Structure of a Multi-Sector OLG Model in an Open Economy and its Simulation Results for Japan
- a Quantitative Analysis of Japan’s Accelerating Demographic Change on the Economic Structure in the Coming Decades

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(Remark) The views expressed in this material are those of the authors and do not necessarily reflect the views of the Japan Ministry of Finance.
Motivation and background

- In Japan, a declining trend of population (Fig.A) and increasing share of the elderly (Fig.B) are apparent under the dwindling birthrate and extended longevity.

- Such a change of demographic structure can affect both the supply and the demand sides of the economy.
  - In the supply side, there can be a drop of domestic production because of labor supply shrinkage with more retired people.
  - In the demand side, there can be soaring demands for healthcare services (including social benefits in kind by the general government) for the elderly.

- Since labor force will be scarcer and demands for healthcare services for the elderly will be on increase in the future, the demographic change must have enormous impacts on the relative prices of goods and services and thus on the economic structure.
[Fig.A] Level of total population in Japan (age of 20-74)

(Note) In this model, the range of age lies in 20-74 so that we can compare our results with those of the preceding study Rausch(2009). The official projection in Japan is issued by the National Institute of Population and Social Security Research.
[Fig.B] Shift of demographic structure

- Younger generation (20–44)
- Middle generation (45–64)
- Elder generation (65–74)
Construction of a multi-sector OLG model

- We construct a multi-sector dynamic CGE model with overlapping generations (OLG) in an open economy so that we can more comprehensively analyze how Japan’s accelerating demographic change will affect the economic structure in the future.

- We introduce 4 production sector aggregates (agriculture and fishery, tradable goods, non-tradable goods, healthcare services sectors).
  - We calibrate the parameters consistent with the Japanese Social Accounting Matrix (SAM) based on the Input-Output table.

- We assume perfectly competitive markets and fully elastic prices.
  - It is justifiable in a sense that short-run frictions can be eliminated in the long run, as we see a life cycle of a household.
Issues to be analyzed in our simulation

What we are going to analyze in our simulation are the effects of Japan’s accelerating demographic change on the following issues:

① Key macroeconomic variables (per-capita variables, factor prices, etc.),
② Industrial structure (sector shares),
③ Saving rate,
④ Government spending and taxation.
The preceding studies with a multi-sector OLG model

  - He analyzes German demographic change on the economic structure, with a multi-sector OLG model.

  - They analyze fiscal consolidation in Japan, with a multi-sector OLG model.
Model description — overview

- Product differentiation employs a CET technology with a elasticity of transformation $\sigma_{HX}(i)$.
- Armington production employs a CES technology with a elasticity of substitution $\sigma_{AR}(i)$.

Output $i(Y_{it})$ is destined for either home ($H_{it}$) or export markets ($EX_{it}$). Combination of home ($H_{it}$) and imported goods ($IM_{it}$) produces Armington goods ($A_{it}$).

Armington goods ($A_{it}$) are then supplied for consumption ($C_{Ait}$), investment ($I_{it}$), government spending ($G_{it}$), and intermediate inputs for the production sectors ($\sum A_{ijt}$).
A cost-minimizing representative firm in the $i$-th sector uses capital ($K_{it}$) and labor ($L_{it}$) to produce value-added aggregate ($VA_{it}$). The firm produces intermediate aggregate ($M_{it}$) by combining intermediate inputs $j$ ($A_{jit}$) in fixed proportions.

Then, value-added aggregate ($VA_{it}$) and intermediate aggregate ($M_{it}$) are merged into gross output ($Y_{it}$).
A household maximizes her lifetime utility \( U_g \) subject to a lifetime budget constraint and a number of constraints. We rule out bequests and Ponzi-game situations.

