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Global Numerical Procedure (Billi, 2011, AEJ Macro)

- A fixed point in the space of policy functions is found with an iterative update rule

\[ \hat{y}^{k+1} = \hat{y}^k + \iota^k (\hat{y}^{*,k+1} - \hat{y}^k), \text{ from step } k \text{ to } k + 1 \]

- Step 1. Assign interpolation nodes and make an initial guess \( \hat{y}^0 \).
- Step 2. Update the state, evaluate the expectations function, and apply update rule above to derive a new guess \( \hat{y}^{+1} \).
- Step 3. Stop if \( \max_{n=1, \ldots, N} ||\hat{y}^{k+1} - \hat{y}^k|| < \tau \) where \( \tau > 0 \) is convergence tolerance. Otherwise, repeat step 2.
Motivation
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Conclusion
## Table: Calibration Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>Economic interpretation</th>
<th>assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>quarterly discount factor</td>
<td>$0.9913 = \left(1 + \frac{3.5%}{4}\right)^{-1}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>real rate elasticity of output</td>
<td>6.25</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>slope of the Phillips curve</td>
<td>0.024</td>
</tr>
<tr>
<td>$\phi_{\pi 1}$</td>
<td>reaction coefficient of inflation under Aggressive regime</td>
<td>2.2</td>
</tr>
<tr>
<td>$\phi_{y 1}$</td>
<td>reaction coefficient of output under Aggressive regime</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_{\pi 2}$</td>
<td>reaction coefficient of inflation under Passive regime</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi_{y 2}$</td>
<td>reaction coefficient of output under Passive regime</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>AR-coefficient Agg Supply shocks</td>
<td>0.0</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR-coefficient Agg Demand shocks</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>S.d. Agg Supply shock innovations (quarterly %)</td>
<td>0.154</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>S.d. Agg Demand shock innovations (quarterly %)</td>
<td>$3.048 = 1.524 \times 2$</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>transition probability from Aggressive to Aggressive</td>
<td>0.7</td>
</tr>
<tr>
<td>$p_{22}$</td>
<td>transition probability from Passive to Passive</td>
<td>0.7</td>
</tr>
</tbody>
</table>
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Figure: Policy Functions w.r.t. Agg Demand Shock $g_t$; non-ZLB vs. ZLB

(a) Under **Aggressive** Policy Regime

(b) Under **Passive** Policy Regime

- **non-ZLB** -> linear, **ZLB** -> non-linear.
Figure: Policy Functions w.r.t. Agg Demand Shock $g_t$; **Stochastic Expectations** vs. **Perfect Foresight**

(a) Under **Aggressive** Policy Regime

(b) Under **Passive** Policy Regime

- **drop of output and inflation under stochastic rational expectation is bigger than under perfect foresight**
Figure: Policy Functions w.r.t. Agg Demand Shock $g_t$: **Aggressive Regime** vs. **Passive Regime**

(a) **Under Perfect Foresight**
- Output
- Inflation
- Nominal Interest Rate

(b) **Under Stochastic Rational Expectations**
- Output
- Inflation
- Nominal Interest Rate

- slope under aggressive regime is more moderate than under passive regime
- the larger size of negative shock is, the closer difference between both regimes
Figure: Impulse Response Functions of Agg Demand Shock $g_t$ under Stochastic Expectations; **Aggressive** vs. **Passive**

(a) Response of **Positive** Shock

- Output
- Inflation
- Nominal Interest Rate

(b) Response of **Negative** Shock

- Output
- Inflation
- Nominal Interest Rate

- In positive shock, big difference between both regimes
- In negative shock hitting zero interest rate, similar impulse between both regimes
Figure: Impulse Response Functions of Agg Demand Shock $g_t$ under Aggressive Regime; **Stoc. Expec** vs. **Perfect Foresight**

(a) Response of **Positive** Shock

(b) Response of **Negative** Shock

- In positive shock, almost same impulse response
- In negative shock, size of decline of output and inflation in stochastic $>$ in perfect
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Without ZLB, the effects under both regime are symmetry between positive and negative areas.

With ZLB, size of declines of output and inflation is similar between both.
Table: Simulation conditional on Specified Regime (100,000 samples)

(a) the Case in **absence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Regime</th>
<th>variables</th>
<th>Aggressive.</th>
<th>Passive</th>
<th>diffe. ( A - P )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>Stoc. Expect.</td>
<td>output</td>
<td>0.00</td>
<td>1.27</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.87</td>
<td>0.00</td>
</tr>
<tr>
<td>Perfect Fore.</td>
<td>output</td>
<td>0.00</td>
<td>1.29</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.89</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- under the **non-ZLB**
- means = steady states (=0.0)
- means under Aggressive = means under Passive
- St.D. under Aggressive < St.D. under Passive
Figure: Simulation conditional on Specified Regime; under Non-ZLB
Table: Simulation conditional on Specified Regime (100,000 samples)

(b) the Case in presence of the ZLB constraint

<table>
<thead>
<tr>
<th>Regime</th>
<th>variables</th>
<th>Aggressive</th>
<th>Passive</th>
<th>diffe. (A - P)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td><strong>Stoc. Expect.</strong></td>
<td>output</td>
<td>-0.93</td>
<td>3.01</td>
<td>-0.54</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.02</td>
<td>0.20</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.13</td>
<td>0.80</td>
<td>0.07</td>
</tr>
<tr>
<td><strong>Perfect Fore.</strong></td>
<td>output</td>
<td>-0.57</td>
<td>2.85</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.19</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.12</td>
<td>0.78</td>
<td>0.06</td>
</tr>
</tbody>
</table>

- under the ZLB
  - means < steady states (=0.0)
  - means of $y_t$ and $\pi_t$ under Aggressive < means under Passive
  - St.D. of $y_t$ and $\pi_t$ under Aggressive < St.D. under Passive
Figure: Simulation conditional on Specified Regime; under ZLB
Figure: Simulations of Regime Switching Policy and Fixed Policy (100 periods); Fixed Policy = one regime fixed under aggressive policy.

