2.2</td>
</tr>
<tr>
<td>( \phi_{y1} )</td>
<td>reaction coefficient of output under <strong>Aggressive regime</strong></td>
<td>0.5</td>
</tr>
<tr>
<td>( \phi_{\pi 2} )</td>
<td>reaction coefficient of inflation under <strong>Passive regime</strong></td>
<td>0.8</td>
</tr>
<tr>
<td>( \phi_{y2} )</td>
<td>reaction coefficient of output under <strong>Passive regime</strong></td>
<td>0.15</td>
</tr>
<tr>
<td>( \rho_u )</td>
<td>AR-coefficient Agg Supply shocks</td>
<td>0.0</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>AR-coefficient Agg Demand shocks</td>
<td>0.8</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>S.d. Agg Supply shock innovations (quarterly %)</td>
<td>0.154</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>S.d. Agg Demand shock innovations (quarterly %)</td>
<td>3.048 (=1.524*2)</td>
</tr>
<tr>
<td>( p_{11} )</td>
<td>transition probability from <strong>Aggressive to Aggressive</strong></td>
<td>0.7</td>
</tr>
<tr>
<td>( p_{22} )</td>
<td>transition probability from <strong>Passive to Passive</strong></td>
<td>0.7</td>
</tr>
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Figure: Policy Functions w.r.t. Agg Demand Shock $g_t$; non-ZLB vs. ZLB

(a) Under **Aggressive** Policy Regime

(b) Under **Passive** Policy Regime

- non-ZLB $\rightarrow$ linear, ZLB $\rightarrow$ non-linear.
**Figure:** Policy Functions w.r.t. Agg Demand Shock $g_t$; **Stochastic Expectations** vs. **Perfect Foresight**

(a) **Under Aggressive** Policy Regime

- Output
- Inflation
- Nominal Interest Rate

(b) **Under Passive** Policy Regime

- Output
- Inflation
- Nominal Interest Rate

- drop of output and inflation under stochastic rational expectation is bigger than under perfect foresight
Policy Functions w.r.t. Agg Demand Shock $g_t$: **Aggressive Regime** vs. **Passive Regime**

(a) Under **Perfect Foresight**

- **Output**
  - Aggressive Regime
  - Passive Regime

- **Inflation**
  - Aggressive Regime
  - Passive Regime

- **Nominal Interest Rate**
  - Aggressive Regime
  - Passive Regime

(b) Under **Stochastic Rational Expectations**

- **Output**
  - Aggressive Regime
  - Passive Regime

- **Inflation**
  - Aggressive Regime
  - Passive Regime

- **Nominal Interest Rate**
  - Aggressive Regime
  - Passive Regime

- **Slope** under aggressive regime is more moderate than under passive regime
- **The larger size of negative shock is**, the closer difference between both regimes.
Figure: Impulse Response Functions of Agg Demand Shock $g_t$ under Stochastic Expectations; **Aggressive** vs. **Passive**

(a) Response of **Positive** Shock

(b) Response of **Negative** Shock

- In positive shock, big difference between both regimes
- In negative shock hitting zero interest rate, similar impulse between both regimes
In positive shock, almost same impulse response
- In negative shock, size of decline of output and inflation in stochastic > in perfect
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Figure: Simulations conditional on Specified Regime (100 periods); Aggressive Regime vs. Passive Regime

(a) the Case in absence of the ZLB

- Output
- Inflation
- Nominal Interest Rate

(b) the Case in presence of the ZLB

- Output
- Inflation
- Nominal Interest Rate

Without ZLB, the effects under both regime are symmetry between positive and negative areas.

With ZLB, size of declines of output and inflation is similar between both.
Table: Simulation conditional on Specified Regime (100,000 samples)

(a) the Case in **absence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Regime</th>
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<th>Aggressive</th>
<th>Passive</th>
<th>differ. (A - P)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td><strong>Stoc. Expect.</strong></td>
<td>output</td>
<td>0.00</td>
<td>1.27</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.87</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Perfect Fore.</strong></td>
<td>output</td>
<td>0.00</td>
<td>1.29</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.89</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- under the **non-ZLB**
  - means = steady states (=0.0)
  - means under Aggressive = means under Passive
  - St.D. under Aggressive < St.D. under Passive
Figure: Simulation conditional on Specified Regime; under Non-ZLB
Table: Simulation conditional on Specified Regime (100,000 samples)

(b) the Case in **presence** of the ZLB constraint

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<tr>
<th>Regime</th>
<th>variables</th>
<th>Aggressive.</th>
<th>Passive</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>Stoc. Expect.</td>
<td>output</td>
<td>-0.93</td>
<td>3.01</td>
<td>-0.54</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.02</td>
<td>0.20</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.13</td>
<td>0.80</td>
<td>0.07</td>
</tr>
<tr>
<td>Perfect Fore.</td>
<td>output</td>
<td>-0.57</td>
<td>2.85</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.12</td>
<td>0.78</td>
<td>0.06</td>
</tr>
</tbody>
</table>

- under the **ZLB**
  - means < steady states (≡0.0)
  - means of \( y_t \) and \( \pi_t \) under Aggressive < means under Passive
  - St.D. of \( y_t \) and \( \pi_t \) under Aggressive < St.D. under Passive
Figure: Simulation conditional on Specified Regime; under ZLB
Figure: Simulations of Regime Switching Policy and Fixed Policy (100 periods); Fixed Policy = one regime fixed under aggressive policy.