She endogenously chooses her optimal retirement age at a time when her reservation wage exceeds her effective market wage.
Model description — the government

- The government can collect labor income taxes (including social security premiums), value-added taxes, capital income taxes and production taxes, with which it covers its spending (government consumption and investment) and net transfers to households.
  - The government sector comprises central government, local government and social security funds (i.e. general government).
  - In the baseline scenario, the rate of VAT is set to zero. In the alternative scenario, we run a simulation with the actual VAT.
  - 62% of the net transfers are distributed to the elderly over the age of 60 as pension payments, and the rest of them to a whole generation, in accordance with its share.

- Tax rate on labor incomes is endogenously determined so that the government budget constraint is satisfied.
Calibration of the model

- We employ the steady-state calibration procedure proposed by Rasmussen and Rutherford (2004, JEDC): using the Japanese Input-Output table for the base year 1985, we assume (contra-factually) that the economy is initially on a balanced growth path.
  - The benchmark real rate of return on domestic asset is set to 6.0%. Risk-free interest rate on the government bond is set to 6.0% in the initial steady state and to 3.0% in the final steady state.
  - A potential GDP growth rate is set to 1.5% (population 0.75% + productivity 0.75%) in the initial steady state and to 0.75% (population 0.0% + productivity 0.75%) in the final steady state.
Scenario of demographic shifts in Japan

- We make a scenario of the demographic shifts in Japan based on the data set of *Population Projections for Japan in 2005* (issued by the National Institute of Population and Social Security Research).
  - Until 2065, we extrapolate the growth rates of total population by utilizing the projected growth rates of new entrant cohort based on the above data set.
  - After 2065, the growth rates of new entrant cohort are all set to zero.

- Since we assume that the households begin to work at the age of 20 and have a deterministic lifetime of 55 years (excluding childhood), our demographic scenario and the official demographic projection are slightly different. However, the both trends seem to coincide very well. (Fig.A, C)
Our demographic scenario  Official projection in Japan

(Note) In this model, the range of age lies in 20-74 so that we can compare our results with those of the preceding study Rausch(2009). The official projection in Japan is issued by the National Institute of Population and Social Security Research.
[Fig.C] the growth rate of total population

Our demographic scenario

Official projection in Japan
Policy scenario for the government spending

- The ratio of the government spending of medical services to GDP is constructed so that it does reflect the increasing proportion of the elderly and much higher per-capita medical cost for them (Fig.D). Detailed calculations are explained later.
  - The ratios of the other government spending (agriculture and fishery goods, tradable goods, non-tradable goods) to GDP are largely kept fixed.
  - The ratio of the government net transfers (including pension payments) to GDP is fixed at the value in the base year.
[Fig.D] Per-capita medical costs by generations (benefits in kind by the general government) in the base year 1985
Calculations of the government spending of medical services

The amount of the government spending of medical services $Gov_{EX}("med", t)$ is calculated as follows:

$$Gov_{EX}("med", t) = \sum_{age\_group} Med\_PC(age\_group, t) \times pop(age\_group, t)$$

where $Med\_PC(age\_group, t)$ denotes the per-capita medical cost (benefits in kind by the general government) belonging to each age group $age\_group$ ($=$the younger, the middle and the elder generations$)$ in year $t$, and $pop(age\_group, t)$ the total population of each age group in year $t$.

We assume that the per-capita medical costs grow as fast as the labor-augmented productivity, which is consistent with the balanced growth path.

The per-capita medical costs are calculated by the *National Cost of Medical Care* and the *IO data*. (Fig.D)
The transitions of the government expenditures by goods and services (to GDP)

- The government spending of medical services (to GDP) expands by around 6% from 2010 to 2050, reflecting the rising proportion of the elderly and much higher per-capita medical cost for them.
Results — key aggregate variables