(a) the Case in absence of the ZLB

(b) the Case in presence of the ZLB

- Without ZLB, the effects under both policies are symmetry between postive and negative areas.
- With ZLB, size of declines of output and inflation is similar between both
### Table: Simulations of Regime Switching and Fixed Policies (100,000 samples)

(a) the Case in **absence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>R.S. Policy</th>
<th>Fixed Policy</th>
<th>diffe ( RS - FP )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>Stoc. Expect.</td>
<td>output</td>
<td>-0.01</td>
<td>2.22</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.76</td>
<td>0.00</td>
</tr>
<tr>
<td>Perfect Fore.</td>
<td>output</td>
<td>-0.01</td>
<td>2.21</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.75</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Note:** Fixed Policy denotes one regime fixed under aggressive policy.

- under the **non-ZLB**
  - means = steady states (=0.0)
  - means under R.S. policy = means under Fixed policy
  - St.D. under R.S. policy > St.D. under Fixed policy
Table: Simulations of Regime Switching and Fixed Policies (100,000 samples)

(b) the Case in **presence** of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th></th>
<th>R.S. Policy</th>
<th>Fixed Policy</th>
<th>diffe ( RS - FP )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
<td>Std Dev</td>
</tr>
<tr>
<td>Stoc. Expect.</td>
<td>output</td>
<td>-0.71</td>
<td>3.60</td>
<td>-0.91</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.21</td>
<td>-0.02</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.12</td>
<td>0.73</td>
<td>0.12</td>
<td>0.74</td>
</tr>
<tr>
<td>Perfect Fore.</td>
<td>output</td>
<td>-0.38</td>
<td>3.40</td>
<td>-0.51</td>
<td>2.64</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.20</td>
<td>-0.01</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.08</td>
<td>0.70</td>
<td>0.09</td>
<td>0.69</td>
</tr>
</tbody>
</table>

- **under the ZLB**
  - means $< \text{steady states} (=0.0)$
  - means of $y_t$ and $\pi_t$ under RS policy $> \text{means under Fixed policy}$
  - St.D. of $y_t$ and $\pi_t$ under RS policy $> \text{St.D. under Fixed policy}$
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Under the ZLB, small difference in dopped sizes of output and inflation between Active Policy regime and Passive Policy Regime. Next, we consider what policy dose work in this situation?

Policy Implication

- The effect of 20% Reduction of St.D (or Uncertainty) of Agg Demand Shock

Figure: Distributions of Stochastic Rational Expectations at -2% Agg Demand Shock
Table: Effects of 20% Reduction of St.D. (or Uncertainty) of Agg Demand Shock

(a) the Case in *absence* of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>20% reduction</th>
<th>Original</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>RS Policy</td>
<td>output</td>
<td>0.00</td>
<td>1.84</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.63</td>
<td>0.00</td>
</tr>
<tr>
<td>Fixed Policy</td>
<td>output</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>0.00</td>
<td>0.12</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.00</td>
<td>0.64</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- under the **non-ZLB**
  - means = steady states (=0.0)
  - just 20 % down for St.D under RS policy and Fixed policy
Table: Effects of 20% Reduction of St.D. (or Uncertainty) of Agg Demand Shock

(b) the Case in presence of the ZLB constraint

<table>
<thead>
<tr>
<th>Policy</th>
<th>variables</th>
<th>20% reduction</th>
<th>Original</th>
<th>differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>Std Dev</td>
<td>mean</td>
</tr>
<tr>
<td>RS Policy</td>
<td>output</td>
<td>-0.41</td>
<td>2.63</td>
<td>-0.71</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.01</td>
<td>0.18</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.08</td>
<td>0.62</td>
<td>0.12</td>
</tr>
<tr>
<td>Fixed Policy</td>
<td>output</td>
<td>-0.57</td>
<td>1.84</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>inflation</td>
<td>-0.02</td>
<td>0.15</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>interest rate</td>
<td>0.09</td>
<td>0.61</td>
<td>0.12</td>
</tr>
</tbody>
</table>

- under the ZLB
- Both of means and St.D of 20% reduction are around 2/3 of Original
Conclusion

- Under the ZLB

1. Small difference in dropped level of output and inflation between Active (or Aggressive) and Passive policy regimes.
2. Non-negligible gap between Stochastic Expectations and Perfect Foresight
   - Perfect Foresight make output and inflation biased upward.
3. Intensifying uncertainty (bigger variance of shocks) would deepen further declines of output and inflation even for the same exogenous shocks, regardless of monetary policy regimes.

- a policy forming expectations would play an important role of recovering an economy by mitigating uncertainty of aggregate demand shock, rather than monetary policy regime should remain aggressively.
- the means of Output and Inflation are biased downward from their steady state.