(a) the Case in absence of the ZLB

- Without ZLB, the effects under both policies are symmetry between positive and negative areas.
- With ZLB, size of declines of output and inflation is similar between both
Table: Simulations of Regime Switching and Fixed Policies (100,000 samples)

(a) the Case in *absence* of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>R.S. Policy</th>
<th>Fixed Policy</th>
<th>diffe ( RS - FP )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td><strong>Stoc. Expect.</strong></td>
<td>output</td>
<td>-0.01</td>
<td>2.22</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.76</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Perfect Fore.</strong></td>
<td>output</td>
<td>-0.01</td>
<td>2.21</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.75</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: Fixed Policy denotes one regime fixed under aggressive policy.

- under the **non-ZLB**
  - means = steady states (=0.0)
  - means under R.S. policy = means under Fixed policy
  - St.D. under R.S. policy > St.D. under Fixed policy
Table: Simulations of Regime Switching and Fixed Policies (100,000 samples)

(b) the Case in **presence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>R.S. Policy</th>
<th>Fixed Policy</th>
<th>diffe (RS - FP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>Stoc. Expect.</td>
<td>output</td>
<td>-0.71</td>
<td>3.60</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.21</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.12</td>
<td>0.73</td>
<td>0.12</td>
</tr>
<tr>
<td>Perfect Fore.</td>
<td>output</td>
<td>-0.38</td>
<td>3.40</td>
<td>-0.51</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.20</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.08</td>
<td>0.70</td>
<td>0.09</td>
</tr>
</tbody>
</table>

- under the **ZLB**
  - means < steady states (=0.0)
  - means of $y_t$ and $\pi_t$ under RS policy > means under Fixed policy
  - St.D. of $y_t$ and $\pi_t$ under RS policy > St.D. under Fixed policy
1 Motivation
   - Previous Works
2 Advantages and Summary
3 Model
   - NK Model with MS Taylor Rule under ZLB
   - Expectations Function
   - Static One-Period Problem of a MS-DSGE subject to ZLB
   - Stochastic, Rational-Expectations Equilibrium (SREE)
4 Calibration Methods
   - Procedure of Calculating Policy Functions
   - Calibration Parameters
5 Calibration Results
   - Policy and Impulse Response Functions
   - Monte Carlo Study
   - Policy Implications
6 Conclusion
Under the ZLB, small difference in dopped sizes of output and inflation between Active Policy regime and Passive Policy Regime. Next, we consider what policy dose work in this situation?

- **Policy Implication**
  - The effect of 20% Reduction of St.D (or Uncertainty) of Agg Demand Shock

**Figure**: Distributions of Stochastic Rational Expectations at -2% Agg Demand Shock
Table: Effects of 20% Reduction of St.D. (or Uncertainty) of Agg Demand Shock

(a) the Case in **absence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>20% reduction</th>
<th>Original</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td><strong>RS Policy</strong></td>
<td>output</td>
<td>0.00</td>
<td>1.84</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest</td>
<td>0.00</td>
<td>0.63</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Fixed Policy</strong></td>
<td>output</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest</td>
<td>0.00</td>
<td>0.64</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- **under the non-ZLB**
  - means = steady states (=0.0)
  - just 20% down for St.D under RS policy and Fixed policy
**Table**: Effects of 20% Reduction of St.D. (or Uncertainty) of Agg Demand Shock

(b) the Case in **presence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>20% reduction</th>
<th>Original</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>RS Policy</td>
<td>output</td>
<td>-0.41</td>
<td>2.63</td>
<td>-0.71</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.18</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.08</td>
<td>0.62</td>
<td>0.12</td>
</tr>
<tr>
<td>Fixed Policy</td>
<td>output</td>
<td>-0.57</td>
<td>1.84</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.02</td>
<td>0.15</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.09</td>
<td>0.61</td>
<td>0.12</td>
</tr>
</tbody>
</table>

- under the **ZLB**
- Both of means and St.D of 20% reduction are around 2/3 of Original
Conclusion

- Under the ZLB

1. Small difference in dropped level of output and inflation between Active (or Aggressive) and Passive policy regimes.
2. Non-negligible gap between Stochastic Expectations and Perfect Foresight
   - Perfect Foresight make output and inflation biased upward.
3. Intensifying uncertainty (bigger variance of shocks) would deepen further declines of output and inflation even for the same exogenous shocks, regardless of monetary policy regimes.

- A policy forming expectations would play an important role of recovering an economy by mitigating uncertainty of aggregate demand shock, rather than monetary policy regime should remain aggressively.
- The means of Output and Inflation are biased downward from their steady state.