<table>
<thead>
<tr>
<th>variables / year</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value-added per capita (%)</td>
<td>1.90</td>
<td>-0.60</td>
<td>-3.56</td>
<td>-8.57</td>
<td>-12.13</td>
</tr>
<tr>
<td>Capital-labor ratio (%)</td>
<td>9.11</td>
<td>18.71</td>
<td>21.86</td>
<td>23.39</td>
<td>16.47</td>
</tr>
<tr>
<td>Consumption to value-added ratio (%)</td>
<td>2.30</td>
<td>6.79</td>
<td>10.08</td>
<td>15.97</td>
<td>18.71</td>
</tr>
<tr>
<td>Investment to value-added ratio (%)</td>
<td>-13.02</td>
<td>-20.02</td>
<td>-28.65</td>
<td>-45.74</td>
<td>-44.65</td>
</tr>
<tr>
<td>Wage rate (%)</td>
<td>5.84</td>
<td>14.67</td>
<td>24.95</td>
<td>36.28</td>
<td>41.15</td>
</tr>
<tr>
<td>User cost of capital (after tax) (%)</td>
<td>-5.70</td>
<td>-4.72</td>
<td>7.79</td>
<td>24.17</td>
<td>46.82</td>
</tr>
</tbody>
</table>

(note) Figures above are percentage changes with respect to the base year 1985 (%).

- **Value-added per capita is projected to decrease**, because with more retired people, labor supply as a factor of production (i.e. numerator) declines more rapidly than the total population (i.e. denominator).

- **Capital-labor ratio is projected to rise**, because the scarcity of labor (i.e. denominator) outweighs the decline of domestic capital stock (i.e. numerator). The latter is due to lower saving rate with the advent of aging society.
Results — key aggregate variables (cont.)

- **Consumption to value-added ratio is projected to rise**, because the share of older people, who are dis-savers and consume more, becomes larger.

- **Investment to value-added ratio is projected to decrease**, because the higher capital-labor ratio reduces the need for firms to invest in new physical capital.

- **Real wage rate is projected to rise**, reflecting the relative scarcity of labor.

- **User cost of capital is initially projected to decrease**, reflecting the relative abundance of capital to labor.
  - However, after 2030, it is projected to rise, because forward-looking capital prices would read the reversal of the population growth after 2050.
Results — transitions of the sector shares

<table>
<thead>
<tr>
<th>sectors / year</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture and fishery sector (%)</td>
<td>2.77</td>
<td>2.73</td>
<td>2.60</td>
<td>2.45</td>
<td>2.33</td>
</tr>
<tr>
<td>Tradable goods sector (%)</td>
<td>41.74</td>
<td>40.48</td>
<td>38.39</td>
<td>35.79</td>
<td>34.57</td>
</tr>
<tr>
<td>Non-tradable goods sector (%)</td>
<td>50.71</td>
<td>51.52</td>
<td>52.33</td>
<td>53.61</td>
<td>55.09</td>
</tr>
<tr>
<td>Medical services sector (%)</td>
<td>4.78</td>
<td>5.26</td>
<td>6.68</td>
<td>8.15</td>
<td>8.01</td>
</tr>
</tbody>
</table>

(note) Figures above are the levels of sector shares of outputs (%).

- The most pronounced is the “medical services” sector, which is projected to expand due to ageing population.
  - Meanwhile, the “tradable goods” sector is projected to contract.

- There arises an influx of labor force from the tradable goods sector to the labor-intensive medical services sector. As a result, tradable goods are substituted by imports, which will contract the share of tradable goods sector. Therefore, in the long run, the trade balance becomes negative.
Results — saving rate of households

<table>
<thead>
<tr>
<th>year</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saving rate of households (%)</td>
<td>8.79</td>
<td>3.44</td>
<td>0.02</td>
<td>-5.51</td>
<td>-6.84</td>
</tr>
</tbody>
</table>

(note) The saving rate of households is calculated in such a manner that the amount of household saving is divided by the sum of labor, asset and transfer incomes (after tax).

- A growing proportion of the elder generation lowers the saving rate of households because of dis-saving behaviors of the elderly.
Results — comparison of the macroeconomic variables under different tax systems

<table>
<thead>
<tr>
<th>variables / year</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value-added per capita (with VAT/without VAT)</td>
<td>1.06</td>
<td>1.08</td>
<td>1.09</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>Capital-labor ratio (with VAT/without VAT)</td>
<td>1.12</td>
<td>1.10</td>
<td>1.12</td>
<td>1.14</td>
<td>1.17</td>
</tr>
</tbody>
</table>

(note) Figures above are calculated in such a manner that the values of the variables with VAT are divided by those without VAT. In the scenario with VAT, the tax rates are set to 3% until 1996, 5% from 1997 to 2013, 8% from 2014 to 2015, and 10% after 2016 according to the law.

- We find that the macroeconomic variables (value-added per capita, capital-labor ratio) with VAT are larger by 10%-17%, in whichever case the tax revenue is kept the same.

- In a transition period of aging, the government can levy the VAT on the consumption of the elderly, leading to more accumulated capital by the younger generations and higher value-added per capita (Ihori (Jpub, 1987)).
  - In other word, we would say that a growth-friendly tax system of VAT stems effectively from taxation on the savings of the elderly.
Future tasks and expansions

- We calibrate the parameters by utilizing the IO data in the base year 1985, as the Japanese economy seems to be relatively stable in that year. Nonetheless, we understand that it is desirable to use more recent data.

- We can expand our model to elaborate the pension block so that we can run a simulation concerning the pension policy (for example, the earnings-related pension benefits, etc.).

- We can also expand our model to allow heterogeneous income classes within each generation so that we can conduct a research on the income redistribution policy (for example, a progressive curve of income taxation, etc.).
The related papers (advertisement!)


Under the assumption that the capital and labor are perfectly mobile across sectors, cost minimization yields the following demand functions for primary factors:

\[
K_{i,t} = VA_{i,t} \left( \frac{c_{VA, i, t}}{p_r^t} \right)^{\alpha_{VA, i}}, \quad L_{i,t} = VA_{i,t} \left( \frac{c_{VA, i, t}}{p_w^t} \right)^{\alpha_{VA, i}}
\]

\[
c_{VA, i, t} = \left[ \beta^L_i \left( p_w^t \right)^{1-\alpha_{VA, i}} + (1-\beta^L_i) \left( p_r^t \right)^{1-\alpha_{VA, i}} \right]^{1/(1-\alpha_{VA, i})}
\]

where
- \(c_{VA, i, t}\) : dual unit cost function for producing value-added aggregate (in present value),
- \(p_r^t\) : user cost of capital (in present value),
- \(p_w^t\) : wage rate (in present value),
- \(\beta_L(i)\) : benchmark value share of L in VA aggregate.
Aggregate capital stock follows the accumulative equation:

\[ K_{t+1} = (1 - \delta) K_t + I_t, \quad K_t = \sum_{i=1}^{t} K_{i,t} \]

where
- \( K_t \): aggregate capital stock at the beginning of period \( t \),
- \( I_t \): aggregate private investment,
- \( \delta \): rate of depreciation.

The aggregate capital stock at the post-terminal date \( K_{T+1} \) should be consistent with the final steady state. Thus, we determine \( K_{T+1} \) so that the growth rate of investment \( I_T \) is equal to that of the potential GDP at the terminal date. This technique is called “the state variable targeting method”, which is proposed by Lau \textit{et. al.} (2002, JEDC).

\[ \text{find } K_{T+1} \text{ such that } \frac{I_T - I_{T-1}}{I_{T-1}} = \text{gdp\_dot}_{T+1} \]

where
- \( \text{gdp\_dot}_t \) : real potential GDP growth rate.
Since the firms incur no adjustment costs with respect to investment, the arbitrage condition for a purchase price of capital is given as follows:

\[ p^k_t = p^r_{at} + (1 - \delta) p^{k}_{t+1}, \quad p^r_t = p^r_{at} + \tau \bar{r} \]

where
- \( p^k_t \): purchase price of capital (in present value),
- \( p^r_{at} \): user cost of capital (capital tax excluded, in present value),
- \( \tau \): time-invariant tax rate on capital incomes,
- \( \bar{r} \): the benchmark real rate of return on domestic asset.

We adopt the Leontief technology to produce aggregate investment \((I_t)\) by combining Armington good \(i(I_{it})\).

\[ I_{i,t} = \beta_{i,IA} I_t \]

where
- \( \beta_{i,IA} \): the benchmark share of Armington good \(i(I_{it})\) in aggregate investment \((I_t)\).
[app.] Model description — household (cont.)

\[\begin{align*}
& \text{Max} : \ u_g(z_{g,t}) = \left[ \sum_{t=g}^{g+N} \theta_{g,t} \left( z_{g,t} \right)^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}} \\
\text{s.t.} & \\
& z_{g,t} = \left[ \alpha \left( c_{g,t} \right)^{\frac{1-v}{v}} + (1-\alpha) \left( \ell_{g,t} \right)^{\frac{1-v}{v}} \right]^{\frac{v}{1-v}}, \quad \theta_{g,t} = \left[ \frac{1}{\kappa} \left( a_{t,g,t} \right)^{\frac{1-\kappa}{\kappa}} + \theta_{NF} \left( c_{g,t}^{NF} \right)^{\frac{1-\kappa}{\kappa}} \right]^{\frac{\kappa}{1-\kappa}}, \\
& c_{g,t}^{NF} = \sum_{i=NF}^{\frac{1-\sigma_{\omega}}{\sigma_{\omega}}} \left[ \sum_{t=g} p_{c,g,t} c_{g,t} \leq \sum_{t=g} \left[ p_{t}^{w} \pi_{g,t} \left( \omega_{g,t} - \ell_{g,t} \right)(1-\tau_{t}^{w}) + p_{t}^{n} \zeta_{g,t} \right] \right], \\
& \ell_{g,t} \leq \omega_{g,t}. 
\end{align*}\]

where
theta(g,t) : share parameter which accounts for discounting,
alpha, theta_F, theta_NF, phi(i,t) : value share of each consumption goods,
p_{cg,t} : price for household-specific consumption goods (in present value),
pi(g,t) : age-dependent inverted U-shaped wage profile over the life cycle,
omega(g,t) : time endowment in each period,
tau_w(t) : tax rate on labor incomes,
p_{n} : reference price (taken as numeraire, in present value),
zeta(g,t) : lump-sum government transfers.
The demand functions of a household and price for leisure (reservation wage) $p_{l,g,t}$ can be written as follows:

$$
\begin{align*}
    z_{g,t} &= u_g \left( \frac{p_{u,g}}{p_{z,g,t}} \right) ^\sigma, \\
    c_{g,t} &= z_{g,t} \left( \frac{p_{z,g,t}}{p_{c,g,t}} \right) ^\nu, \\
    \ell_{g,t} &= z_{g,t} \left( \frac{p_{z,g,t}}{p_{l,g,t}} \right) ^\nu,
\end{align*}
$$

$$
\begin{align*}
    a_{i,g,t} &= c_{g,t} \left( \frac{p_{c,g,t}}{p_{A,i,t}} \right) ^\kappa \text{ for } i = 1 \text{ (food)}, \\
    a_{i,g,t} &= c_{g,t} \left( \frac{p_{c,g,t}}{p_{c,g,t}^{NF}} \right) ^\kappa \left( \frac{p_{c,g,t}^{NF}}{p_{A,i,t}} \right) ^{\sigma_{NF}} \text{ for } i \in \text{NF (non-food)}.
\end{align*}
$$

where

$p_{u,g}$, $p_{z,g,t}$, $p_{c,g,t}^{NF}$ : dual unit cost functions for the corresponding CES productions (in present value),

$p_{A,i,t}$ : price for Armington good $i$ (in present value),

$p_{l,g,t}$ : price for leisure (in present value),

$\phi_{g,t}$ : Lagrange multiplier with respect to inequality $\ell_{g,t} \leq \omega_{g,t}$.
Complementarity slackness implies that

(1) if the constraint holds with strict inequality (i.e. $\ell_{g,t} < \omega_{g,t}$), labor supply $\text{lab}_{g,t} (\equiv \omega_{g,t} - \ell_{g,t})$ is strictly positive and the Lagrange multiplier $\phi_{a,g,t}$ becomes zero. In this case, the reservation wage $p_{l,g,t}$ coincides with the effective market wage.

(2) if the constraint holds with equality (i.e. $\ell_{g,t} = \omega_{g,t}$), labor supply $\text{lab}_{g,t} (\equiv \omega_{g,t} - \ell_{g,t})$ is zero (endogenous retirement!) and the Lagrange multiplier $\phi_{a,g,t}$ becomes strictly positive. In this case, the reservation wage $p_{l,g,t}$ exceeds the effective market wage.

As a household has an age-dependent inverted U-shaped wage profile $p_i(g,t)$, she endogenously chooses her optimal retirement age at a time when her reservation wage $p_{l,g,t}$ exceeds her effective market wage.
Using the data set of *Basic Survey on Wage Structure in 1985* (Ministry of Health, Labor and Welfare), we first estimate an age-dependent inverted U-shaped wage profile in Japan.
We then calibrate a consumption profile of the reference generation in the initial steady state, which finds a rate of time preference of 0.014. We can also see the endogenous retirement decision at the age of around 64.
After the optimal age of retirement (64 years old) of the reference generation, we can confirm that the reservation wage actually exceeds the effective market wage.
## Setting the deep parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{VA(i)}$</td>
<td>Elasticity of substitution between capital and labor (i=1) (i=2, 3, 4)</td>
<td>GTAP model</td>
<td>0.560</td>
</tr>
<tr>
<td>$\sigma_{AR(i)}$</td>
<td>Elasticity of substitution between home and imported goods (i=1) (i=2) (i=3, 4)</td>
<td>GTAP model</td>
<td>1.260</td>
</tr>
<tr>
<td>$\sigma_{HX(i)}$</td>
<td>Elasticity of transformation between home and export (for $\forall i$)</td>
<td>Rausch (2009)</td>
<td>2.000</td>
</tr>
<tr>
<td>$\beta_{L(i)}$</td>
<td>Share of labor income in the value-added (i=1) (i=2) (i=3) (i=4)</td>
<td>IO Table (1985)</td>
<td>0.170</td>
</tr>
<tr>
<td>$\sigma_{Inter-temporal}$</td>
<td>Elasticity of substitution</td>
<td>Rausch (2009)</td>
<td>0.8</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of substitution between commodity and leisure</td>
<td>Rausch (2009)</td>
<td>0.6</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Elasticity of substitution between food and non-food consumption</td>
<td>set by authors</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{NF}$</td>
<td>Elasticity of substitution between non-food consumption</td>
<td>set by authors</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Value share of commodity consumption aggregate</td>
<td>set by authors</td>
<td>0.5</td>
</tr>
<tr>
<td>$\theta_{F}$</td>
<td>Value share of food consumption (i=1)</td>
<td>IO Table (1985)</td>
<td>0.042</td>
</tr>
<tr>
<td>$\theta_{NF}$</td>
<td>Value share of non-food consumption (i=2,3,4)</td>
<td>IO Table (1985)</td>
<td>0.958</td>
</tr>
<tr>
<td>$\tau_{w}$</td>
<td>Tax rate on capital incomes (in the baseline)</td>
<td>IO Table (1985)</td>
<td>0.263</td>
</tr>
<tr>
<td>$\tau_{r}$</td>
<td>Tax rate on capital incomes (in the baseline)</td>
<td>set by authors</td>
<td>0.240</td>
</tr>
</tbody>
</table>

(Note) $i=1$ refers to agriculture and fishery goods, $i=2$ tradable goods, $i=3$ non-tradable goods and services, and $i=4$ healthcare services.